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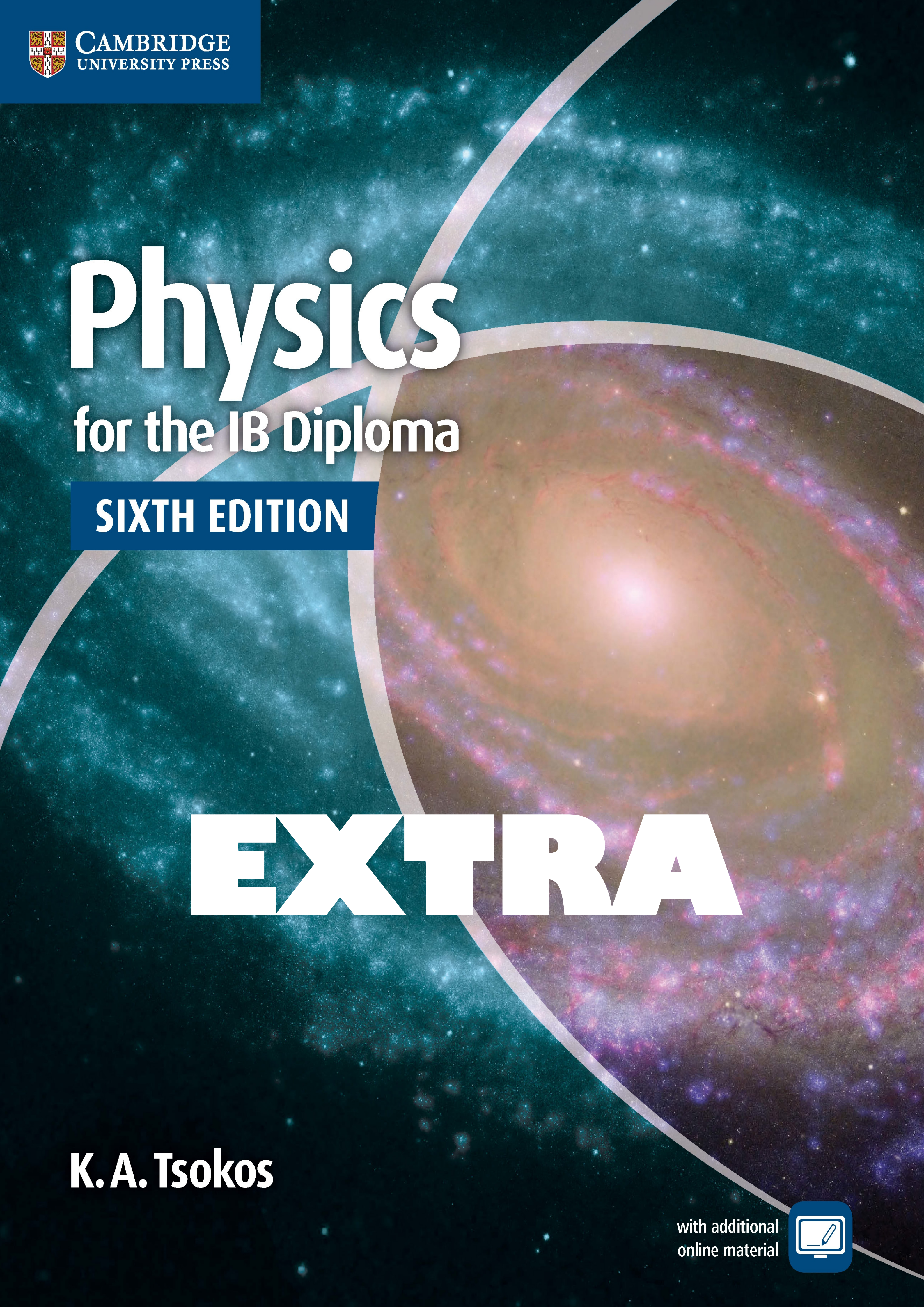


Physics

for the IB Diploma

SIXTH EDITION

EXTRA

K. A. Tsokos

with additional
online material

Self-test questions

Topic 1

1 The speed of sound in air is 340 m s^{-1}. The thunder from a lightning strike is heard 10 seconds after the lightning is seen. What is the distance to the point where lightning struck?

A 170 m
B 340 m
C 1700 m
D 3400 m

2 Which of the following is the best estimate for the angular frequency of rotation of the Earth around the Sun in radians per second?

A $\frac{2\pi}{365}$

B $\frac{2\pi}{365 \times 24}$

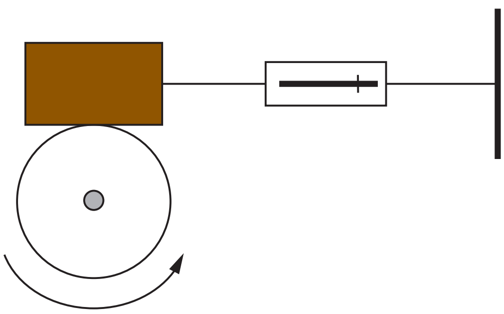
C $\frac{2\pi}{365 \times 24 \times 60}$

D $\frac{2\pi}{365 \times 24 \times 3600}$

3 The resistance force experienced by a sphere of radius r falling in a liquid with speed v is given by $F = 6\pi\eta rv$ where η is a constant. What is the unit of η?

A $\text{kg m}^{-2}\text{s}^{-2}$
B $\text{kg m}^{-1}\text{s}^{-2}$
C $\text{kg m}^{-2}\text{s}^{-1}$
D $\text{kg m}^{-1}\text{s}^{-1}$

4 A block connected by a string to a spring balance is placed on top of a rotating disc as shown in the diagram.



This apparatus may be used to measure:

A the force of static friction between the block and the disc
B the force of kinetic friction between the block and the wheel
C the normal reaction between the block and the disc
D the weight of the block

5 Two lengths, x and y, are measured as $x = (11 \pm 1)$ cm and $y = (9 \pm 1)$ cm. In which of the following calculated quantities is the percentage uncertainty the greatest?

A $x + y$

B $x - y$

C xy

D $\frac{x}{y}$

6 The volume of a sphere of radius R is given by $V = \frac{4\pi R^3}{3}$. The radius is measured with a percentage uncertainty of 3%. What is the percentage uncertainty in V?

	V
A	9%
B	12%
C	27%
D	36%

7 In an experiment it is expected that y depends on x according to $y = ax^2 + bx$ where a and b are constants. A straight line would be obtained by plotting:

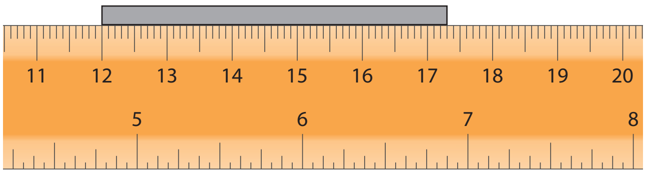
A y versus x

B y versus x^2

C y/x versus x

D y/x versus $1/x$

8 The ruler is marked in cm. What is the best estimate of the length of the grey object including it's uncertainty?



A (4.75 ± 0.05) cm

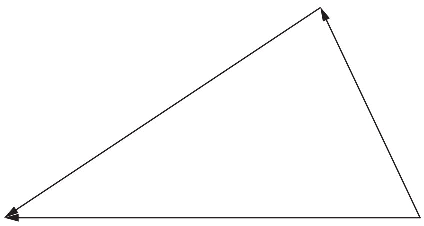
B (4.8 ± 0.1) cm

C (4.8 ± 0.10) cm

D (4.8 ± 0.05) cm

9 In which of the following is the net force zero?

A



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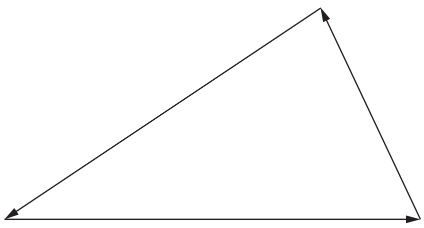

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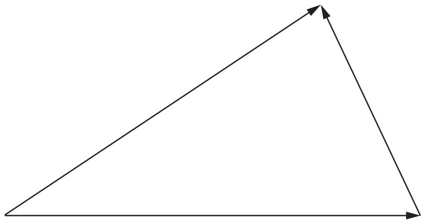
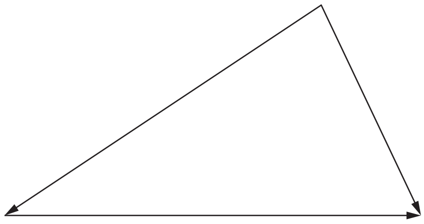
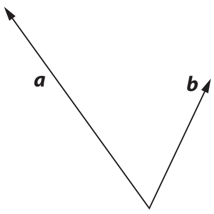


D



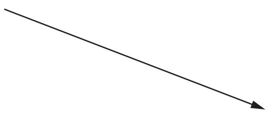
A
B
C
D

10 The diagram shows two vectors ***a*** and ***b***.



Which vector represents the difference ***a*** − ***b***?

A



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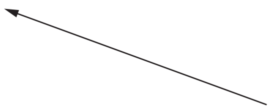

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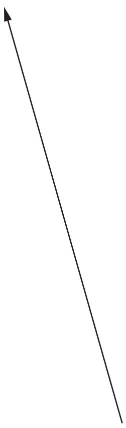

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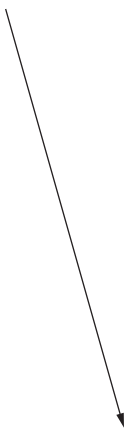
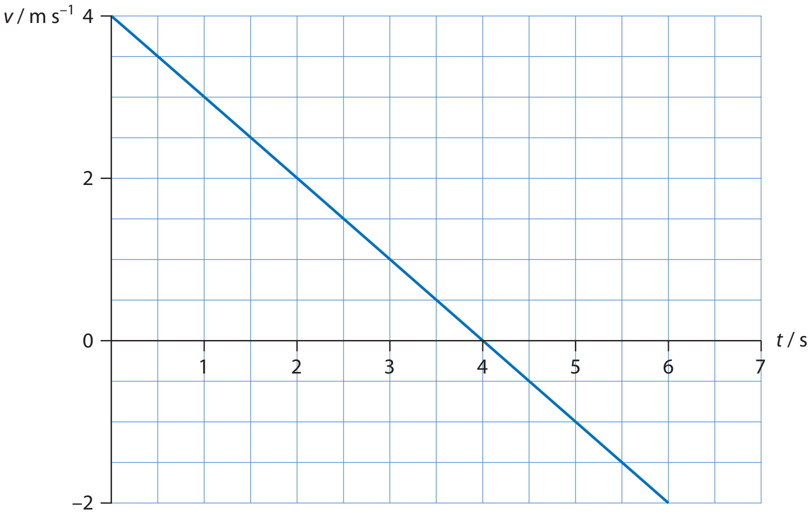


A
B
C
D

Self-test questions

Topic 2

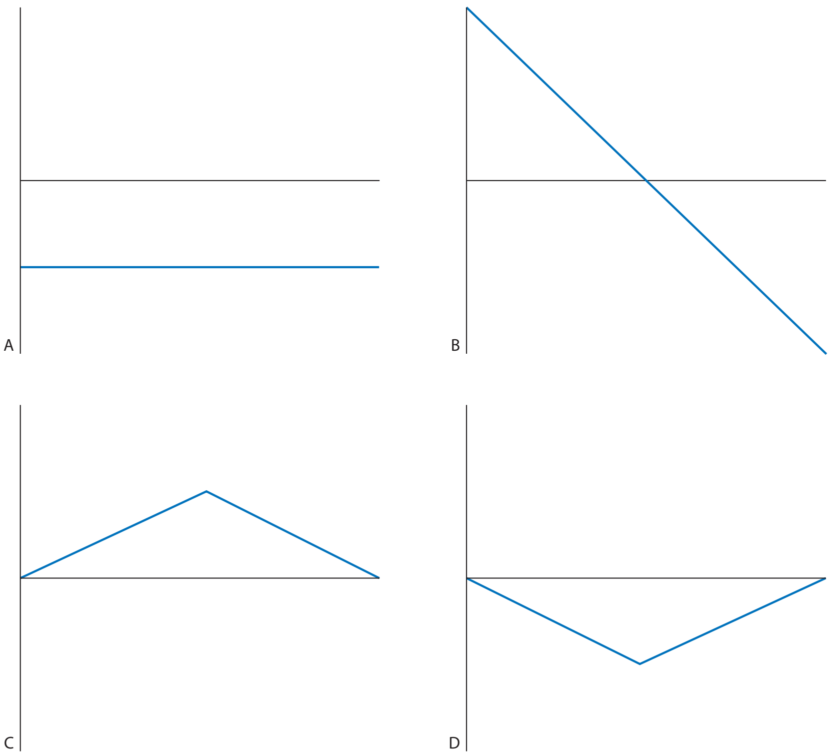
1 The graph shows the variation with time of the velocity of an object.



What is the distance travelled in 6.0 s?

A 6.0 m
B 8.0 m
C 10 m
D 12 m

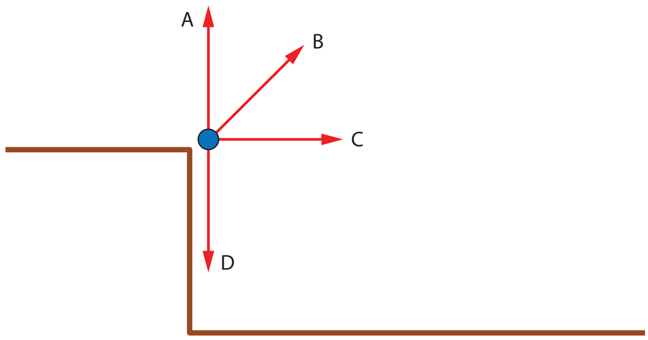
2 A ball is thrown vertically upwards. The ball returns to the ground some time later. Which graph shows the variation with time of the acceleration of the ball? (Air resistance is ignored.)



A
B
C
D

CORRECT

3 Four balls are projected from the top of a building with equal speeds but in different directions. Which ball will get to the level ground with the least vertical component of velocity?



A
B
C
D

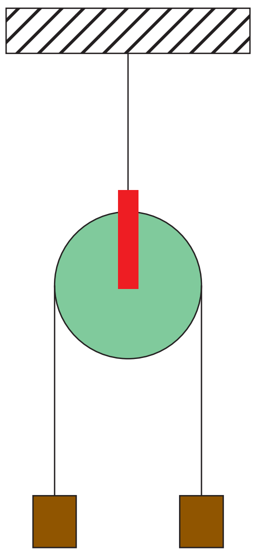
4 Three forces act on a particle. The particle is in equilibrium. Consider the statements:

I The vector sum of the forces is zero
II The sum of the magnitudes of the forces is zero
III The forces have the same magnitude

Which of the following is always correct?

A I only
B III only
C I and II only
D I, II and III

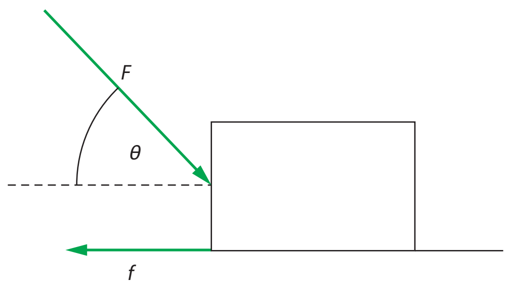
5 A pulley of negligible mass is hung from the ceiling by a string. Two blocks of weight W are attached to another string that goes around the pulley as shown in the diagram. The blocks are at rest.



What is the tension in the string connecting the pulley to the ceiling?

A 0
B $\frac{W}{2}$
C W
D $2W$

6 A force of magnitude F is applied on a block of mass M as shown in the diagram. A frictional force f acts on the block as shown. The block does not move. What is the normal reaction force from the ground on the block?



A $F \sin\theta + Mg$
B $F \cos\theta + Mg$
C $f \sin\theta$
D $f \cos\theta$

7 A block of mass 4.0 kg is placed on top of a block of mass 6.0 kg. The static coefficient of friction between the two blocks is 0.60 and the kinetic coefficient is 0.50. What is the largest force that can be applied to the 6.0 kg block so that the 4.0 kg block does not slip?

A 50 N
B 60 N
C 70 N
D 80 N

8 Two objects of mass m and $2m$ are travelling in opposite directions with the same speed v. The objects collide and stick together. What is the kinetic energy lost in the collision?

A zero
B $\frac{3mv^2}{2}$
C $\frac{5mv^2}{6}$
D $\frac{4mv^2}{3}$

9 A body of mass 2.0 kg and speed 8.0 m s^{-1} is brought to rest in 4.0 s. What is the average force responsible for stopping the body?

A 1.0 N
B 4.0 N
C 8.0 N
D 16 N

10 A net force of 8.0 N accelerates a 4.0 kg body from rest to a speed of 5.0 m s^{-1}. What is the work done by the force?

A 20 J
B 32 J
C 40 J
D 50 J

Self-test questions

Topic 3

1 How many grams of helium (^4_2He) contain the same number of atoms as 24 g of $^{12}_6C$?

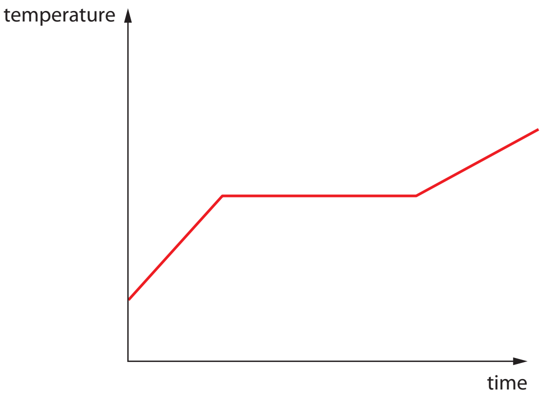
A 4
B 8
C 12
D 24

2 Forty (40) grams of water at $20\,^{\circ}C$ is mixed with eighty (80) g of water at $80\,^{\circ}C$. What is the final temperature of the mixture?

A $50\,^{\circ}C$
B $55\,^{\circ}C$
C $60\,^{\circ}C$
D $65\,^{\circ}C$

CORRECT

3 The graph shows the variation with time of the temperature of a constant mass of a solid that is being heated by a heater of known constant power.



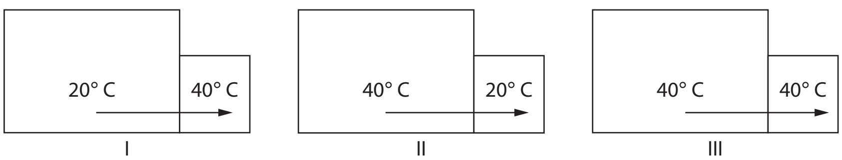
Quantities that may need to be measured in order to determine the specific latent heat of fusion of the solid include:

I the melting temperature
II the time interval needed for melting
III the mass of the solid

The list of required quantities is:

A I only
B II only
C II and III
D I, II and III

4 The diagrams show two blocks of unequal mass made of the same material. The temperatures of the blocks are indicated. In which case or cases is the flow of heat correctly indicated?



A II only
B III only
C I and II only
D II and III only

5 A quantity of helium gas ($^{4}_{2}He$) and a quantity of argon ($^{40}_{18}Ar$) are kept at the same temperature. What is an estimate of the ratio of the rms speed of helium molecules to that of argon?

A $\sqrt{10}$

B 10

C $\sqrt{9}$

D 9

6 A real gas cannot be approximated by an ideal gas under conditions of high pressure, small volume and low temperature. Suggested reasons for this include:

I The collisions of the molecules are no longer elastic

II The forces between molecules are no longer negligible

III The volume of the molecules is no longer negligible

A correct explanation is:

A I only

B II only

C II and III

D I, II and III

7 The pressure of a fixed quantity of an ideal gas is 2.2×10^{5} Pa. The temperature of the gas is increased from $30\,^{\circ}\mathrm{C}$ to $330\,^{\circ}\mathrm{C}$ at constant volume. What is an approximate value of new pressure of the gas?

A 2.4×10^{6} Pa

B 2.0×10^{4} Pa

C 4.4×10^{5} Pa

D 1.1×10^{5} Pa

8 An ideal gas is in a container with a movable piston. The piston is moved in rapidly decreasing the volume of the gas. What is the reason for the increase in the temperature of the gas?

A the molecules are closer together

B the molecules collide with each other more frequently

C the molecules collide with the container walls more frequently

D the piston transfers kinetic energy to the molecules

9 A container holds n moles of an ideal gas at kelvin temperature T. The number of moles is doubled without changing the temperature. What are the changes in the internal energy of the gas and the average kinetic energy of the molecules of the gas?

	Internal energy	Average kinetic energy
A	doubles	doubles
B	doubles	stays the same
C	stays the same	doubles
D	stays the same	stays the same

10 The temperature of an ideal gas is doubled at constant pressure. What is the change in the rms speed of the molecules and the density of the gas?

	rms speed	density
A	doubles	doubles
B	increases by $\sqrt{2}$	doubles
C	doubles	halves
D	increases by $\sqrt{2}$	halves

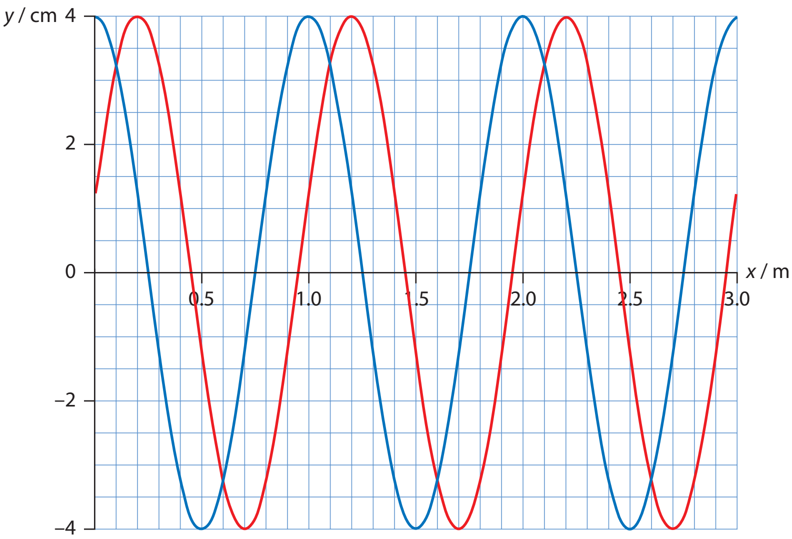
Self-test questions

Topic 4

1 A body of mass *m* is suspended at the end of a vertical spring. When the body is displaced from equilibrium by an amount *A* and then released, the body executes simple harmonic oscillations with period *T*. A second body of mass 4 m is suspended form an identical spring. What will be the period of oscillations of this body when it is displaced by an amount 2*A*?

A T
B $2T$
C $4T$
D $8T$

2 The diagram shows the displacement of a wave at time zero (in blue) and at time 0.5 s (in red).

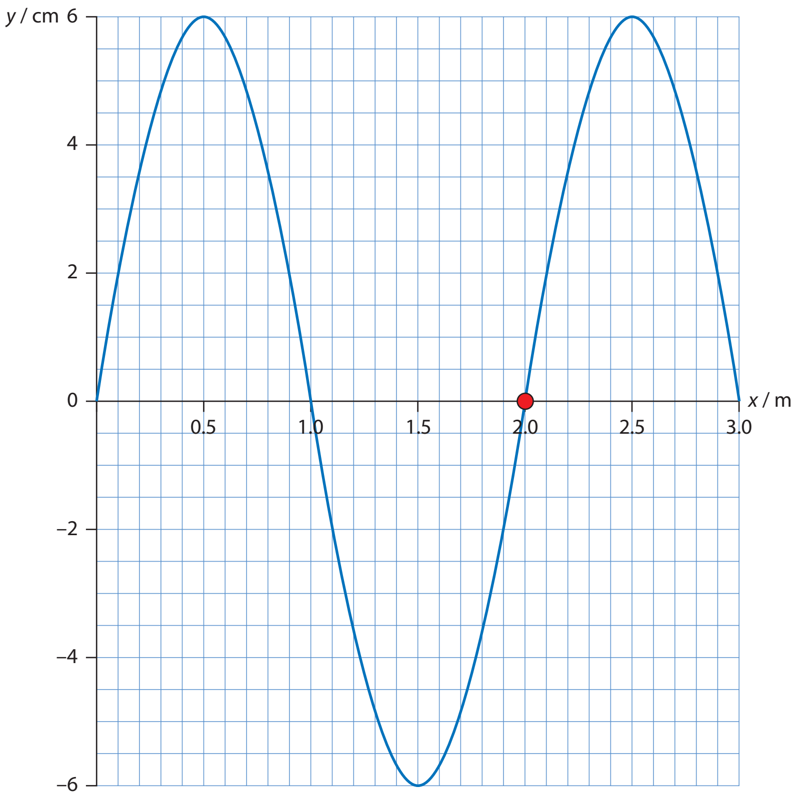


What is the speed of the wave?

A 0.08 cm s^{-1}
B 5.0 m s^{-1}
C 0.40 m s^{-1}
D 0.04 cm s^{-1}

CORRECT

3 The diagram shows the variation with distance of the displacement of a longitudinal wave travelling from right to left. Positive displacements indicate motion to the right.



A point in the medium has been marked. What is the direction of velocity of the marked point?

A Up
B Down
C Right
D Left

4 Which of the following wave phenomena cannot be observed for sound waves?

A diffraction
B refraction
C interference
D polarisation

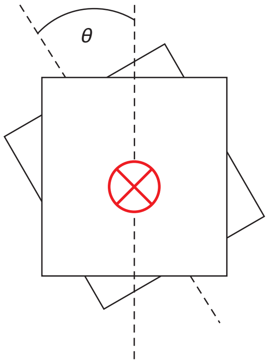
5 A ray of light has wavelength in air. The ray passes into a medium of refractive index $\frac{4}{3}$. What is the wavelength of light in the new medium?

A λ
B 4λ
C $\frac{3\lambda}{4}$
D $\frac{4\lambda}{3}$

6 A ray of light passes though the eye of a needle. Which phenomenon is the transmitted light most likely to show?

A diffraction
B polarisation
C refraction
D absorption

7 Vertically polarised light of intensity I_0 is incident on an arrangement of two parallel polarisers. The first polariser has its transmission axis vertical.



The angle between the transmission axes of the polariser is ϑ.

What is the intensity of the light transmitted through the second polariser?

A 0

B $\dfrac{I_0}{2}$

C $\dfrac{I_0}{2}\cos^2\vartheta$

D $I_0\cos^2\vartheta$

8 A tube X of length L_X has both ends open. A tube Y of length L_Y has one end closed and the other open. The frequency of the first harmonic in X is the same as the frequency of the first harmonic in Y. What is the ratio $\dfrac{L_X}{L_Y}$?

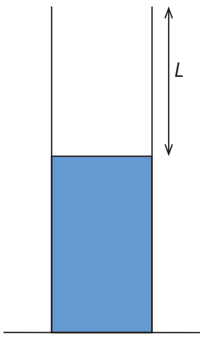
A $\dfrac{2}{3}$

B $\dfrac{3}{2}$

C 2

D $\dfrac{1}{2}$

9 A tuning fork is placed above a tube that is partially filled with water. The level of the water is slowly rising. A loud sound is heard from the tube when L = 49 cm and again when L = 35 cm.



At which value of L will a loud sound be heard again?

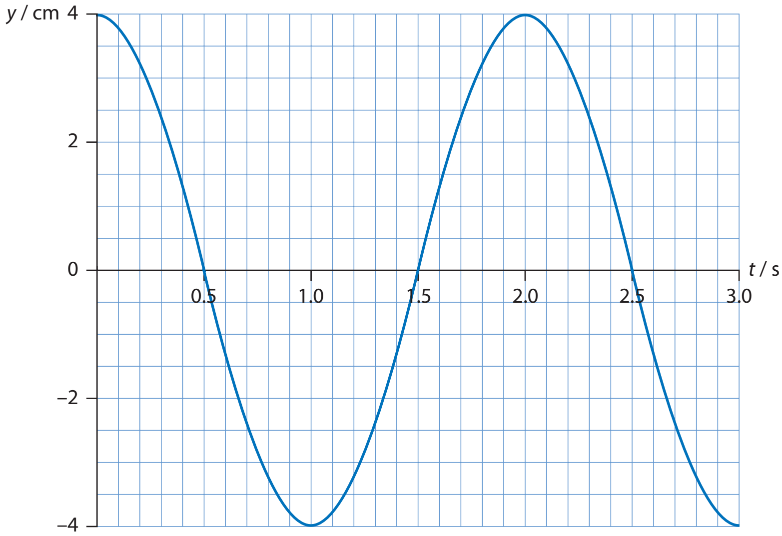
A 42 cm

B 28 cm

C 14 cm

D 7.0 cm

10 The diagram shows the displacement of a medium as a transverse wave of wavelength 5.0 m travels from left to right. What is the average speed of a point in the medium during one period of the wave and what is the speed of the wave?

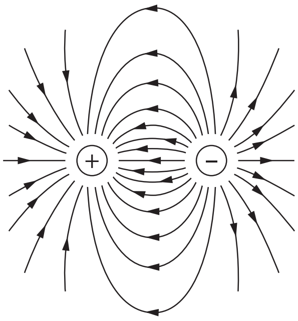


	Average speed of point in medium	Wave speed
A	4.0 cm s^{-1}	2.5 m s^{-1}
B	2.5 m s^{-1}	8.0 cm s^{-1}
C	2.5 m s^{-1}	8.0 cm s^{-1}
D	8.0 cm s^{-1}	2.5 m s^{-1}

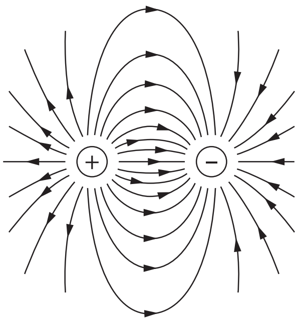
Self-test questions

Topic 5

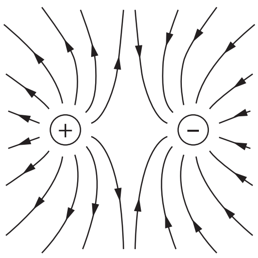
1 Which of the following diagrams correctly represents the electric field around two equal and opposite point charges?



A



B



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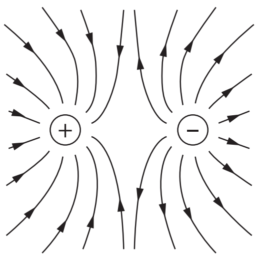


D

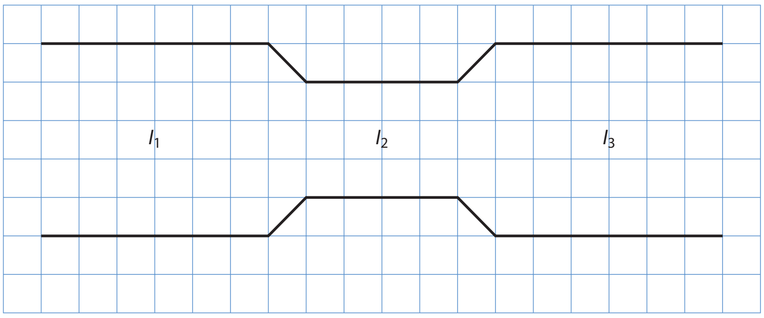
A
B
C
D

2 Two identical particles have mass m and charge q. When they are separated by a distance d the electric force between the particles has magnitude F. The mass and charge of each particle is doubled and so is the separation. What is the magnitude of the force between the particles now?

A F
B $2F$
C $4F$
D $8F$

CORRECT

3 Electric current flows in a conductor of variable cross sectional area.



The currents in each section are indicated. Which is the correct relationship between the currents?

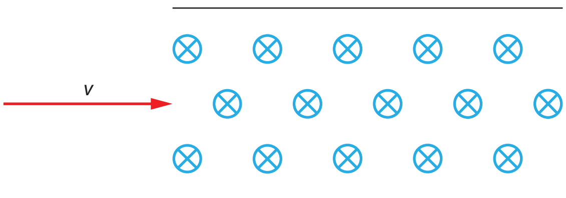
A $I_1 = I_2 = I_3$

B $I_1 = I_3 > I_2$

C $I_1 = I_3 < I_2$

D $I_1 > I_2 > I_3$

4 An electron enters the region in between two oppositely charged parallel plates with speed v. The electric field in between the plates has magnitude E. A magnetic field of magnitude B is directed into the page as shown.



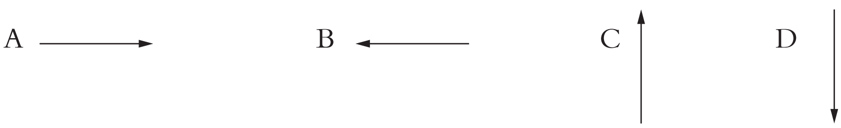
The electron is undeflected. What can be deduced about the magnitude and direction of the electric field?

	Magnitude	Direction
A	vB	Down
B	$\frac{B}{v}$	Down
C	vB	Up
D	$\frac{B}{v}$	Up

5 The diagram shows three wires carrying equal currents.

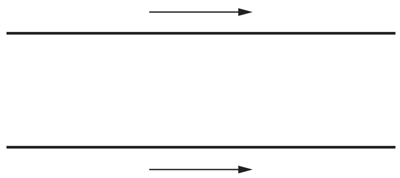
$\otimes$ $\otimes$ $\odot$

What is the direction of the force on the middle wire?



A
B
C
D

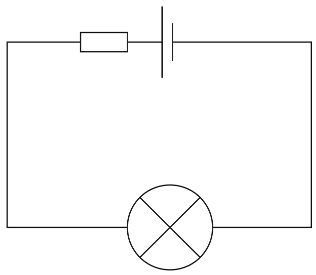
6 Two parallel wires have unequal currents in the same direction. Each wire produces a magnetic field at the position of the other wire.



Which of the following is correct about the magnetic forces on 1 m of each wire and the magnetic fields they produce?

	Forces	Magnetic fields
A	equal	equal
B	equal	unequal
C	unequal	equal
D	unequal	unequal

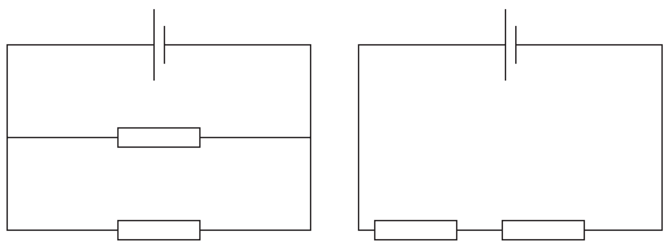
7 A cell with a non-negligible internal resistance is connected to a lamp.



The current in the circuit is 3.0 A. The power dissipated in the lamp is 12 W and that in the internal resistance is 6.0 W. What is the emf of the cell?

A 2.0 V
B 3.0 V
C 4.0 V
D 6.0 V

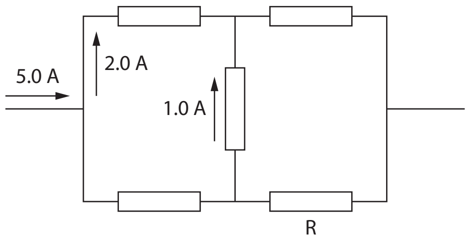
8 In the circuit below the cells have negligible internal resistances and identical emf. The resistances are identical.



What is the ratio of the total power dissipated in the left circuit to that in the right?

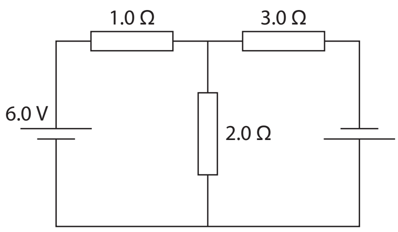
A $\frac{1}{4}$
B $\frac{1}{2}$
C 2
D 4

9 What is the current through resistor R?



A 1.0 A
B 2.0 A
C 3.0 A
D 4.0 A

10 In the circuit below the cells have negligible internal resistance. The current through the 2.0 Ω resistor is zero.



What is the emf of the cell to the right?

A 2.0 V
B 3.0 V
C 6.0 V
D 18 V

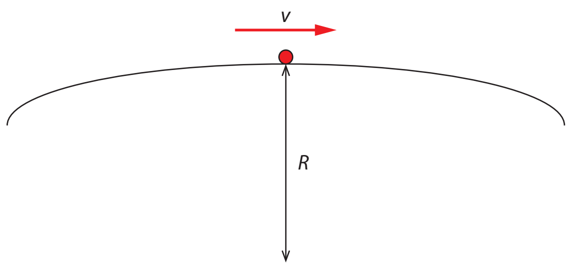
Self-test questions

Topic 6

1 Two objects X and Y are placed on a horizontal disc of radius R that rotates about a vertical axis through its centre at constant speed. Particle X is placed at a distance $\frac{R}{2}$ from the centre of the disc. Particle Y is placed at the rim of the disc. The linear speed of X is v and its centripetal acceleration is a. Which is the linear speed and centripetal acceleration of Y?

	speed	acceleration
A	v	a
B	v	$2a$
C	$2v$	a
D	$2v$	$2a$

2 A car moves along a path that is part of a vertical circle of radius R. The speed of the car at the highest point is v.



What is the maximum value of v so that the car does not lose contact with the road?

A $\frac{\sqrt{gR}}{2}$

B $\sqrt{gR}$

C $\sqrt{2gR}$

D $2\sqrt{gR}$

CORRECT

3 A particle moves along a horizontal circle with constant speed. Which of the following is correct about the magnitude and direction of the acceleration?

	Magnitude	Direction
A	constant	changing
B	constant	constant
C	changing	changing
D	changing	constant

4 A probe orbits a planet in a circular orbit of radius r. It completes one revolution in T seconds. What is the mass of the planet?

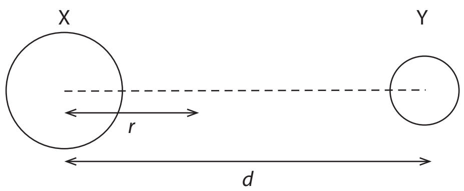
A $\frac{4\pi^2 r^3}{GT^2}$

B $\frac{4\pi^2 r}{GT^2}$

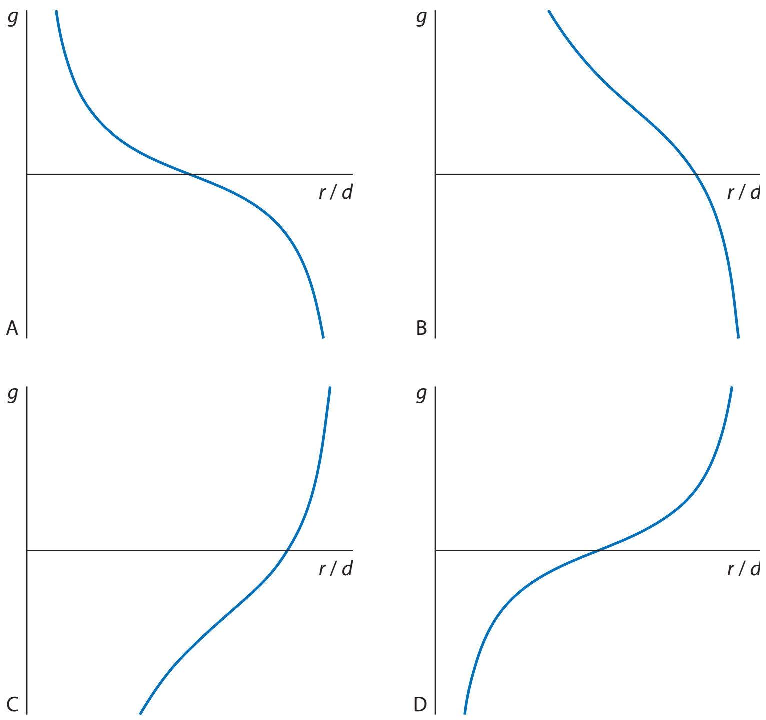
C $\frac{4G\pi^2 r^3}{T^2}$

D $\frac{4G\pi^2 r}{T^2}$

5 Two stars are separated by a distance d. The mass of star X is 4 times the mass of star Y.



The net gravitational field strength along the dotted line a distance r from the centre of star X is g. Positive g means the field is directed to the right. Which graph shows the variation of g with r/d?



A
B
C
D

6 A planet has three times the mass and three times the radius of Earth. The gravitational field strength at the surface of Earth is g. What is the gravitational field strength at the surface of the planet?

A g

B $\frac{g}{3}$

C $\frac{g}{9}$

D $\frac{g}{27}$

7 Two identical satellites, X and Y, orbit the Earth in circular orbits. The radius of the orbit of satellite Y is double that of satellite X. What is the ratio $\frac{v_X}{v_Y}$ of the orbital speeds of the two satellites?

A $\frac{1}{2}$

B 2

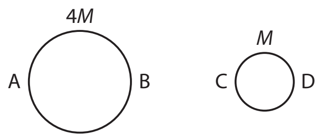
C $\frac{1}{\sqrt{2}}$

D $\sqrt{2}$

8 Which of the following is correct about a probe that orbits the Earth in a circular orbit at constant speed?

A the acceleration is constant
B the velocity is constant
C the kinetic energy is constant
D the momentum is constant

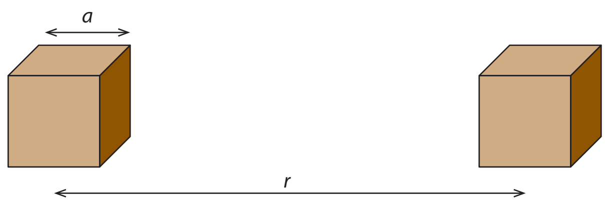
9 Two stars have masses $4M$ and M.



At which point is the magnitude of the combined gravitational field strength the least?

A
B
C
D

10 A student attempts to use the formula $F = \frac{Gm^2}{r^2}$ to calculate the gravitational force between two uniform cubes of side a and mass m.



The result will be:

A correct
B approximately correct
C approximately correct only if $a = r$
D approximately correct only if $a << r$

Self-test questions

Topic 7

1 Two elements with atomic (proton) numbers Z_1 and Z_2 and neutron numbers N_1 and N_2 are isotopes if which of the following conditions applies?

A $Z_1 = Z_2$ and $N_1 = N_2$

B $Z_1 = Z_2$ and $N_1 \neq N_2$

C $Z_1 \neq Z_2$ and $N_1 = N_2$

D $Z_1 \neq Z_2$ and $N_1 \neq N_2$

2 Which of the following is a correct about radioactive decay?

A it is random and spontaneous

B the half-lives of unstable elements depend on temperature

C unstable elements with roughly the same nucleon number have roughly the same half-lives

D activity is proportional to nucleon number

CORRECT

3 The half life of a radioactive sample is 8.0 minutes. Which of the following is a correct statement of the radioactive decay of the sample?

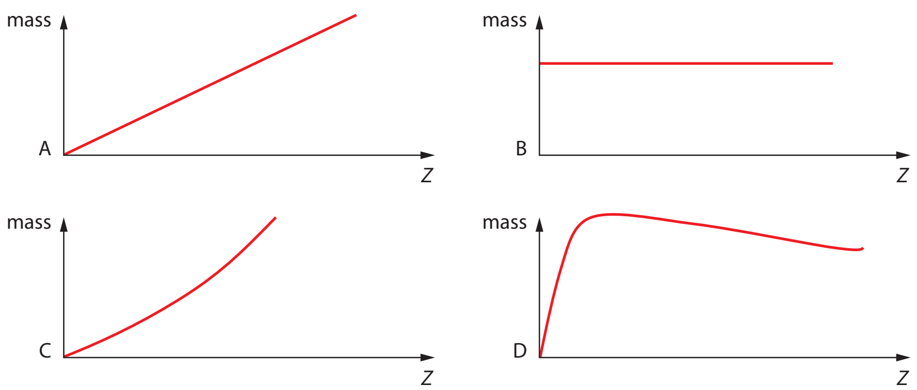
A in 16 minutes the activity will drop to zero

B in 16 minutes the mass of the sample will be reduced by a factor of 2

C a particular nucleus will decay with probability 0.50 in an interval of 8.0 minutes

D a randomly chosen nucleus will decay with probability 0.50 in an 8.0 minute interval

4 Which of the following graphs best represents the variation with atomic number of the atomic mass of stable nuclei?



A

B

C

D

5 For the decay $X \to Y + Z$ the following data on binding energies (BE) are available:

BE of X = 10 MeV

BE of Y = 7 MeV

What is the minimum binding energy of Z for the decay to be possible?

A 0 MeV

B 3 MeV

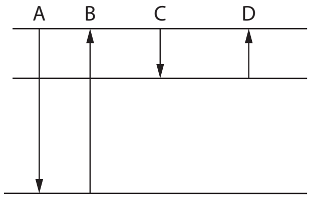
C 17 MeV

D there is no minimum value; the decay is impossible

6 What are the two missing particles in the beta decay $^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + \_ + \_$?

A a neutrino and an electron
B an antineutrino and an electron
C a neutrino and a positron
D an antineutrino and a positron

7 The energy level diagram shows four possible transitions in a hypothetical atom. In which one is a photon of the longest wavelength emitted?

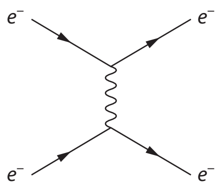


A
B
C
D

8 The reaction $e^- \rightarrow \bar{\mu} + \gamma$ does not occur because if it did it would violate the law of conservation of

A baryon number
B momentum
C charge
D energy

9 The figure shows a Feynman diagram for the reaction $e^- + e^- \rightarrow e^- + e^-$.



The wavy line could represent

A a photon
B a meson
C a baryon
D a quark

10 The strong nuclear force acts on:

A quarks
B leptons
C all charged particles
D all particles with mass

Self-test questions

Topic 8

1 Which is a correct description of primary energy sources?
A They are practically inexhaustible
B They are being consumed at a rate that is faster than that at which they can be produced
C They are sources that can be directly used to power machines
D They are sources that have not been processed in any way

2 Which is the function of the moderator in a nuclear fission reactor?
A to cool the reactor
B to prevent core meltdown
C to slow down neutrons
D to control the rate of reactions

CORRECT

3 Which is a correct comparison of active solar devices and photovoltaic panels in relation to energy conversions?

	Active solar devices	Photovoltaic panels
A	solar energy to thermal energy	solar energy to electrical energy
B	solar energy to thermal energy	solar energy to thermal energy
C	solar energy to electrical energy	solar energy to electrical energy
D	solar energy to electrical energy	solar energy to thermal energy

4 A wind generator extracts power P when the wind speed is v. What is the power extracted when the wind speed doubles?
A P
B $2\,P$
C $4\,P$
D $8\,P$

5 In an hydroelectric power station water of density ρ leaves a dam and flows through turbines an average height h below the dam at a rate of Q m^3 per second. What is an estimate of the power generated by this station?
A $\rho\, Qh$
B $\rho\, Qgh$
C $\dfrac{Qgh}{\rho}$
D $\dfrac{\rho gh}{Q}$

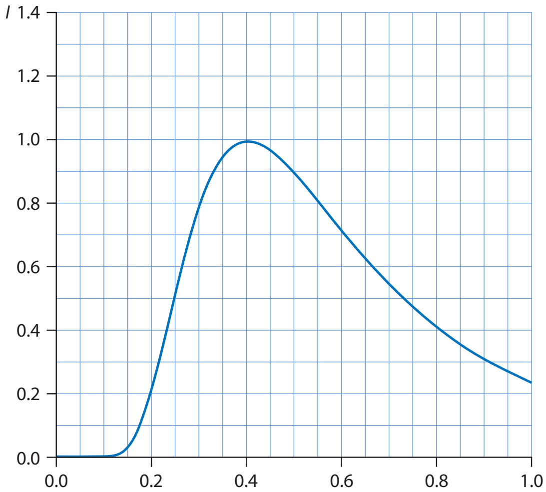
6 Solids are generally better conductors of heat than gases because they have:
A more free electrons per unit volume
B stronger bonds between their molecules
C higher densities
D higher specific heat capacities

7 A body that is a good approximation to a black body would be described as:
A a good radiator, a good absorber and a good reflector
B a good radiator, a good absorber and a poor reflector
C a poor radiator, a poor absorber and a good reflector
D a poor radiator, a poor absorber and a poor reflector

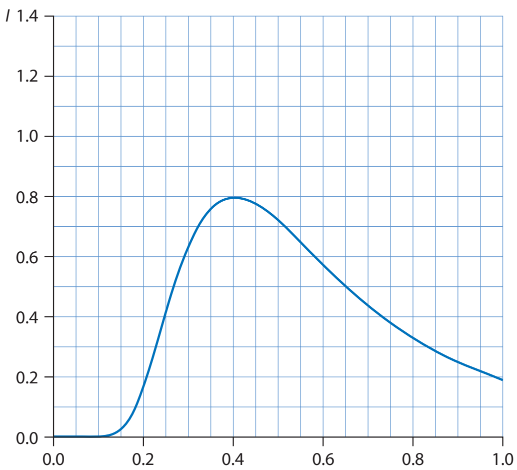
8 A black body has kelvin temperature T and surface area A. The power radiated by an area of 1 m^2 of the body is P. The temperature and area of the body are both doubled. What is the new power radiated by 1 m^2 of the body?

A $4P$
B $16P$
C $32P$
D $64P$

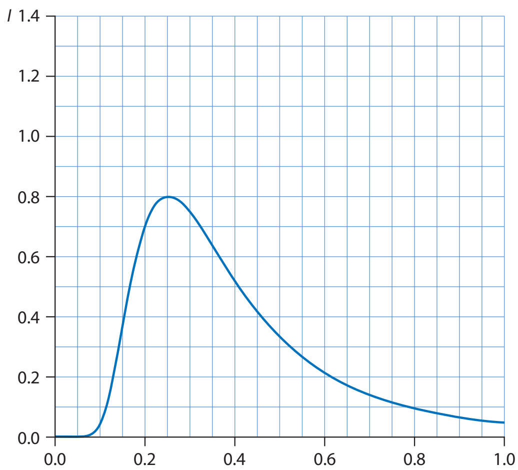
9 The graph shows the variation of intensity with wavelength of the radiation emitted by a spherical body of radius R and surface temperature T. The units on the graph are arbitrary.



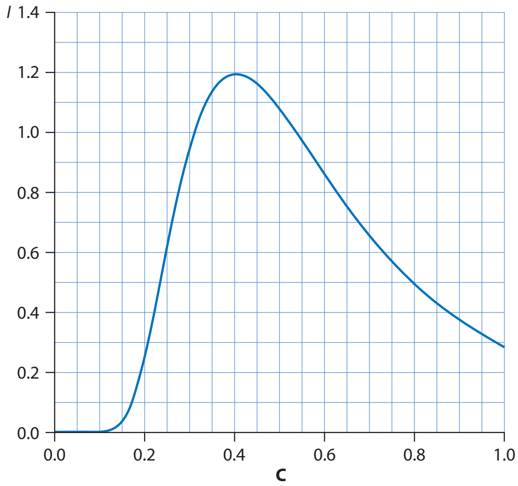
Which of the following graphs shows the variation of intensity with wavelength for another spherical body of the same radius and temperature but of higher emissivity?

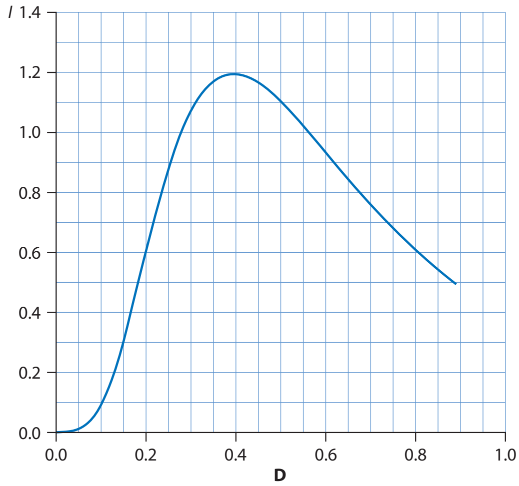


A



B





A
B
C
D

10 The greenhouse effect would be described as:

A infrared radiation emitted by the Earth is absorbed by greenhouse gases in the atmosphere and is then re-emitted in all directions including the Earth

B ultraviolet radiation emitted by the Earth is absorbed by greenhouse gases in the atmosphere and is then re-emitted in all directions including the Earth

C infrared radiation emitted by the Sun is absorbed by greenhouse gases in the atmosphere and is then re-emitted in all directions including the Earth

D ultraviolet radiation emitted by the Sun is absorbed by greenhouse gases in the atmosphere and is then re-emitted in all directions including the Earth

Self-test questions

Topic 9

1 A block of mass m is suspended from a vertical spring of spring constant k. At equilibrium the spring is extended by a distance x_0. The block is pulled down a further distance y_0 from the equilibrium position and is then released. What is the maximum speed of the block during the oscillations that take place?

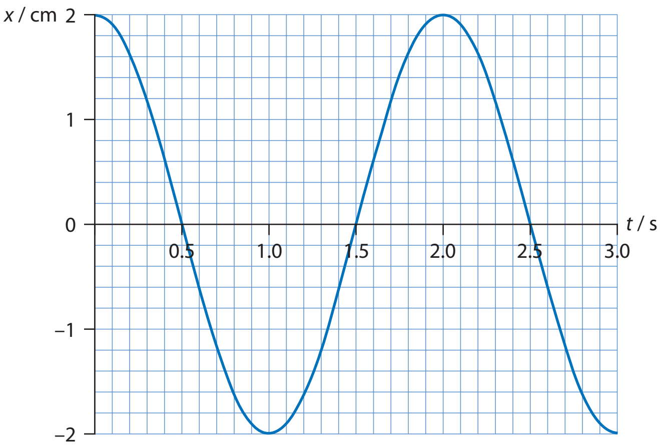
A $y_0\sqrt{\dfrac{k}{m}}$

B $y_0\sqrt{\dfrac{m}{k}}$

C $(x_0 + y_0)\sqrt{\dfrac{k}{m}}$

D $(x_0 + y_0)\sqrt{\dfrac{m}{k}}$

2 The graph shows the variation with time of the displacement of a particle executing simple harmonic oscillations.



What is the velocity, in m s^{-1}, of the particle at time t = 0.50 s?

A 6.28

B –6.28

C 6.28×10^{-2}

D -6.28×10^{-2}

CORRECT

3 In simple harmonic motion what is the phase difference between the graphs of displacement versus time and acceleration versus time?

A 0

B $\dfrac{\pi}{4}$

C $\dfrac{\pi}{2}$

D π

4 Light is incident normally on a thin film of soap water of thickness d that is suspended in air. The wavelength of light in air is λ. The refractive index of soap water is n. The light reflected from the film suffers destructive interference. Which of the following gives a possible thickness of the film?

A $\dfrac{\lambda}{4}$

B $\dfrac{\lambda}{2}$

C $\dfrac{\lambda}{4n}$

D $\dfrac{\lambda}{2n}$

5 In a diffraction experiment with a single slit the intensity of the central maximum of the diffraction pattern is I and the angle at which the first minimum is observed is θ. What is the effect on I and θ of reducing the slit width? The light source stays the same.

	I	θ
A	increases	increases
B	increases	decreases
C	decreases	increases
D	decreases	decreases

6 In a Young type two slit interference experiment the third maximum away from the central maximum of the two slit interference pattern coincides with the first minimum of the single slit diffraction pattern. What is the relation between the slit separation d and the slit width b?

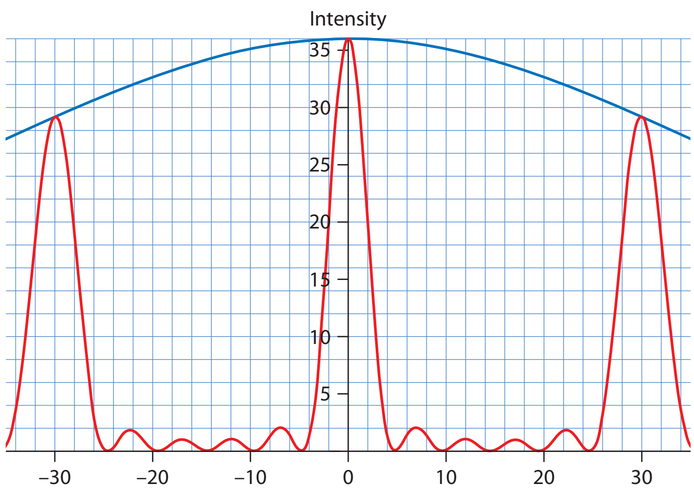
A $d = \dfrac{b}{3}$

B $d = 3b$

C $d = \dfrac{b}{6}$

D $d = 6b$

7 The graph shows the intensity pattern from interference by N slits (red curve) modulated by the single slit diffraction pattern (blue curve).



What is the value of N and what is the relationship between the slit separation d and the wavelength λ?

	N	Relationship
A	4	$d = 2\lambda$
B	4	$d = \frac{\lambda}{2}$
C	6	$d = 2\lambda$
D	6	$d = \frac{\lambda}{2}$

8 Two stars are observed by a number of radio telescopes of different dish diameters. The stars emit a range of wavelengths. The images of the stars by a particular telescope are not well resolved. Which of the changes below would be most likely to result in resolution of the star images?

	Wavelength	Dish diameter
A	Observe stars at shorter wavelengths	Use larger dish telescope
B	Observe stars at shorter wavelengths	Use smaller dish telescope
C	Observe stars at longer wavelengths	Use larger dish telescope
D	Observe stars at longer wavelengths	Use smaller dish telescope

9 A diffraction grating has 500 lines per mm and is 2.0 cm wide. Light of average wavelength 600 nm is incident on the diffraction grating. What is the least difference in wavelength that can be resolved b this diffraction grating in the third order?

A 0.010 nm
B 0.020 nm
C 0.030 nm
D 0.040 nm

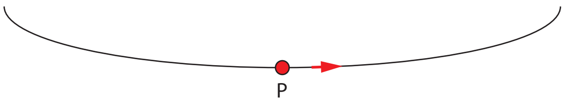
10 A jet plane is moving away from a stationary observer at a speed that is half the speed of sound in still air. The jet emits sound of frequency f. What is the frequency received by the observer?

A $2f$
B $\frac{f}{2}$
C $\frac{2f}{3}$
D $\frac{3f}{2}$

Self-test questions

Topic 10

1 The diagram shows a gravitational field line. A point mass is placed at point P.



Which arrow represents the gravitational force on the particle?

A
B
C
D

2 The escape speed form the surface of planet X is v. Planet Y has the same density as planet X and has double the radius. What is the escape speed from the surface of planet Y?

A $\frac{v}{\sqrt{2}}$

B $v\sqrt{2}$

C $2v$

D $4v$

CORRECT

3 A mass of 2.0 kg is moved at a very small constant speed from a point where the gravitational potential is -5.0×10^{6} J kg^{-1} to a point where the potential is -2.0×10^{6} J kg^{-1}. What is the work done by the gravitational force acting on the mass?

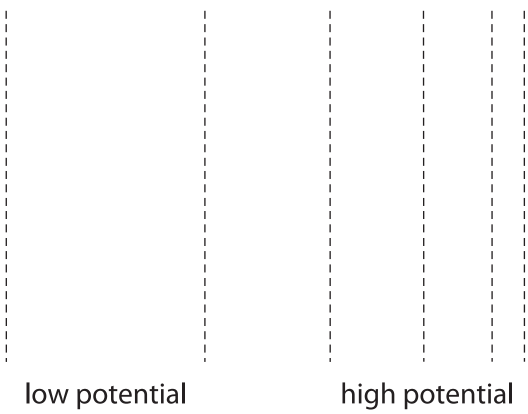
A -6.0×10^{6} J

B 6.0×10^{6} J

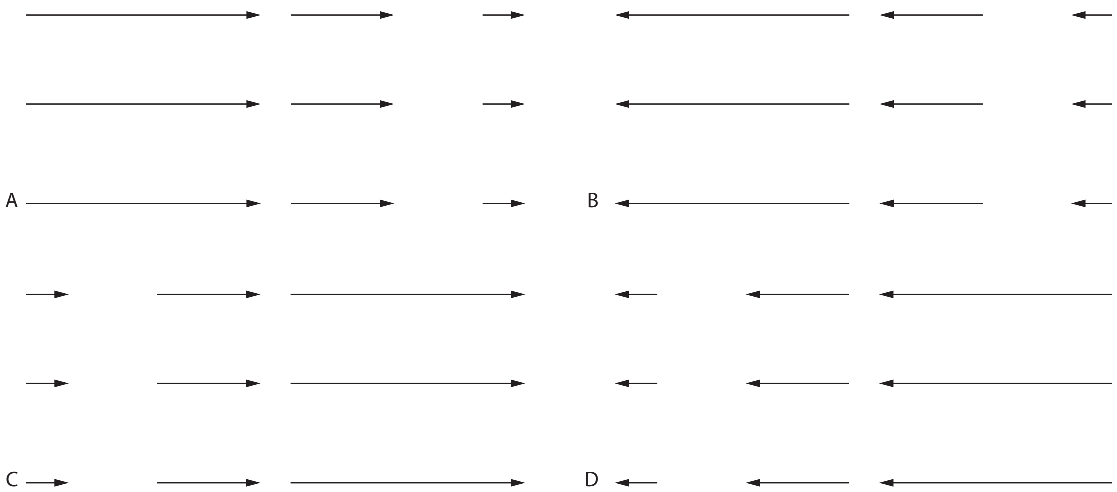
C -1.5×10^{6} J

D 1.5×10^{6} J

4 The diagram shows a series of electric equipotential lines. The difference in potential between consecutive lines is the same. The potential increases as we move to the right.

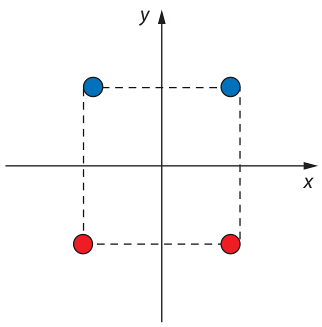


Which diagram best represents the electric field?



A
B
C
D

5 The diagram shows four charges of equal magnitude. The top charges are positive and the lower ones are negative. The charges placed at the vertices of a square whose centre is at the origin of the axes shown.



Where is the electric potential zero?

A along the x-axis only
B along the y-axis only
C along both axes
D at the origin only

6 A probe is orbiting a planet in a low circular orbit. It is desired to move the probe into a higher orbit. What are the changes in the probe's kinetic and gravitational potential energy in doing so?

	Kinetic energy	Potential energy
A	increases	increases
B	increases	decreases
C	decreases	increases
D	decreases	decreases

7 The gravitational potential at the surface of a spherical planet of radius R and uniform density is V. What is an expression for the magnitude of the gravitational field strength at the surface of the planet?

A $-VR$

B $-\frac{V}{R}$

C $-VR^2$

D $-\frac{V}{R^2}$

8 Two stars of the same mass M orbit a common centre with the same speed. The distance separating the stars is d. What is the total energy of the system?

A $-\frac{3GM^2}{4d}$

B $\frac{3GM^2}{4d}$

C $-\frac{GM^2}{2d}$

D $\frac{GM^2}{2d}$

9 A probe of mass m is launched from the surface of a planet of mass M and radius R with kinetic energy $\frac{3GMm}{4R}$. The probe settles into a circular orbit around the planet. What is the radius of the orbit?

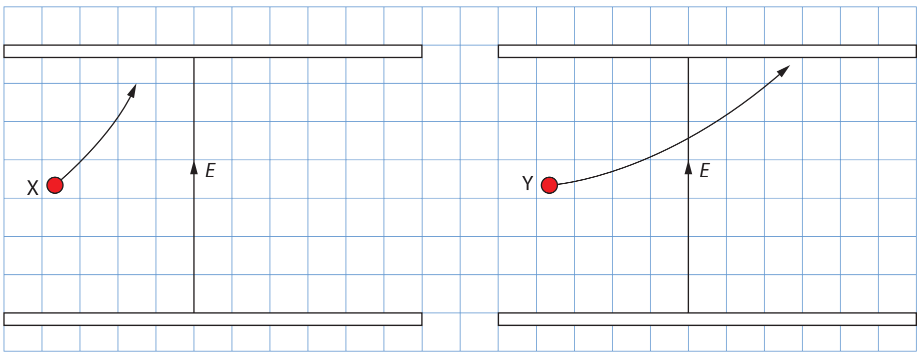
A R

B $2\,R$

C $3\,R$

D $4\,R$

10 Two charged particles, X and Y, enter the same region of uniform electric field with the same velocity. They follow different paths as shown. Gravity is negligible.



What can be deduced about the particles?

A the charge of X is greater than that of Y

B the charge of X is smaller than that of Y

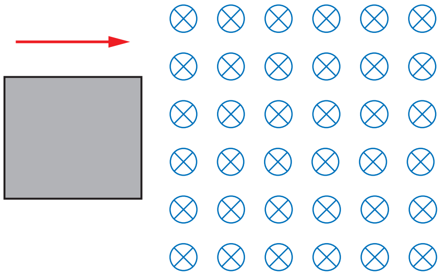
C the charge to mass ratio for X is greater than that of Y

D the charge to mass ratio for X is smaller than that of Y

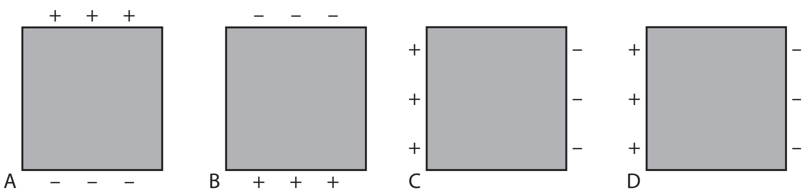
Self-test questions

Topic 11

1 A solid conducting disc is made to move through a region of a uniform magnetic field directed into the plane of the page.

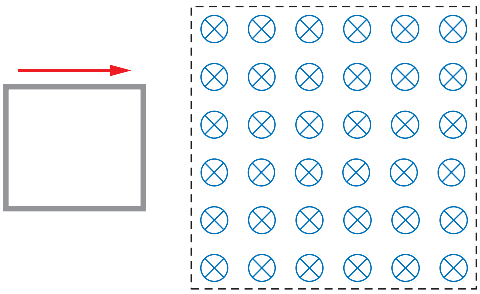


Which diagram shows the correct charge distribution in the disc while the disc is in the region of magnetic field?

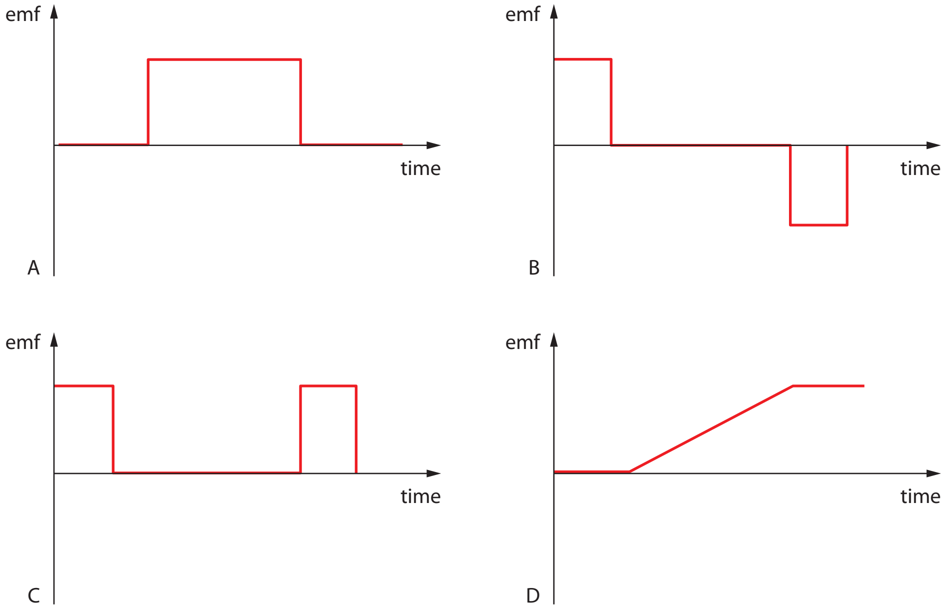


A
B
C
D

2 A loop of wire is moved at constant speed such that it will pass through a region R of magnetic field directed into the plane of the page.



Which graph correctly shows the variation with time of the induced emf in the loop before, during and after the loop enters region R?



A
B
C
D

CORRECT

3 A dc current of 5.0 A causes 20 W of power to be dissipated in a resistor. What is the rms value of an ac current that would cause an average power dissipation of 20 W in the same resistor?

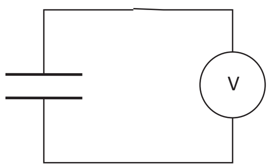
A 2.5 A

B 5.0 A

C $\frac{5.0}{\sqrt{2}}$ A

D $5.0\sqrt{2}$ A

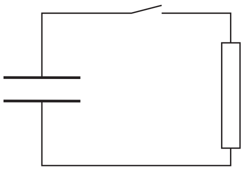
4 A dielectric is placed in between the plates of a charged parallel plate capacitor in vacuum. The capacitor is connected to an ideal voltmeter.



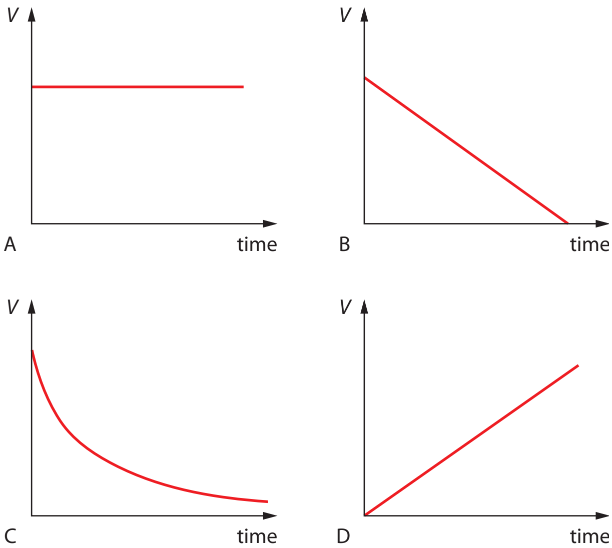
Which of the following best describes the changes in the charge, voltage and capacitance of the capacitor as a result of inserting the dielectric?

	Charge	Voltage	Capacitance
A	stays the same	decreases	stays the same
B	stays the same	decreases	increases
C	increases	increases	stays the same
D	increases	increases	increases

5 The diagram shows a circuit that includes a capacitor that is initially fully charged, a switch and a resistor.

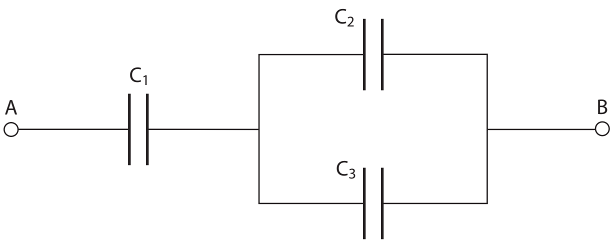


The switch is closed. Which graph best represents the variation with time of the potential difference, *V*, between the ends of the resistor?



A
B
C
D

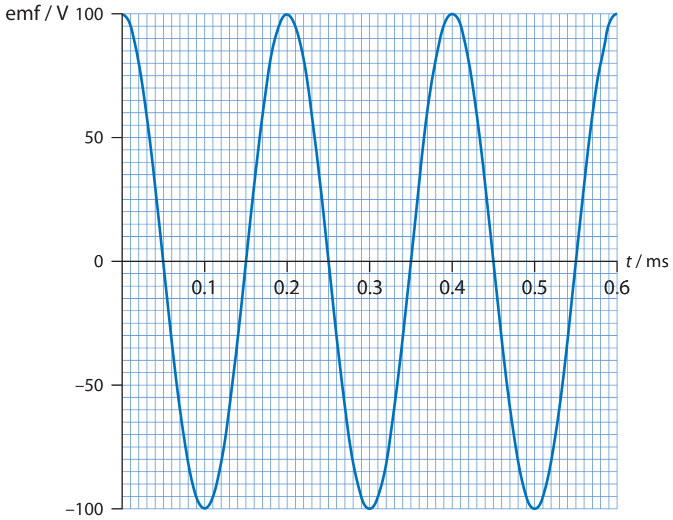
6 Each of the capacitors in the figure below has a value of 12 pF.



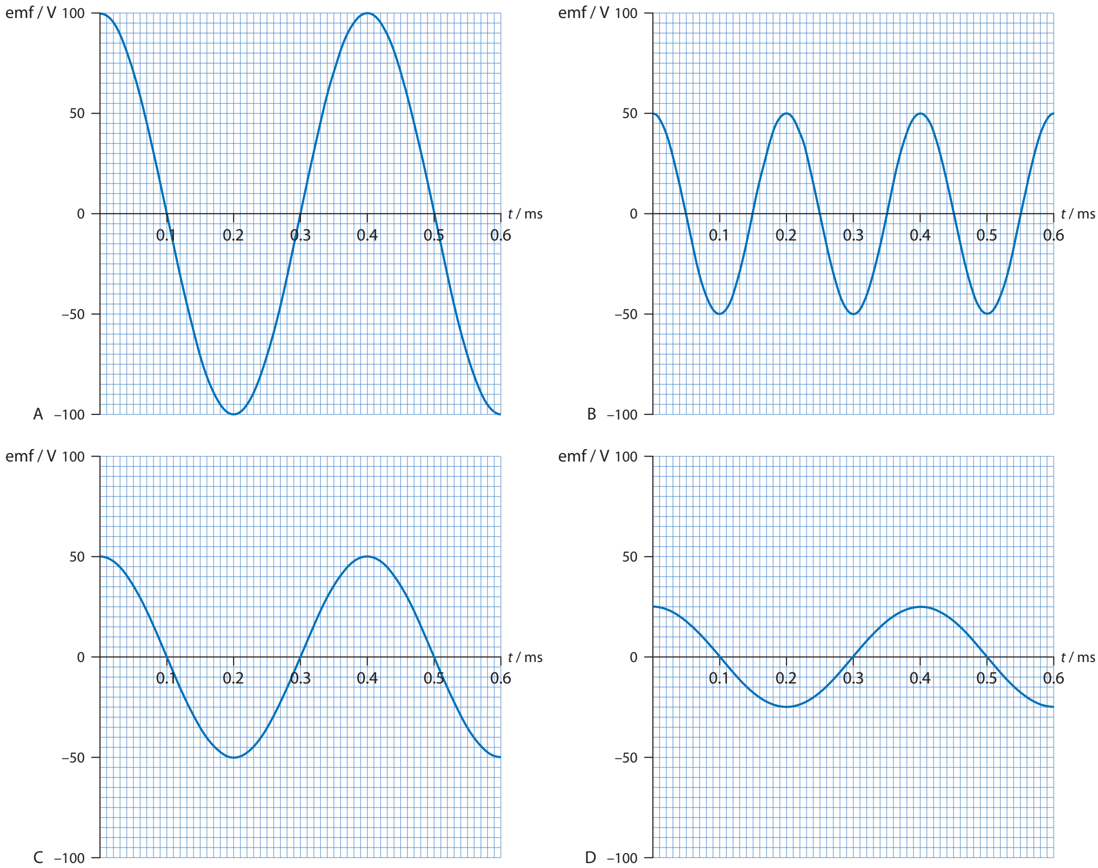
Points A and B are connected to a source of potential difference 3.0 V. What is the charge on one of the plates of capacitor C_1?

A 12 pC
B 24 pC
C 27 pC
D 54 pC

7 The graph shows the variation with time of the induced emf in a coil rotating in a uniform magnetic field.

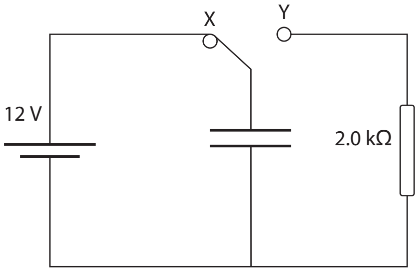


Which graph shows the variation of the induced emf with time when the same coil is rotated in the same magnetic field at half the speed?



A
B
C
D

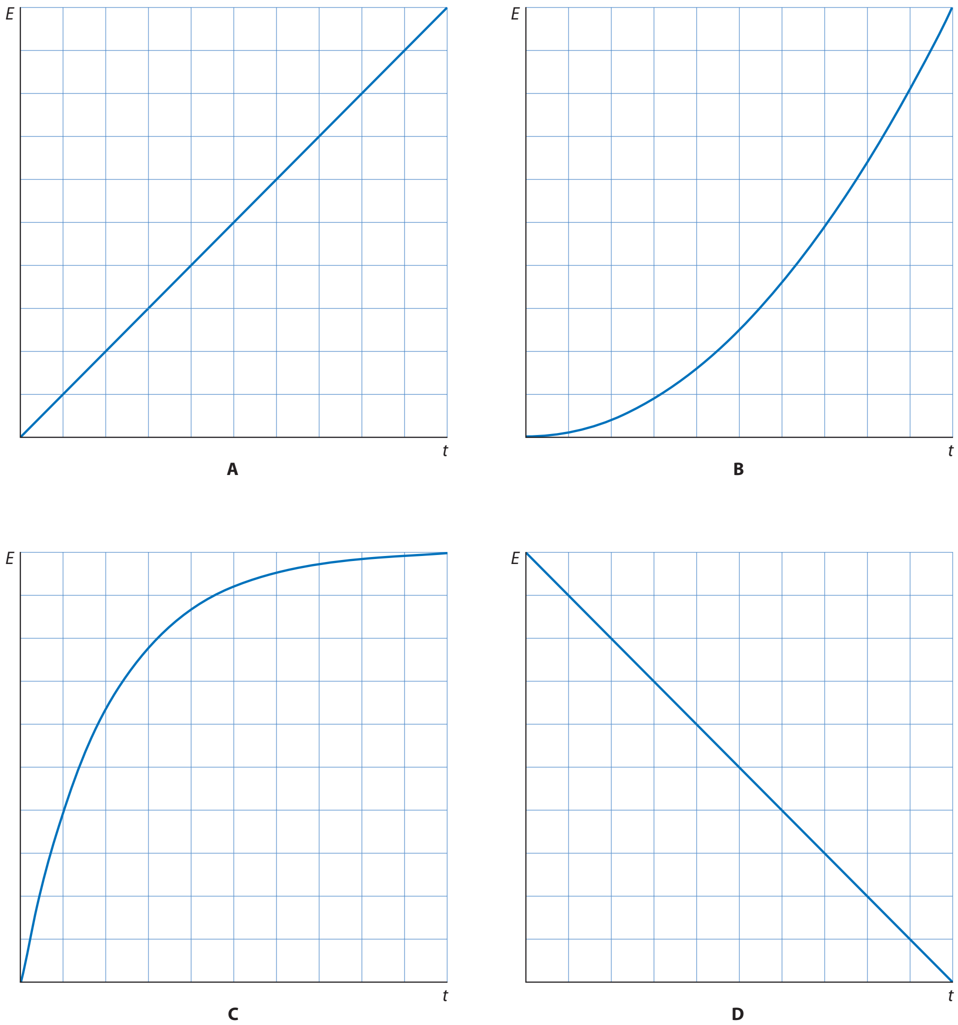
8 In the circuit shown the switch is in position X for a long time so that the capacitor is fully charged.



The switch is then moved to position Y. What is the initial current and voltage across the resistor?

	Current/mA	Voltage/V
A	0	0
B	0	12
C	6.0	0
D	6.0	12

9 A capacitor that is initially uncharged is connected to a source of constant potential difference. Which graph shows how the energy, *E*, stored in the capacitor varies with time *t*?



A
B
C
D

10 In a diode rectifying bridge circuit the role of the capacitor is to make the current

A smoother

B constant

C positive

D larger

Self-test questions

Topic 12

1 Light, incident on a metallic surface, causes electrons to be emitted. It is suggested that the following changes will increase the energy of the emitted electrons:

I increasing the intensity of the light
II increasing the frequency of the light
III increasing the work function of the surface

Which change or changes will actually increase the energy of the electrons?

A I only
B II only
C III only
D I and III

2 Light of frequency f is incident on a photosurface. The work function of the photosurface is ϕ. What is the critical frequency for this photosurface?

A $hf - \phi$
B $\dfrac{h}{\phi}$
C $\dfrac{\phi}{h}$
D $f - \dfrac{\phi}{h}$

CORRECT

3 An electron is confined within a region of linear size L. What is an **estimate** of the electron's kinetic energy?

A $\dfrac{mL^2}{h^2}$
B $\dfrac{h^2}{mL^2}$
C $\dfrac{mc^2}{hL}$
D $\dfrac{hL}{mc^2}$

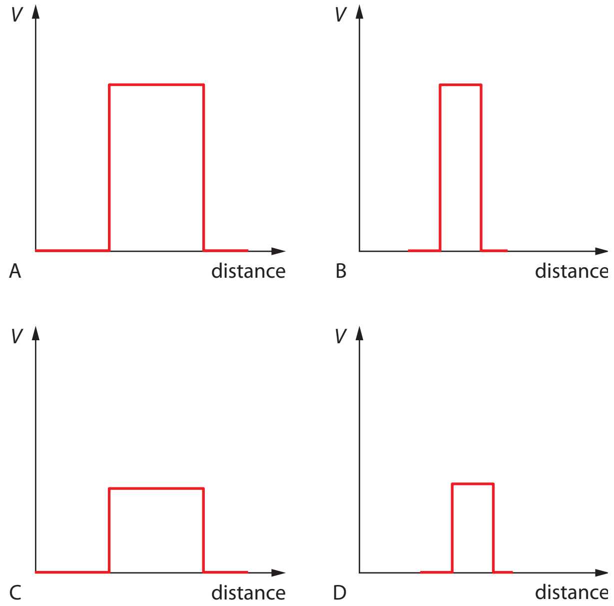
4 Evidence for de Broglie hypothesis comes from:

A diffraction experiments with electrons and neutrons
B alpha particle scattering experiments
C the existence of nuclear energy levels
D the existence of isotopes

5 An alpha particle and a proton have the same de Broglie wavelength. What is an approximate value of the ratio $\dfrac{v_p}{v_\alpha}$ of the speed of the proton to that of the alpha?

A 2
B 4
C $\dfrac{1}{2}$
D $\dfrac{1}{4}$

6 In which of the following potential barriers would the probability of tunneling for protons be the greatest?



A
B
C
D

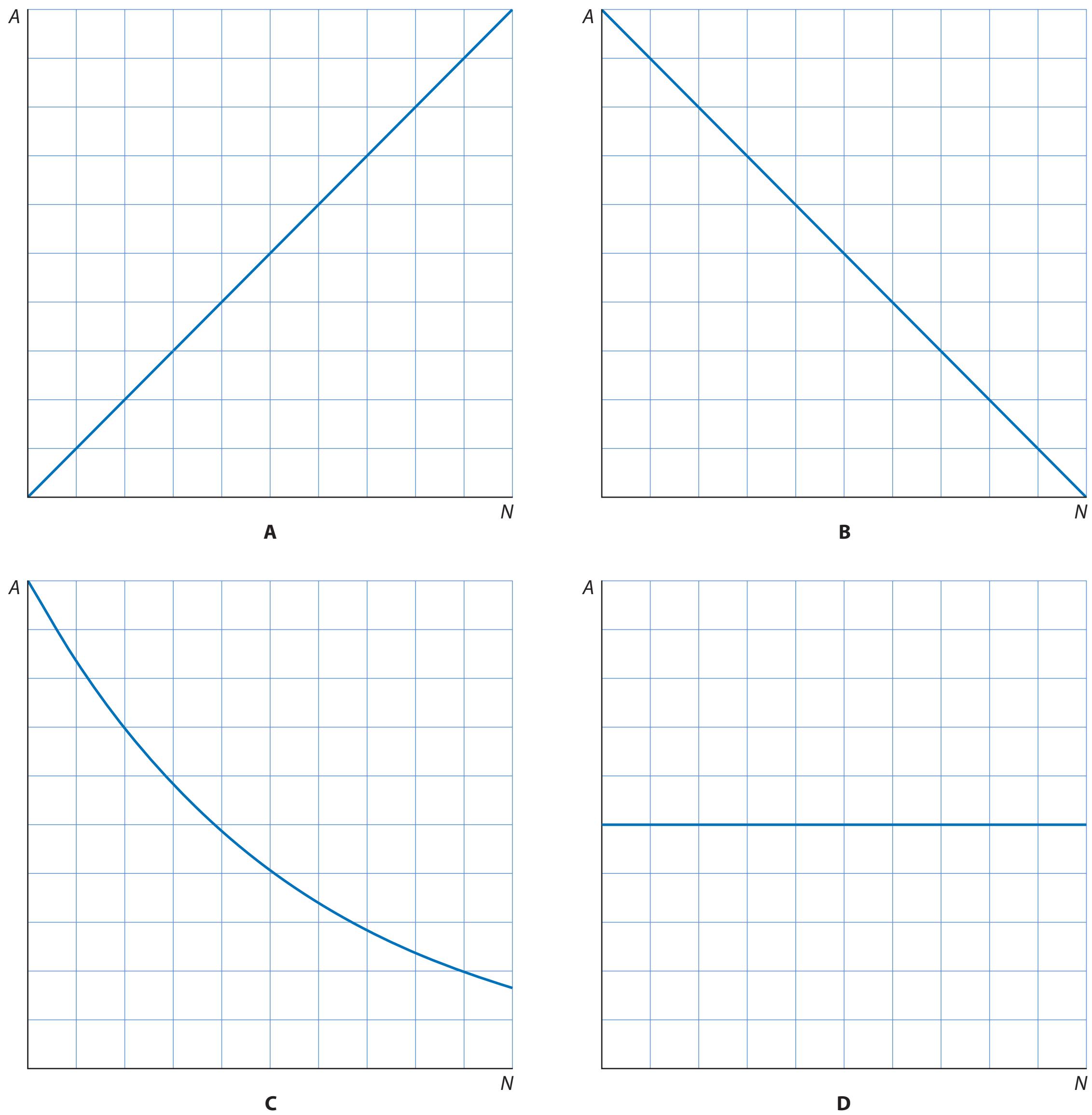
7 At high energies, deviations from Rutherford scattering are observed because:

A the de Broglie wavelength of the alpha particles increases
B the de Broglie wavelength of the alpha particles decreases
C the weak nuclear force starts to acts on the alpha particles
D the strong nuclear force starts to act on the alpha particles

8 Which of the following is a correct statement about nuclei?

A they have the same binding energy
B they have the same binding energy per nucleon
C they have the same density
D they have the same ratio of neutrons to protons

9 The activity of a radioactive sample is *A* and, at that time, the number of radioactive nuclei present is *N*. Which graph shows the correct variation of *A* with *N*?



A
B
C
D

10 An electron and a positron at rest annihilate. What is the best estimate of the wavelength of one of the photons produced?

A $\dfrac{h}{mc}$

B $\dfrac{h}{mc^2}$

C $\dfrac{mc}{h}$

D $\dfrac{mc^2}{h}$

Answers to exam-style questions

Topic 1

Where appropriate, 1 ✓ = 1 mark

1 B

2 A

3 D

4 B

5 A

6 D

7 C

8 A

9 C

10 A

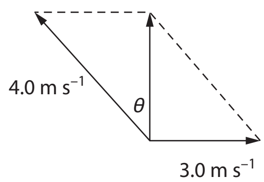
11 Use a smaller heavier ball. ✓
In order to minimise the effect of air resistance. ✓
Let the ball drop from various heights. ✓
In order to draw a graph of height versus time and get the acceleration through the gradient of the graph. ✓
If a stopwatch is to be used measure the time for each height many times and get an average. ✓
In order to get a more accurate value for the time. ✓

12 **a** It will take $\frac{30}{4.0} = 7.5\ \text{s}$ to get across. ✓
And he will move $3.0 \times 7.5 = 22.5 \approx 22\ \text{m}$ to the right of P. ✓

b Correct diagram. ✓

$\sin\theta = \frac{3.0}{4.0} = 0.75$ ✓

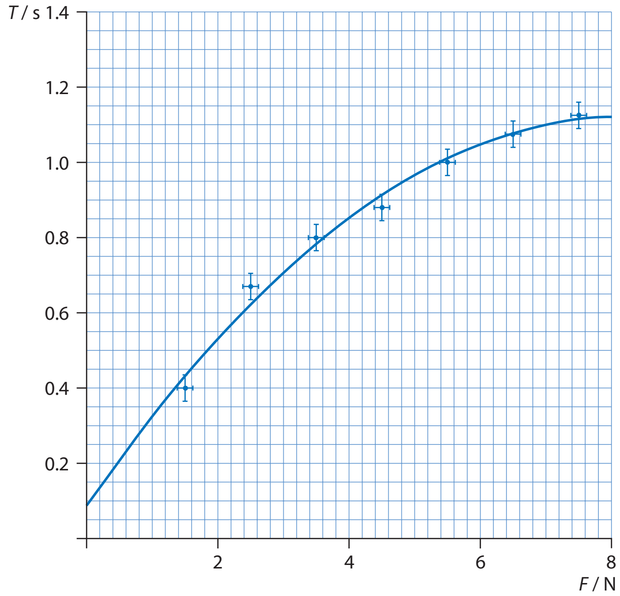
$\theta = \sin^{-1} 0.75 = 48.6°$ ✓



c The woman moves across with a speed of $\sqrt{4.0^2 - 3.0^2} = 2.6458\ \text{m s}^{-1}$. ✓

So she will take a time of $\frac{30}{2.6458} = 11.3 \approx 11\ \text{s}$, so will be longer than the man. ✓

13 **a** Smooth curve. ✓
Through all the error bars. ✓



b The vertical intercept is about 0.1 s. ✓

c For T to be proportional to F requires a straight line graph through the origin. ✓

And here neither of these conditions are satisfied. ✓

d The uncertainty in T is about ± 0.035 s . ✓

$$\frac{\Delta T^2}{T^2} = 2\frac{\Delta T}{T} \Rightarrow \Delta T^2 = 2T\Delta T$$ ✓

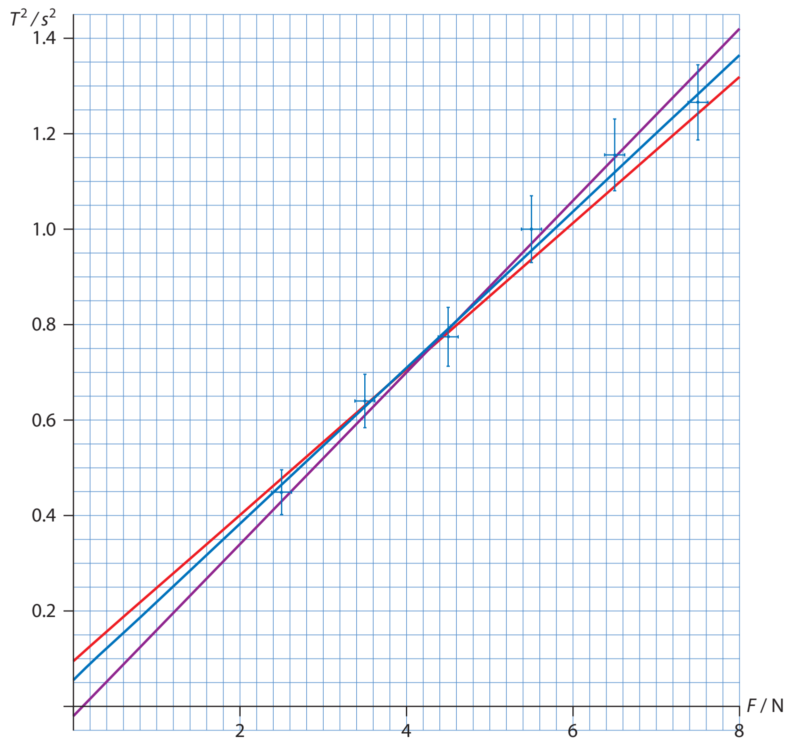
Hence $\Delta T^2 = \pm 2 \times 1.0 \times 0.035 = \pm 0.07$ s^2 ✓

e Correct plotting of points. ✓

Correct error bars and lines of maximum and minimum slope. ✓

Line of best-fit is straight and within uncertainties passes through origin. ✓

Hence claim is correct. ✓



f Slope of line of best fit 0.164 $s^2\,N^{-1}$. ✓

Max/min slopes 0.153 $s^2\,N^{-1}$ and 0.180 $s^2\,N^{-1}$ so uncertainty is $0.0135 \approx 0.01$ $s^2\,N^{-1}$. ✓

So (0.164 ± 0.001) $s^2\,N^{-1}$. ✓

Answers to exam-style questions

Topic 2

Where appropriate, 1 ✓ = 1 mark

1 D

2 C

3 C

4 D

5 A

6 D

7 D

8 A

9 C

10 A

11 **a** **i** The equation applies to straight line motion with acceleration g. Neither condition is satisfied here. ✓

ii This equation is the result of energy conservation so it does apply since there are no frictional forces present. ✓

b From $v = \sqrt{2gh}$ we find $h = \frac{v^2}{2g} = \frac{4.8^2}{2 \times 9.81} = 1.174 \approx 1.2$ m. ✓

c **i** The kinetic energy at B is $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4.8^2 = 28.8$ J. ✓

The frictional force is $f = \mu_K N = \mu_K mg = 0.45 \times 25 \times 9.81 = 110.36$ N and so the work done by this force is the change in the kinetic energy of the block, and so $110.36 \times d = 28.8 \Rightarrow d = 0.261 \approx 0.26$ m. ✓

ii The deceleration is $\frac{f}{\mu} = \frac{110.36}{25} = 4.41$ m s^{-2}, ✓

and so $0 = 4.8 - 4.41 \times t$ giving 1.1 s for the time. ✓

d The speed at B is independent of the mass. ✓

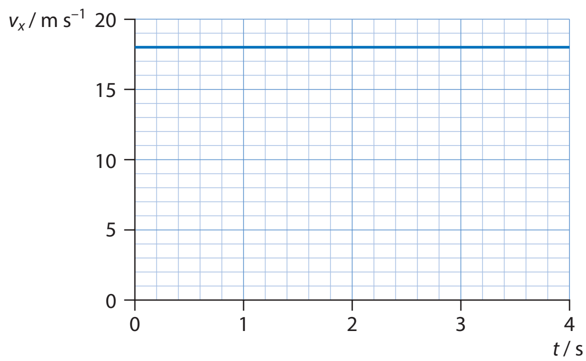
$fd = \frac{1}{2}mv^2 \Rightarrow \mu_K mgd = \frac{1}{2}mv^2 \Rightarrow d = \frac{v^2}{2\mu_K}$, ✓

and so the distance is also independent of the mass. ✓

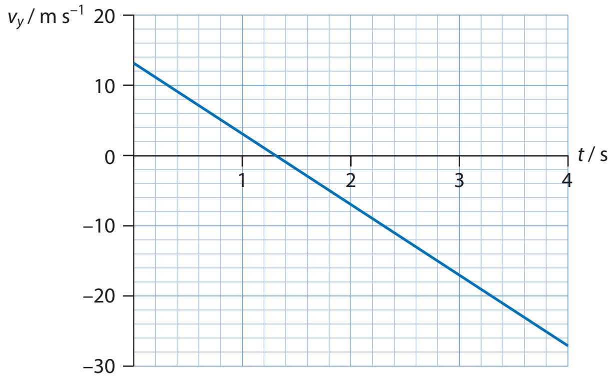
12 **a** **i** $v_x = v\cos\theta = 22 \times \cos 35° = 18.0 \approx 18$ m s^{-1} ✓

$v_y = v\sin\theta = 22 \times \sin 35° = 12.6 \approx 13$ m s^{-1} ✓

ii Graph as shown. ✓



Graph as shown. ✓



b i At maximum height: $v_y^2 = 0 = u_y^2 - 2gy$. ✓

$y = \frac{u_y^2}{2g}$ ✓

and so $y = \frac{12.6^2}{2 \times 9.8} = 8.1$ m ✓

OR

$v_y = 0 = v\sin\theta - gt$ $12.6 - 9.8t = 0$ ✓

so $t = 1.29$ s ✓

Hence $y = 12.6 \times 1.29 - \frac{1}{2} \times 9.8 \times 1.29^2 = 8.1$ m ✓

ii The force is the weight, i.e. $F = 0.20 \times 9.8 = 1.96 \approx 2.0$ N. ✓

c i $\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$ hence $v = \sqrt{u^2 + 2gh}$ ✓

$v = \sqrt{u^2 + 2gh} = \sqrt{22^2 + 2 \times 9.8 \times 32} = 33.3 \approx 32$ m s^{-1} ✓

ii $v^2 = v_x^2 + v_y^2 \Rightarrow v_y = -\sqrt{v_x^2 - v_x^2} = -\sqrt{33.3^2 - 18.0^2} = -28.0$ m s^{-1} ✓

Now $v_y = u_y \sin\theta - gt$ so $-28.0 = 12.6 - 9.8 \times t$ hence $t = 4.1$ s ✓

d i Smaller height. ✓

Smaller range. ✓

Steeper impact angle. ✓

ii The angle is steeper because the horizontal velocity component tends to become zero. ✓

Whereas the vertical tends to attain terminal speed and so a constant value. ✓

13 a i In 1 second the mass of air that will move down is $\rho(\pi R^2 v)$. ✓

The change of its momentum in this second is $\rho(\pi R^2 v)v = \rho\pi R^2 v^2$. ✓

And from $F = \frac{\Delta p}{\Delta t}$ this is the force. ✓

ii $\rho\pi R^2 v^2 = mg$ ✓

And so $v = \sqrt{\dfrac{mg}{\rho\pi R^2}} = \sqrt{\dfrac{0.30\times 9.8}{1.2\times\pi\times 0.25^2}} = 3.53 \approx 3.5\ \text{m s}^{-1}$. ✓

b The power is $P = Fv$ where $F = \rho\pi R^2 v^2$ is the force pushing down on the air and so $P = \rho\pi R^2 v^2$. ✓
So $P = 1.2\times\pi\times 0.25^2 \times 3.53^2 = 2.936 \approx 3.0\ \text{W}$ ✓

c i From $F = \rho\pi R^2 v^2$ the force is now 4 times as large, i.e. $4mg$ and so the **net** force on the helicopter is $3mg$. ✓

And so the acceleration is constant at $3g$. Hence $s = \dfrac{1}{2}\times 3g\times t^2 \Rightarrow t = \sqrt{\dfrac{2s}{3g}} \approx 0.90$ s. ✓

ii $v = 3gt = \sqrt{\dfrac{2s}{3g}}$ ✓

$v \approx 26\ \text{m s}^{-1}$ ✓

iii The work done by the rotor is $W = Fd = 4mgd = 4\times 0.30\times 9.8\times 12 = 141$ J. ✓

14 a i The area is the impulse i.e. 2.0×10^3 N s. ✓

ii The average force is found from $\bar{F}\Delta t = 2.0\times 10^3$ N s. ✓

And so $\bar{F} = \dfrac{2.0\times 10^3}{0.20} = 1.0\times 10^4$ N. ✓

Hence the average acceleration is $\bar{a} = \dfrac{1.0\times 10^4}{8.0} = 1.25\times 10^3\ \text{m s}^{-2}$. ✓

iii The final speed is $\bar{v} = \bar{a}t = 1.25\times 10^3 \times 0.20 = 250\ \text{m s}^{-1}$. ✓
And so the average speed is $125\ \text{m s}^{-1}$. ✓

iv $s = \dfrac{1}{2}\bar{a}t^2 = \dfrac{1}{2}\times 1.25\times 10^3 \times 0.20^2$ ✓

$s = 25$ m ✓

b i The final speed is $\bar{v} = \bar{a}t = 1.25\times 10^3 \times 0.20$, ✓
$\bar{v} = 250\ \text{m s}^{-1}$. ✓

ii The kinetic energy is $E_K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\times 8.0\times 250^2$ ✓

$E_K = 2.5\times 10^5$ J ✓

iii $P = \dfrac{E_K}{t} = \dfrac{2.5\times 10^5}{0.20}$ ✓

$P = 1.25\times 10^6$ W ✓

15 a i It is zero (because the velocity is constant). ✓

ii $F - mg\sin\theta - f = 0$ ✓

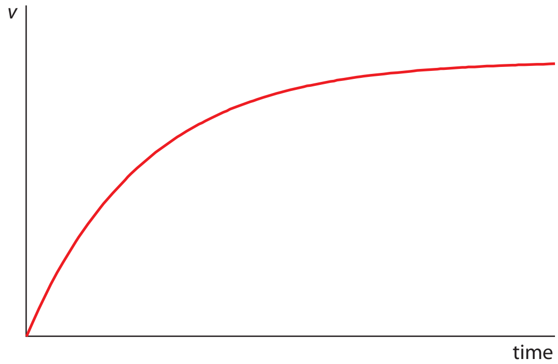
$F = mg\sin\theta + f = 1.4\times 10^4 \times \sin 5.0° + 600$ ✓

$F = 1820$ N ✓

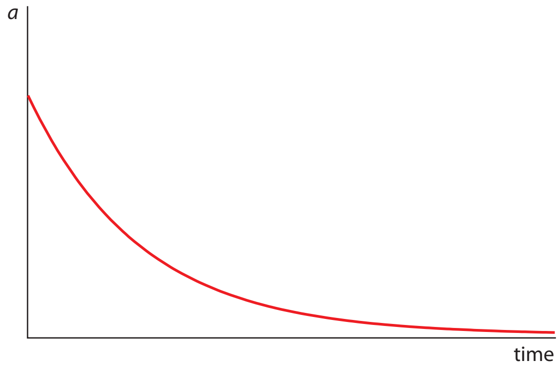
b The power used by the engine in pushing the car is $P = Fv = 1820\times 6.2 = 1.13\times 10^4$ W, ✓
$P = 11.3$ kW. ✓

The efficiency is then $e = \dfrac{11.3}{15} = 0.75$ ✓

c i Initial speed zero. ✓
Terminal speed. ✓



ii Initial acceleration not zero. ✓
And approaching zero. ✓



16 a i The change in momentum is $\Delta p = 0.090 \times (90 - 130)$, ✓
$\Delta p = -3.6$ N s. ✓

ii This is also the negative change in the momentum of the block and so $1.20v = 3.6$ N s giving $v = 3.0$ m s^{-1}. ✓

iii The initial kinetic energy is $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.090 \times 130^2 = 422.5$ J. ✓

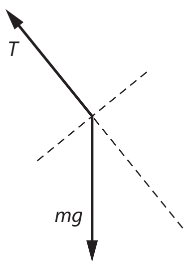
The final kinetic energy is $E = \frac{1}{2} \times 0.090 \times 90^2 + \frac{1}{2} \times 1.20 \times 3.0^2 = 369.9$ J. The change is then $\Delta E = 369.9 - 422.5 = -52.6 \approx -53$ J. ✓

b We have conservation of energy and so $\frac{1}{2} \times m \times 3.0^2 = m \times 9.8 \times h$ and so $h = 0.459$ m. ✓

But $h = L - L\cos\theta$ and so $0.459 = 0.80 \times (1 - \cos\theta)$ ✓

giving $\cos\theta = 0.426$ and so $\theta = 64.77° \approx 65°$ ✓

c i It is not because there is a net force on it. ✓



ii From the diagram, $T - mg\cos\theta = m\frac{v^2}{L}$. ✓

But $v = 0$ and so $T = mg\cos\theta = 1.20 \times 9.8 \times \cos 64.77° = 5.0$ N. ✓

$T = 5.0$ N. ✓

Answers to exam-style questions

Topic 3

Where appropriate, 1 ✓ = 1 mark

1 A

2 B

3 C

4 D

5 B

6 A

7 B

8 A

9 D (the question should have specified equal moles for each gas)

10 A

11 a Use $pV = nRT \Rightarrow V = \frac{nRT}{p}$ ✓

To find $V = \frac{1.0 \times 8.31 \times 273}{1.0 \times 10^5} = 2.27 \times 10^{-2}\,\text{m}^3$ ✓

b i There are $N_A = 6.02 \times 10^{23}$ molecules. ✓

So to each molecule corresponds a volume $\frac{2.27 \times 10^{-2}}{6.02 \times 10^{23}} = 3.77 \times 10^{-26}\ \text{m}^3$. ✓

ii Assuming a cube of this volume the side is $\sqrt[3]{3.77 \times 10^{-26}} = 3.35 \times 10^{-9}$ m, which is therefore an estimate of the separation of the molecules. ✓

This separation is much larger than the diameter of the helium atom and so the ideal gas approximation is good. ✓

c One mole of lead has a mass of 0.207 kg and a volume of $V = \frac{m}{\rho} = \frac{0.207}{11.3 \times 10^3} = 1.83 \times 10^{-5}\ \text{m}^3$. ✓

To each molecule corresponds a volume $\frac{1.83 \times 10^{-5}}{6.02 \times 10^{23}} = 3.04 \times 10^{-29}\ \text{m}^3$. ✓

Assuming a cube of this volume the side is $\sqrt[3]{3.04 \times 10^{-29}} = 3.12 \times 10^{-10}$ m which is therefore an estimate of the separation of the molecules. ✓

d The ratio is then $\frac{3.35 \times 10^{-9}}{3.12 \times 10^{-10}}$, ✓

≈ 10 . ✓

12 a Specific heat capacity is the amount of energy required to change the temperature of a 1 kg of a substance by 1 K. ✓

b One mole of any substance contains the same number of molecules; to raise the temperature by 1 K the internal energy will increase by the same amount and so the same heart must be provided. ✓

One kg of different substances contains different numbers of molecules and so different amounts of energy are required to increase the temperature by 1 K. ✓

c From $\frac{\Delta Q}{\Delta t}=\frac{\Delta m}{\Delta t}c\Delta T$ we find $600=\frac{\Delta m}{\Delta t}\times 990\times(40-20)$. ✓

So that $\frac{\Delta m}{\Delta t}=3.0\times10^{-2}\ \text{kg s}^{-1}$. ✓

d Then $\frac{\Delta V}{\Delta t}=\rho\frac{\Delta V}{\Delta t}=1.25\times3.0\times10^{-2}=3.8\times10^{-2}\ \text{m}^3\ \text{s}^{-1}$. ✓

e The energy required is $Q=mL=180\times2200=3.96\times10^5$ J. ✓

$t=\frac{3.96\times10^5}{750}=528\ \text{s}=8.8$ min. ✓

13 a i The graph is a curve. ✓
If there was no air resistance the acceleration would have been constant and the velocity – time graph a straight line. ✓

ii We must estimate the area under the graph by counting squares with one small square equal in area to 0.5 m. ✓
There about 370 small squares so the height is about 185 m. ✓

iii Applying $mgh=\frac{1}{2}mv^2$ gives $v=\sqrt{2gh}=\sqrt{2\times9.8\times185}$, ✓

$v=60.2\approx60\ \text{m s}^{-1}$. ✓

b The impact speed is about $18.1\ \text{m s}^{-1}$ implying a loss of mechanical energy of
$\frac{1}{2}\times8.0(60.2^2-18.1^2)=1.32\times10^4$ J. ✓
Assuming all of this goes into heating the ball and that this amount of energy warms the entire body uniformly. ✓
$mc\Delta T=1.32\times10^4$, ✓

and so $\Delta T=\frac{1.32\times10^4}{8.0\times320}\approx5$ K. ✓

14 a The internal energy is the sum of the total random kinetic energy of the molecules and the intermolecular potential energy of the molecules of tungsten. ✓

b The tungsten loses heat $0.050\times132\times(T-31)$. ✓
This heat is absorbed by the water and the calorimeter:
$0.300\times4200\times(31-22)+0.120\times900\times(31-22)=1.23\times10^4$ J ✓

Hence $0.050\times132\times(T-31)=1.23\times10^4$ or $T-31=\frac{1.23\times10^4}{0.050\times132}=1864$ and finally $T=1895\approx1900\,°\text{C}$. ✓

c The calculated temperature is $T=\frac{Q}{m_W c_W}+31$ where Q is the heat that went into the water and calorimeter.
The actual Q would have been higher because some was transferred into the air during the move of the metal into the water. ✓
Hence the calculated value is smaller than the actual temperature. ✓

15 a The internal energy is the sum of the total random kinetic energy of the molecules and the intermolecular potential energy of the molecules of the substance. ✓

b During melting energy is supplied to the substance melting increasing its internal energy but not its temperature. ✓
Hence the student's statement is false. ✓

c The liquid is losing heat to the surroundings because the container is not insulated. ✓
When the rate of heat loss is equal to the rate at which energy is being provided the temperature will remain constant. ✓

d The rate of heat loss is equal to the rate at which energy was being provided when the heater was on i.e. 35 W. ✓

Since $\frac{\Delta Q}{\Delta t} = mc\frac{\Delta T}{\Delta t}$ we have that $35 = 0.240 \times c \times \frac{3.1}{60}$. ✓

And so $c = \frac{35 \times 60}{0.240 \times 3.1} = 2.8 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$. ✓

16 a $pV = nRT \Rightarrow n = \frac{pV}{RT}$ to find $n = \frac{250 \times 10^3 \times 1.50 \times 10^{-2}}{8.31 \times 273} = 1.653$. ✓

So that $N_1 = nN_A = 1.653 \times 6.02 \times 10^{23} = 9.95 \times 10^{23} \approx 1.0 \times 10^{24}$ molecules. ✓

b As the tyre rolls on the road the rubber lining of the tyre expands and contracts generating thermal energy that heats the air in the tyre. ✓

The volume will increase.

And so will the pressure and temperature. ✓

c $p = \frac{nRT}{V} = \frac{1.653 \times 8.31 \times (273 + 35)}{1.60 \times 10^{-2}} = 2.64 \times 10^5 \text{ Pa} \approx 260 \text{ kPa}$. ✓

d i Assuming the volume and temperature stay the same we must have that $\frac{p_1}{n_1} = \frac{p_2}{n_2}$ and so $\frac{250}{1.653} = \frac{230}{n_2}$ giving $n_2 = 1.52$. The number of molecules is then $N_2 = 1.52 \times 6.02 \times 10^{23} = 9.15 \times 10^{23}$. ✓

The number of molecules that left is therefore $N_1 - N_2 = 9.95 \times 10^{23} - 9.15 \times 10^{23} = 8.0 \times 10^{22}$. ✓

The rate of loss is then $\frac{8.0 \times 10^{22}}{8 \times 60 \times 60} = 2.8 \times 10^{18} \text{ s}^{-1}$. ✓

ii The number of moles lost is $1.65 - 1.52 = 0.13$ ✓

And so the lost mass of air is $0.13 \times 29 = 3.8$ g. ✓

Answers to exam-style questions

Topic 4

Where appropriate, 1 ✓ = 1 mark

1 A

2 C

3 B

4 A

5 D

6 D

7 D

8 C

9 B

10 A

11 **a** In a longitudinal wave the displacement is along the direction of energy transfer (DOET) ✓ whereas in a transverse wave it is at right angles to the DOET. ✓

b **i** The amplitude is 4.0 mm. ✓

ii The wavelength is 0.20 m. ✓

iii The period is 10 s and so the frequency is $f = \frac{1}{T} = \frac{1}{10} = 0.10$ Hz. ✓

c The speed is $v = \lambda f = 0.20 \times 0.10$. ✓

$v = 0.020 \text{ m s}^{-1}$ ✓

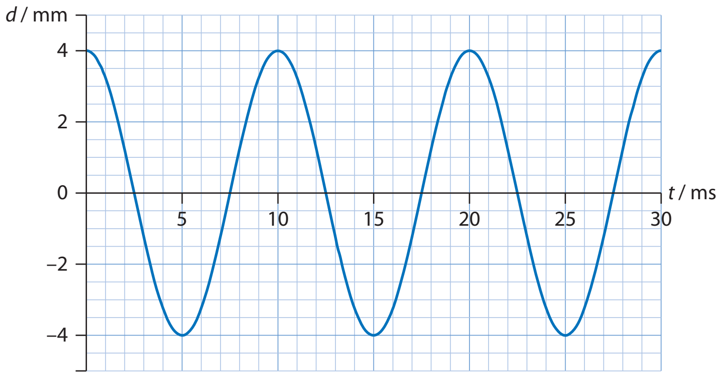
d Particle P has zero displacement at t = 10 s. ✓

A short time later the displacement becomes positive (we look at the second graph). ✓

To make the displacement of the point at 0.20 m positive a short time after 10 s the first graph must be shifted to the right, so the wave moves to the right. ✓

e At t = 10 s point Q has displacement 4.0 mm. ✓

Hence we must have the following graph. ✓



f **i** The wavelength of the first harmonic is $4L$, ✓

and so $4L = 0.20 \Rightarrow L = 0.050$ m. ✓

ii Standing waves do not transfer energy; travelling waves do. ✓

Standing waves have variable amplitude; travelling waves have a constant amplitude. ✓

iii It is the speed of one of the travelling waves, ✓

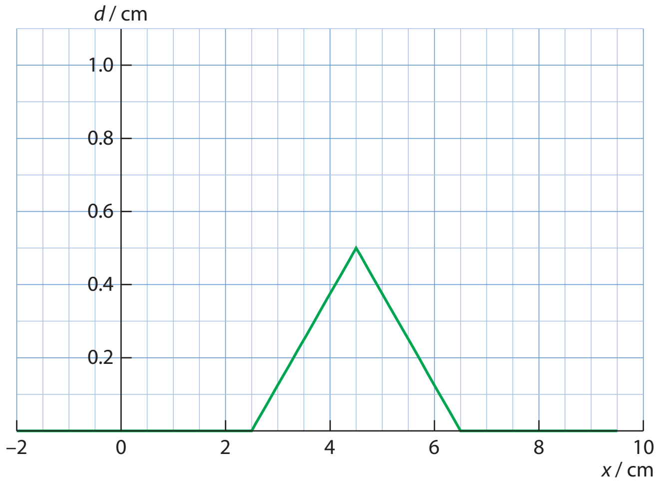
making up the standing wave. ✓

12 **a** When two waves (of the same type) meet, ✓
the resultant displacement is the algebraic sum of the individual displacements. ✓

b The speed of the black pulse is the same as that of the grey pulse since the medium is the same. ✓

c i The centres of the pulses are separated by a distance of 5.0 cm. The relative speed of the pulses is 30 m s^{-1} and so will completely overlap at a time of $\frac{5.0}{30} = 0.167 \approx 0.17$ s. ✓

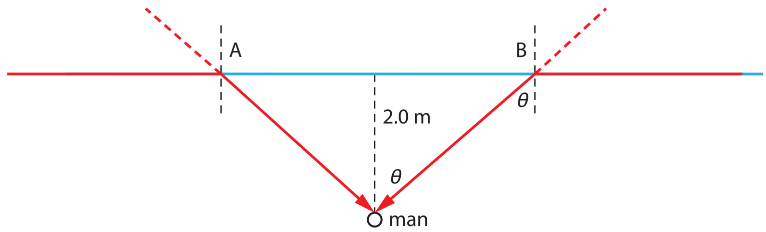
ii In 0.167 s each pulse will move a distance of 2.5 m, ✓
and so the resulting pulse has the shape of the following graph. ✓



d i The pulses have the same shape after the collision. ✓
So no energy is lost (the collision of the pulses is elastic). ✓

ii The energy carried by a pulse is proportional to the (square of the) height of the pulse. ✓
The pulse is short during overlap. ✓
But the string is moving vertically during overlap and so makes up for the apparently missing energy. ✓

13 **a** The diagram shows how rays of light coming in parallel to the water surface will refract. ✓



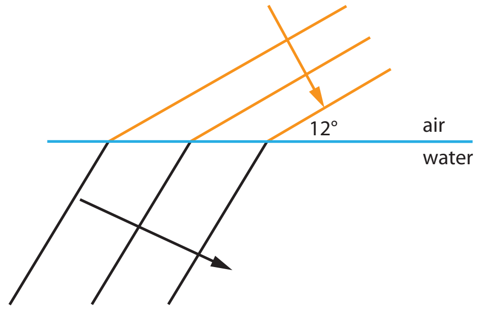
So the rays that can enter the man's eyes lie within a circle of diameter AB. ✓

b From the diagram above and Snell's law $1.00 \times \sin 90° = 1.33 \times \sin\theta$ so that $\theta = 48.8°$. ✓
Hence $R = 2.0 \tan\theta = 2.0 \times \tan 48.8° = 2.28 \approx 2.3$ m. ✓

c The angle θ will be the same. ✓
But since the depth is greater so will the radius. ✓

d i Snell's law says that $\frac{\sin 12°}{340} = \frac{\sin\theta}{1500}$ ✓
so that $\theta = 66.5° \approx 67°$. ✓

ii Three wavefronts as shown:
Rays bending away from normal. ✓
Wavelength greater. ✓



iii The sound tends to move parallel to the surface of the water, ✓
and not to penetrate deeper into the water where a swimmer might be. ✓

14 a Light in which the electric field oscillates on only one plane. ✓

b The intensity transmitted through the first polariser will be 160 W m^{-2}. ✓
The intensity through the second will be $160\cos^2\theta$ W m^{-2} and through the third $160\cos^4\theta$ W m^{-2}. ✓
Hence $160\cos^4\theta = 10$ giving $\theta = 60°$. ✓

c Let the intensities of the polarised and unpolarised components be I_P, I_U respectively: at maximum transmitted intensity the polariser's axis will be parallel to the polarised light's electric field and the transmitted intensity will then be $I_P + \frac{I_U}{2}$; at minimum intensity the polarised component will not be transmitted and so the intensity will be $\frac{I_U}{2}$. ✓

We have that $\frac{I_P + \frac{I_U}{2}}{\frac{I_U}{2}} = 7$ and so $\frac{I_P}{I_U} = 3$. ✓

The required fraction is then $\frac{3}{4}$. ✓

d The wall is vertical and so the reflected light is partially polarised. ✓
In a direction that is parallel to the wall, i.e. vertical. ✓
And so a polariser with a horizontal transmission axis will cut off the reflected glare. ✓

15 a Light leaving each of the slits diffracts at each slit, ✓
and so light from each slit will arrive at the middle of the screen. ✓

b With both slits open light arrives at the middle of the screen in phase and so the amplitude is twice the amplitude due to one slit. ✓
The intensity is proportional to the amplitude squared. ✓
So with one slit open the amplitude will be half and the intensity one quarter, i.e. 1 W m^{-2}. ✓

c The intensity of the side maxima is not the same as that of the central maximum. ✓

d The separation of the maxima on the screen is 0.60 cm and the separation is given by $s = \frac{\lambda D}{d}$ and so $\lambda = \frac{sd}{D}$. ✓

Hence $\lambda = \frac{0.60\times10^{-2}\times0.39\times10^{-3}}{3.2} = 7.3\times10^{-7}$ m. ✓

e Blue light has a smaller wavelength than red light. ✓
Hence the separation of the maxima will be less. ✓

16 a A standing wave is formed when two identical travelling waves moving in opposite directions. ✓

Meet and superpose. ✓

b i The travelling wave from the source reflects off the water surface. ✓

The reflected wave superposes with the incoming wave creating a standing wave in the tube. ✓

ii The standing wave will have a wavelength equal to $\frac{4L}{n}$ where L is the length of the air column and n is an odd integer. ✓

So for a given wavelength λ this will happen only when $L = \frac{\lambda n}{4}$, i.e. for specific values of the air column length. ✓

iii The difference in air column lengths is half a wavelength (explained in the next part) and so the next length is 42 cm. ✓

iv The difference in air column lengths is $\frac{\lambda n}{4} - \frac{\lambda(n-2)}{4} = \frac{\lambda}{2}$, i.e. half a wavelength and the wavelength is $\lambda = 2 \times 0.12 = 0.24$ m. ✓

So $v = f\lambda = 1400 \times 0.24 = 336 \approx 340 \text{ m s}^{-1}$. ✓

Answers to exam-style questions

Topic 5

Where appropriate, 1 ✓ = 1 mark

1 B

2 C

3 B

4 C

5 A

6 D

7 C

8 C

9 C

10 C

11 B

12 a It does not because the graph is not a straight line (through the origin). ✓

b The current is 2.35 mA. ✓

And so $R = \frac{V}{I} = \frac{4.0}{2.35 \times 10^{-3}} = 1.70 \times 10^3 \approx 1.7 \times 10^3\ \Omega$. ✓

c $R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} \Rightarrow L = \frac{\pi r^2 R}{\rho}$ ✓

$L = \frac{\pi \times (0.25 \times 10^{-3})^2 \times 1.70 \times 10^3}{3.0 \times 10^{-7}} = 1.1\ \text{km}$ ✓

d i Both lamps take the same current, 1.0 mA and so the potential difference across each is 1.0 V and hence the emf of the battery is 1.0 V. ✓

ii The power in each lamp is $P = VI = 1.0 \times 1.0 \times 10^{-3} = 1.0 \times 10^{-3}$ W. ✓

e The electric field established inside the wires and lamps forces electrons to accelerate. ✓

The accelerated electrons collide with atoms of the lamp filament transferring energy to them and increasing their random kinetic energy. ✓

The increased kinetic energy of the atoms shows up macroscopically as increased temperature (since the average random kinetic energy is proportional to temperature). ✓

13 a From $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$. ✓

$R = \frac{230^2}{1500} = 35.3 \approx 35\ \Omega$. ✓

b i The top right device is short circuited and no current passes through the lower device. Hence $P = 1500$ W. ✓

ii The top right device is short circuited and the top left and lower devices are connected n parallel so $P = 3000$ W. ✓

iii The lower device takes no current and the upper two are now in series. The voltage across each is 115 V and so the power in each is 1500/4 for a total of 750 W. ✓

iv The lower device dissipates 1500 W and the upper two 750 W for a total of 2250 W. ✓

c i We use $V = E - Ir$ to find $11.5 = E - 9.80 \times 0.0500$. ✓

Hence $E = 11.99 \approx 12$ V. ✓

ii We must decide if the current changes when the switch closes. If the source had no internal resistance there would be no change. But here it does so the current will change.

With the switch open the lamp takes current 9.80 A. Its resistance is $R = \frac{11.5}{9.80} \approx 1.17\ \Omega$. ✓

With the switch closed the total resistance of the circuit is $R = \frac{1.17 \times 25}{25 + 1.17} + 0.050 = 1.17\ \Omega$. ✓

iii The current in the circuit is then 10.3 A. The current in the lamp is then $I = \frac{25}{25 + 1.17} \times 10.3 = 9.83$ A and is larger than before, so the lamp is brighter. ✓

$I = 10.3 - 9.83 = 0.46$ A ✓

14 a i To the left. ✓

ii Upwards. ✓

b In 1 second the electrons that will cross a cross sectional area of the conductor are at most a distance v from the cross section. ✓

The number of electrons in this volume is vAn. ✓

The current is the charge that will cross the cross sectional area in 1 s and so this is $qvAn$. ✓

c The magnetic force on the electrons pushes electrons towards the top of the conductor. ✓

The bottom side of the conductor has a net positive charge and so a potential difference is established between B and T. ✓

d i Electrons will stop moving upwards when $qE = qvB$. ✓

Hence $\frac{V}{d} = vB$. ✓

ii From $I = qvAn$ we find $v = \frac{I}{qAn}$ and so $v = \frac{0.50}{1.6 \times 10^{-19} \times 4.2 \times 10^{-6} \times 3.2 \times 10^{28}} = 2.3 \times 10^{-5}\ \mathrm{m\,s^{-1}}$. ✓

Assuming a square cross section $d = \sqrt{4.2 \times 10^{-6}} = 2.1 \times 10^{-3}$ m. ✓

Hence $V = 2.3 \times 10^{-5} \times 0.20 \times 2.1 \times 10^{-3} = 9.7 \times 10^{-9}$ V. ✓

e With negative charge carriers. ✓

The polarity of the Hall voltage is such that the top side of the conductor will be negative. ✓

15 a The work done by the magnetic force on the charge is zero because the force is at right angles to the velocity. ✓

Since the work done by the net force is the change in kinetic energy, and hence in speed, is zero. ✓

b The magnetic force on the proton is always at right angles to the velocity. ✓

Which is the condition for circular motion. ✓

c i From Newton's second law $qvB = \frac{mv^2}{R}$ ✓

Cancelling one power of the speed gives the answer. ✓

ii $R = \frac{mv}{qB} = \frac{1.673 \times 10^{-27} \times 3.6 \times 10^{6}}{1.60 \times 10^{-19} \times 0.25}$ ✓

$R = 1.505 \times 10^{-1}$ m ✓ (you must show the answer to at least one more significant figure than what the answer requires)

iii The time for one full revolution is $T = \frac{2\pi R}{v} = 2.63 \times 10^{-5}$ s. ✓

And we need a quarter of this, so 6.6×10^{-6} s. ✓

d i From $R = \frac{mv}{qB}$ we find $\frac{R_1}{R_2} = \frac{m_1}{m_2}$. ✓

And so $\frac{38.0}{41.8} = \frac{3.32 \times 10^{-26}}{m_2}$ leading to $m_2 = \frac{41.8 \times 3.32 \times 10^{-26}}{38.0} = 3.65 \times 10^{-26}$ kg. ✓

ii The extra mass is due to some of the atoms having extra neutrons in the nucleus. ✓

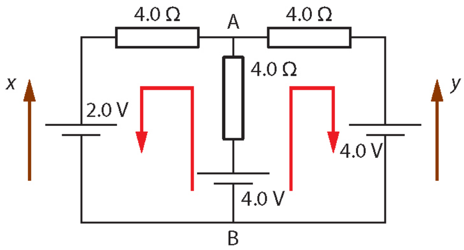
This confirms the existence of isotopes. ✓

16 **a** A and B take the same current and so are equally bright. ✓
C takes half the current of A and B so is 4 times less bright. ✓

b The potential difference across A and B before C burns out is $\frac{\varepsilon}{2}$ where ε is the emf of the source. ✓
After C burns out the potential difference is still ε so there is no change. ✓

c With B burnt out A will not light. ✓
C still has potential difference ε across it so its brightness will be unaffected. ✓

17 **a** We denote the currents as shown in the diagram. The current in the middle cell is then $x + y$ directed downwards. ✓



Kirchhoff's loop law states that:
$4.0 - 2.0 = -4(x + y) - 4x$ or $4x + 2y = -1$ ✓
$4.0 - 4.0 = -4(x + y) - 4y$ or $x + 2y = 0$ ✓
Solving gives $x = -0.34$ A, $y = 0.17$ A ✓
(This means that the current through the left cell is down (0.34 A) in the middle cell is up (0.14 A) and in the right cell up (0.17 A).)

b $V = 4.0 - 4.0 \times 0.17$ ✓
$V = 3.4$ V ✓

c In the cell to the right and the middle, the power is $P = VI = 4.0 \times 0.17 = 0.68$ W. ✓
In the left cell, the power is $P = VI = 2.0 \times (-0.34) = -0.68$ W. ✓
The negative sign indicates that this cell is being charged. ✓

Answers to exam-style questions

Topic 6

Where appropriate, 1 ✓ = 1 mark

1 A

2 C

3 B

4 C

5 C

6 B

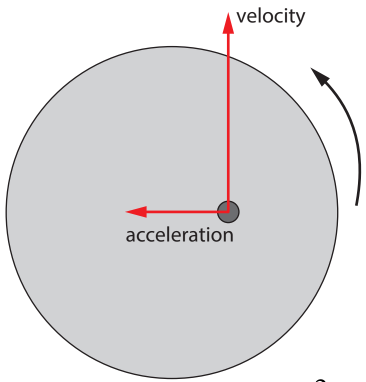
7 D

8 D

9 C

10 A

11 **a** Velocity arrow. ✓
Acceleration arrow. ✓



b The angular speed is $\omega = \frac{2\pi}{1.40} = 4.488 \approx 4.5\ \text{rad s}^{-1}$. ✓

The linear speed is $v = \omega r = 4.488 \times 0.22 = 0.987 \approx 0.99\ \text{m s}^{-1}$. ✓

c At maximum distance the frictional force will be the largest possible, i.e. $f_{\text{max}} = \mu_s N = \mu_s mg (= 0.434\ \text{N})$. ✓

$\mu_s mg = m\frac{v^2}{r} = m\frac{\omega^2 r^2}{r}$, hence $r = \frac{\mu_s g}{\omega^2}$ ✓

$r = \frac{0.82 \times 9.8}{4.488^2} = 0.399 \approx 0.40\ \text{m}$ ✓

d **i** Using $r = \frac{\mu_s g}{\omega^2}$ we find $\omega = \sqrt{\frac{\mu_s g}{r}}$ ✓

$\omega = \sqrt{\frac{0.82 \times 9.8}{0.22}} = 6.0\ \text{rad s}^{-1}$ ✓

ii The static frictional force can no longer supply the larger centripetal force required. ✓
The body will then slide and the static frictional force is now replaced by the even smaller sliding frictional force; hence the disc will slide off the rotating platform. ✓

12 a From energy conservation: $\frac{1}{2}mv^2 = mgL$ so $v = \sqrt{2gL}$, ✓

$v = \sqrt{2 \times 9.8 \times 2.0} = 6.26 \approx 6.3\ \text{m s}^{-1}$. ✓

b $a = \frac{v^2}{L} = \frac{6.26^2}{2.0} = 19.6 \approx 20\ \text{m s}^{-2}$. ✓

c Weight vertically downwards. ✓
Larger arrow for tension upwards. ✓

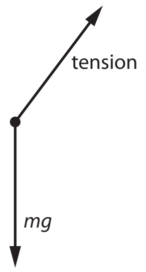
d i A particle is in equilibrium if it moves with constant velocity. ✓
This particle moves on a circle and so cannot be in equilibrium. ✓

ii $T - mg = \frac{mv^2}{L}$ ✓

$T = \frac{mv^2}{L} + mg = \frac{5.0 \times 6.26^2}{2.0} + 5.0 \times 9.8 = 147 \approx 150\ \text{N}$ ✓

(or better: $T = \frac{mv^2}{L} + mg = \frac{m \times 2gL}{L} + mg = 3mg = 3 \times 5.0 \times 9.8 = 147 \approx 150\ \text{N}$)

13 a Correct arrows for tension. ✓
Correct arrow for weight. ✓



b A particle is in equilibrium if it moves with constant velocity. ✓
This particle moves on a circle and so cannot be in equilibrium. ✓

c i The vertical component of the tension equals the weight and so $T\cos\theta = mg$, i.e. $T = \frac{mg}{\cos\theta}$. ✓

The horizontal component of the tension is $T\sin\theta$ and $T\sin\theta = m\frac{v^2}{r} = m\frac{v^2}{L\sin\theta}$ ✓

Combining gives the answer $v = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}$.

ii The angular and linear speeds are related by $v = \omega r = \omega L\sin\theta$. ✓

So $\omega = \frac{\sqrt{\frac{gL\sin^2\theta}{\cos\theta}}}{L\sin\theta}$. ✓

Which is the answer $\omega = \sqrt{\frac{g}{L\cos\theta}}$.

d i $v = \sqrt{\frac{9.8 \times 0.45 \times \sin^2 60°}{\cos 60°}} = 2.57 \approx 2.6\ \text{m s}^{-1}$ ✓

ii $\theta = \sqrt{\frac{9.8}{0.45 \times \cos 60°}} = 6.5997 \approx 6.6\ \text{rad s}^{-1}$ ✓

e i The air resistance force will reduce the speed of the ball. ✓

ii A graph of $\frac{\sin^2\theta}{\cos\theta}$ shows that because the speed decreases, the angle will also decrease. ✓

iii The cosine of the angle will increase and hence the angular speed will decrease. ✓
(**Note:** These questions are best answered by considering the total energy of the ball:

$$E = \frac{1}{2}mv^2 + mgh = \frac{1}{2}m\frac{gL\sin^2\theta}{\cos\theta} + mgL(1-\cos\theta) = \frac{1}{2}mgL\left(\frac{\sin^2\theta + 2\cos\theta - 2\cos^2\theta}{\cos\theta}\right)$$

The air resistance will reduce the total energy; graphing the total energy as a function of angle θ shows that for the energy to decrease the angle must decrease.)

14 a Measuring distances from the top of the sphere and using energy conservation shows that:

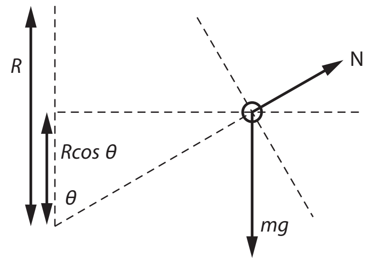
$0 = \frac{1}{2}mv^2 - mgh$ where h is the vertical distance the marble falls. ✓

From trigonometry: $h = R(1-\cos\theta)$. ✓ (see diagram that follows in **b**)

And so $0 = \frac{1}{2}mv^2 - mgR(1-\cos\theta)$. ✓

Manipulating gives $v = \sqrt{2gR(1-\cos\theta)}$.

b The forces on the marble are the weight mg and the normal reaction force N:



Taking components of the weight gives $mg\cos\theta - N = \frac{mv^2}{R}$. ✓

Hence $N = mg\cos\theta - \frac{mv^2}{R}$. ✓

Substituting the expression for the speed from above gives $N = mg\cos\theta - 2mgR(1-\cos\theta)$. ✓
And the result $N = mg(3\cos\theta - 2)$ follows.

c The marble will lose contact when $N \to 0$, i.e. when $\cos\theta = \frac{2}{3}$ or $\theta \approx 48°$. ✓

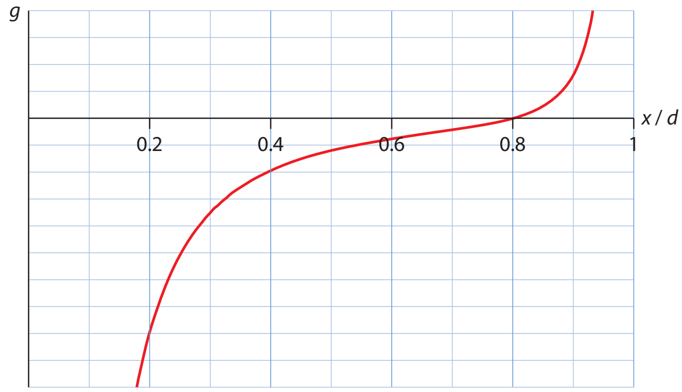
15 a Calling this distance x we have that:

$\frac{G16M}{x^2} = \frac{GM}{(d-x)^2}$ ✓

$16(d-x)^2 = x^2$ or $4(d-x) = \pm x$ ✓

Only the plus sign gives a positive distance and so $x = \frac{4d}{5}$. ✓

b Correct sign. ✓
Correct intersection. ✓
(The negative of this graph is also acceptable)

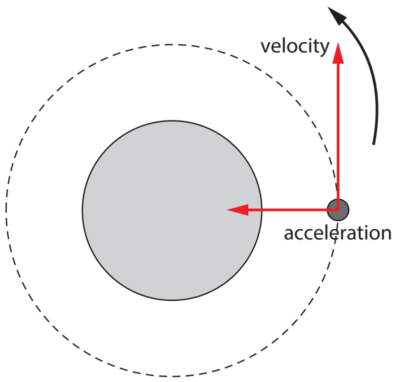


c **i** The force is zero. ✓

ii The force from the larger mass will be larger because the particle will be closer to it. ✓
Hence the net force will be directed towards the large mass. ✓

d It will move to the left. ✓
With increasing speed and increasing acceleration. ✓

16 **a** **i** Velocity arrow. ✓
Acceleration arrow. ✓



ii Acceleration is the rate of change of the velocity vector. ✓
Here the velocity vector is changing because its direction is so we have acceleration. ✓

b The force on the satellite is $\frac{GMm}{r^2} = m\frac{v^2}{r}$ i.e. $\frac{GM}{r} = v^2$. ✓

Using $v = \omega r$, ✓

gives $\frac{GM}{r} = \omega^2 r^2$. ✓

From which the result $\omega^2 r^3 = GM$ follows.

c **i** Since r decreases, from $\omega^2 r^3 = GM$ the angular speed will increase. ✓

ii From $\frac{GM}{r} = v^2$, as r decrease v increases. ✓

d **i** Using $\omega^2 r^3 = GM$ we find $M = \frac{\omega^2 r^3}{G}$ ✓

And so $M = \frac{(5.31\times 10^{-5})^2 \times (2.38\times 10^8)^3}{6.67\times 10^{-11}} = 5.70\times 10^{26}$ kg. ✓

ii Again using $\omega^2 r^3 = GM$ we find $\omega_T^2 r_T^3 = \omega_E^2 r_E^3$. ✓

Hence $\omega_T = \omega_E\sqrt{\frac{r_E^3}{r_T^3}} = 5.31\times 10^{-5} \times \sqrt{\left(\frac{2.38\times 10^8}{1.22\times 10^9}\right)^3} = 4.58\times 10^{-6}$ rad s^{-1} ✓

Hence $T = \frac{2\pi}{\omega_T} = \frac{2\pi}{4.58\times 10^{-6}} = 1.37\times 10^6$ s $= \frac{1.37\times 10^6}{24\times 3600}$ d $= 15.856 \approx 15.9$ d ✓

Answers to exam-style questions

Topic 7

Where appropriate, 1 ✓ = 1 mark

1 B

2 B

3 C

4 A

5 B

6 D

7 C

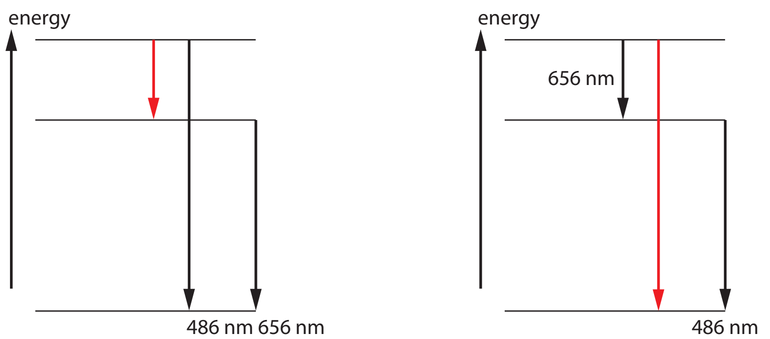
8 D

9 C

10 C

11 **a** The wavelength of a photon is determined by its wavelength (frequency). ✓
Emission spectra show lines at specific wavelengths. ✓
This is consistent with transitions between energy levels of specific energies. ✓

b There are two possibilities as shown in the diagram below. The red line represents the transition with unknown wavelength.



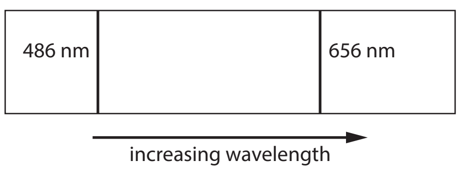
c In the first case the third transition corresponds to an energy difference of

$$\frac{hc}{486\times10^{-9}}-\frac{hc}{656\times10^{-9}}\left(=\frac{6.63\times10^{-34}\times3.0\times10^{8}}{486\times10^{-9}}-\frac{6.63\times10^{-34}\times3.0\times10^{8}}{656\times10^{-9}}=1.06\times10^{-19}\text{ J}\right).\ ✓$$

The third transition has wavelength $\frac{hc}{\lambda}=\frac{hc}{486\times10^{-9}}-\frac{hc}{656\times10^{-9}}\Rightarrow\lambda=1.88\times10^{-6}$ m. ✓

In the second case $\frac{hc}{\lambda}=\frac{hc}{486\times10^{-9}}+\frac{hc}{656\times10^{-9}}\Rightarrow\lambda=2.79\times10^{-7}$ m. ✓

d Only 2 lines shown. ✓
With right wavelengths and in the right order. ✓



12 **a** **i** In order to avoid collisions of the alpha particles with air molecules which would have scattered the alpha particles. ✓

ii To avoid multiple scatterings of alpha particles within the foil. ✓

iii In order to have well defined scattering angles. ✓

b The electrostatic force. ✓

c **i** A very small fraction of the incident alpha particles were scattered at very large scattering angles. ✓

ii This required a very large electric force. ✓

This force could be provided if the positive charge of the atom was concentrated in a very small volume so that the alpha particle could come very close to it. ✓

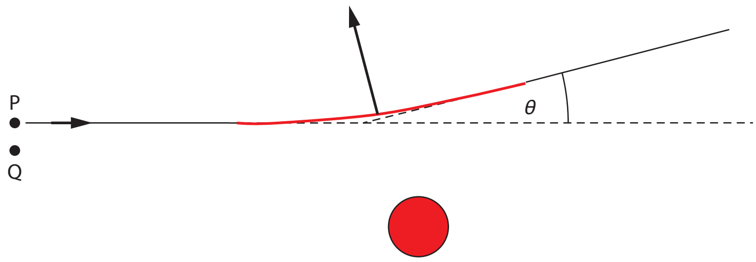
So most of the atomic volume is empty and most of the mass and all the positive charge is concentrated in the tiny nucleus. ✓

d **i** Smooth curve joining incident and scattered path. ✓

ii Extensions of incident and scattered paths. ✓

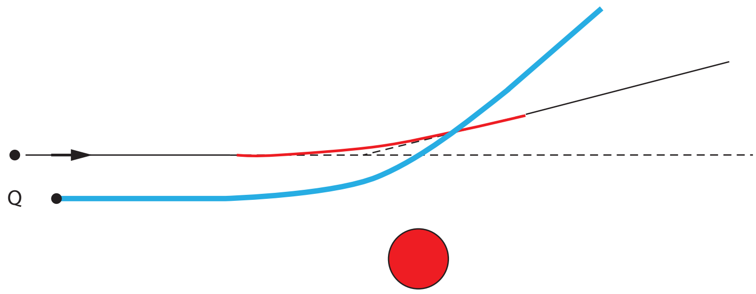
Angle delineated as shown. ✓

iii Arrow as shown passing through centre of nucleus. ✓



e **i** Comes closer to the nucleus. ✓

Has a larger scattering angle. ✓



ii The new nucleus would have the same nuclear charge. ✓

So there would be no difference. ✓

13 **a** **Random**: It is not possible to predict which nucleus will decay. ✓

Or when it will decay. ✓

Spontaneous: A nucleus cannot be prevented from decaying. ✓

The rate of decay cannot be modified in any way. ✓

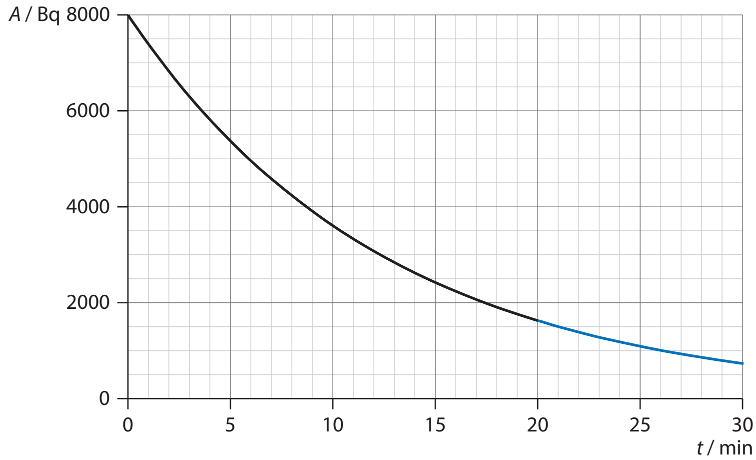
b **i** A nucleus of a specific element (so with a specific atomic number) but with a different number of neutrons (so a different mass number). ✓

ii Locating the point with activity 4000 Bq. ✓

8.7/8.8 min. ✓

iii That the background radiation is negligible. ✓

iv Smooth joining. ✓
To correct value within 1 square at 30 min. ✓



c i $^{225}_{90}\text{Th} \rightarrow {}^{221}_{88}\text{Ra} + {}^{4}_{2}\alpha$
Correct numbers for alpha. ✓
Correct numbers for radium. ✓

ii $\Delta m = 226.024903 - (221.013917 + 4.0026603) = 1.0083$ u ✓
$Q = 1.0083 \times 931.5 = 939$ MeV ✓

d The alpha and the radium nucleus have equal and opposite momenta, each of magnitude p. ✓

The energies are $E_{\text{Ra}} = \dfrac{p^2}{2 \times 221}$ and $E_{\alpha} = \dfrac{p^2}{2 \times 4.0}$. ✓

Hence the alpha to radium energy ratio is energy is $\dfrac{221}{4.0} = 55$. ✓

14 a i $x = 236 - 90 - 143 = 3$ ✓

ii $Q = BE_{\text{right}} - BE_{\text{left}}$ ✓
$BE_{\text{right}} \approx 143 \times 8.4 + 90 \times 8.7 = 1984$ MeV and $BE_{\text{left}} \approx 235 \times 7.6 = 1786$ MeV ✓
$Q = 1984 - 1768 = 198 \approx 200$ MeV ✓

b Because the nuclear force is short range only the immediate neighbours of any given nucleon prevent the nucleon from being ejected from a nucleus. ✓
Nuclei with $A > 20$ are large nuclei with many protons and neutrons so any one nucleon is surrounded by roughly the same number of nucleons. ✓
Since the binding energy per nucleon is a measure of the energy needed to eject one nucleons, this energy is roughly constant. ✓

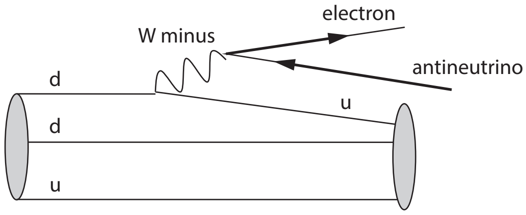
c In fusion we start with light nuclei and produce heavier nuclei. ✓
According to the binding energy curve this increases the binding energy than the reactants and hence energy is released. ✓

15 a A baryon is a particle made of 3 quarks. ✓
Whereas a meson is made of a quark and an antiquark. ✓

b Hadrons correct. ✓
Leptons correct. ✓

	strong	weak
hadrons	✓	✓
leptons		✓

c ddu for neutron. ✓
d to u. ✓
W minus. ✓
Electron. ✓
Antineutrino. ✓



d **i** K and π are mesons so $B = 0$ and p is a baryon so $B = 1$. ✓
To conserve baryon number Σ^- must have B = 1 and so is baryon. ✓

ii The reaction violates strangeness conservation. ✓
And so must happen via the weak interaction since the other interactions conserve strangeness. ✓

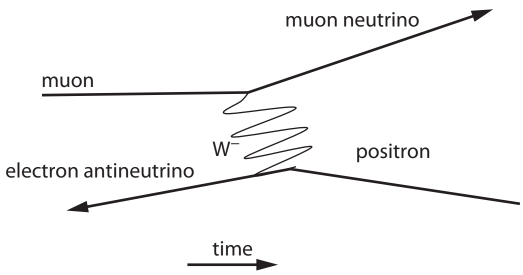
iii In order to conserve family lepton number. ✓
It has to be an electron antineutrino. ✓

16 **a** **i** Similarities: both are leptons/both have charge − 1. ✓
Differences: have different mass/belong to different families. ✓

ii It violates electron lepton number conservation. ✓
It violates muon lepton number conservation. ✓

b **i** Top line: muon neutrino. ✓
Middle line: electron. ✓
Lower line: electron antineutrino. ✓

ii Top vertex correct. ✓
Lower vertex correct. ✓



iii Positron. ✓
Correct neutrinos. ✓
$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

c It has very large mass. ✓

Answers to exam-style questions

Topic 8

Where appropriate, 1 ✓ = 1 mark

1 D

2 B

3 D

4 B

5 D

6 C

7 A

8 C

9 B

10 C

11 a i $\Delta m = 235.044 + 1.009 - (139.922 + 93.915 + 2 \times 1.009) = 0.198$ u ✓
$Q = 0.198 \times 931.5 = 184$ MeV ✓
Which is about 180 MeV.

ii 184 MeV, i.e. $184 \times 10^{6} \times 1.6 \times 10^{-19} = 2.9 \times 10^{-11}$ J are produced by a mass of 235.044 u of uranium, i.e. by about 3.9×10^{-25} kg. ✓

So the specific energy is $\dfrac{2.9 \times 10^{-11}}{3.9 \times 10^{-25}} = 7.4 \times 10^{13}$ J kg^{-1}. ✓

iii The energy produced by the power plant in a year is $800 \times 10^{6} \times 365 \times 24 \times 3600 = 2.52 \times 10^{16}$ J. ✓

The energy produced in the nuclear reactions must then be $\dfrac{2.52 \times 10^{16}}{0.32} = 7.88 \times 10^{16}$ J. ✓

So the mass of uranium used is $\dfrac{7.88 \times 10^{16}}{2.9 \times 10^{-11}} \times 3.9 \times 10^{-25} = 1060 \approx 1100$ kg. ✓

b i The produced neutrons are very fast and cannot be absorbed by uranium nuclei. ✓
Collisions with moderator atoms slow down the neutrons so they can be absorbed. ✓

ii Control rods control the rate of reactions in various part of the reactor core by being lowered or raised from the core. ✓
They absorb neutrons when they are lowered decreasing the rate or increase it when they are raised. ✓

iii The thermal energy is produced in the moderator by collisions of neutrons with moderator atoms. ✓
The heat exchanger removes this energy by, for example circulating cold water through the moderator. ✓

c Without a moderator neutrons would not be slowed down. ✓
And so could not be used to produce fission and no energy would be produced. ✓

d Advantage: very large specific energy of fuels. ✓
Disadvantage: radioactive nuclear waste difficult to dispose of safely. ✓

12 a The mass is $M = \rho V = 1000 \times 4.8 \times 10^{4} \times 38 = 1.8 \times 10^{9}$ kg ✓

b $Mgh = 1.8 \times 10^{9} \times 9.8 \times 225$ ✓
Which equals 4.0×10^{12} J. ✓

c The time to empty the reservoir is $\dfrac{4.8 \times 10^{4} \times 38}{350} = 5.2 \times 10^{3}$ s. ✓

And so the power developed is $\dfrac{4.0 \times 10^{12}}{5.2 \times 10^{3}} = 768 \approx 770$ MW. ✓

d The electrical energy supplied is $0.60\times4.0\times10^{12}\ \text{J}=2.4\times10^{12}\ \text{J}=\dfrac{2.4\times10^{12}}{3.6\times10^{6}}=6.7\times10^{5}\ \text{kWh}$. ✓

So the income from this energy is $6.7\times10^{5}\times0.12=8.0\times10^{4}$ \$. ✓

The cost to refill the reservoir is $\dfrac{4.0\times10^{12}}{0.64\times3.6\times10^{6}}\times0.07=7.3\times10^{4}$ \$ leading to a profit of 7000\$. ✓

13 a Primary energy refers to energy that is available but has not been processed in any way like the kinetic energy of air. ✓
Secondary energy refers to energy that has become available as a result of processing as in the case of electrical energy produced in a wind turbine. ✓

b i All the air incident on the wind turbine has been stopped. ✓

ii Turbulence in the air. ✓
Makes some of the air's kinetic energy "wasted" in eddies resulting in a smaller net power output. ✓

c $P=\frac{1}{2}\pi\rho_1R^2v_1^3-\frac{1}{2}\pi\rho_2R^2v_2^3$ ✓

$P=\frac{1}{2}\pi\times1.2\times12^2\times8.2^3-\frac{1}{2}\pi\times1.9\times12^2\times5.3^3=8.568\times10^4\ \text{W}$ ✓

The extracted power is then $0.30\times8.568\times10^4=2.6\times10^4\ \text{W}$. ✓

14 a The air above the land is very warm, ✓
and so rises, ✓
giving its place to cooler air form the sea. ✓

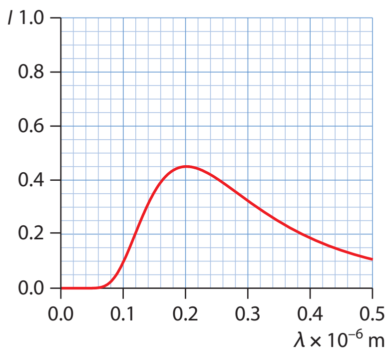
b The air has smaller thermal conductivity than water (and one usually wears clothes when walking!). ✓
And so heat is removed from the body faster in water. ✓

c i A black body is a theoretical body that absorbs all the radiation incident on it. ✓
Reflecting none. ✓

ii The peak wavelength is 2.0×10^{-7} m. ✓

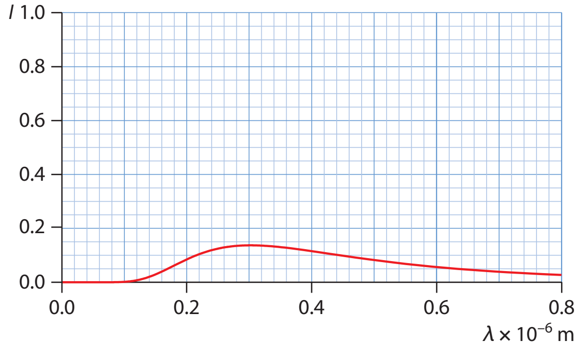
And by Wien's law $T=\dfrac{2.9\times10^{-3}}{2.0\times10^{-7}}=1.4\times10^{4}$ K. ✓

d i Same peak wavelength. ✓
Half the max height. ✓



ii Peak wavelength shifted to 3.0×10^{-7} m. ✓
Lower in height. ✓

(but ignore amount by which peak is reduced – the area under this curve must be $\left(\frac{3}{2}\right)^4 \approx 5$ times smaller than the original curve)



15 a i σT_1^4 ✓

ii $e\sigma T_2^4$ ✓

iii $e\sigma T_1^4$ ✓

iv $(1-e)\sigma T_1^4$ ✓

b The net power leaving the black body is $\sigma T_1^4 - e\sigma T_2^4 - (1-e)\sigma T_1^4 = \sigma(T_1^4 - T_2^4)$. ✓

At equilibrium this is zero and so $T_1 = T_2$. ✓

16 a i Intensity is the power received per unit area. ✓

The power radiated is received over an area $4\pi d^2$. ✓

Which gives the result.

ii Albedo is the ratio of the scattered intensity to the incident intensity of radiation. ✓

b i Radiation that falls on the earth surface has to pass through a disc of radius R and so the power through the disc is $\pi R^2 S$. ✓

Of this a fraction α is reflected and so the incident power is $\pi R^2 S(1-\alpha)$. ✓

The average power per unit area is then $\frac{\pi R^2 S(1-\alpha)}{4\pi R^2}$. ✓

Which, after simplification, is the result.

ii $S = \frac{3.9 \times 10^{26}}{4\pi \times (1.5 \times 10^{11})^2} = 1379\ \text{W m}^{-2}$ ✓

$\frac{S(1-\alpha)}{4} = \sigma T^4 \Rightarrow T = \sqrt[4]{\frac{S(1-\alpha)}{4\sigma}} = \sqrt[4]{\frac{1379 \times (1-0.30)}{4 \times 5.67 \times 10^{-8}}} = 256\ \text{K}$ ✓

c The calculation ignores the greenhouse effect i.e. that greenhouse gases in the atmosphere absorb infrared radiation radiated by the earth. ✓

The gases subsequently re-radiate this radiation in all directions. ✓

Including back down to the surface of the earth warming it further. ✓

Answers to exam-style questions

Topic 9

Where appropriate, 1 ✓ = 1 mark

1 B

2 D

3 A

4 D

5 A

6 B

7 A

8 C

9 A (The question should have referred to the wavelength in air)

10 C

11 **a** In simple harmonic motion the acceleration is opposite to and proportional to the displacement from the equilibrium position. ✓
This means that a graph of acceleration against time should a straight line through the origin with a negative slope. ✓
Which is what this graph is. ✓

b **i** The amplitude is 2.6 cm. ✓

ii The gradient is $-\omega^2 = -\dfrac{12}{5.2\times10^{-2}} \Rightarrow \omega = 15.19\ \text{rad s}^{-1}$ ✓

$\omega = 2\pi f \Rightarrow f = \dfrac{15.19}{2\pi} = 2.4\ \text{Hz}$ ✓

c **i** $E_{\text{max}} = \dfrac{1}{2}m\omega^2 x_0^2 = \dfrac{1}{2}\times0.25\times15.19^2\times(2.6\times10^{-2})^2$ ✓

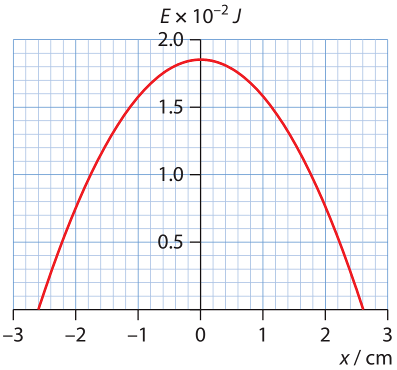
$E_{\text{max}} = 1.9479\times10^{-2} \approx 1.9\times10^{-2}\ \text{J}$ ✓

ii $E_K = E_P \Rightarrow E_K = \dfrac{1}{2}E_{\text{max}}$ ✓

$E_K = \dfrac{1}{2}\times1.9497\times10^{-2} = 9.75\times10^{-3}\ \text{J}$ ✓

$\dfrac{1}{2}\times0.25\times v^2 = 9.75\times10^{-3} \Rightarrow v = \sqrt{\dfrac{2\times9.75\times10^{-3}}{0.25}} = 0.279 \approx 0.30\ \text{m s}^{-1}$ ✓

d Correct shape of parabola. ✓
Correct intercepts. ✓



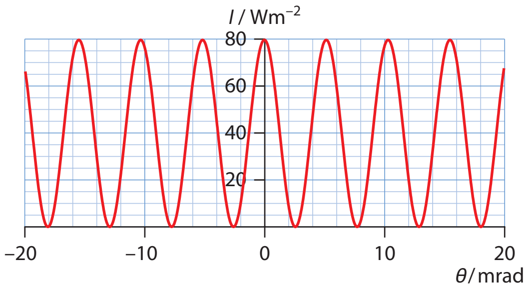
12 a i Light diffracting from each slit arrives at the screen. ✓
At those positions where the phase difference between the 2 waves is 0 the resulting amplitude is twice that of the wave from just one slit and we have bright fringes (constructive interference). ✓

ii The separation of the bright fringes is given by $s = \frac{\lambda D}{d}$ and so $\lambda = \frac{sd}{D}$. ✓

$$\lambda = \frac{1.86\times10^{-2}\times0.120\times10^{-3}}{3.60}$$ ✓

$\lambda = 6.20\times10^{-7}$ m ✓

b Correct overall shape. ✓
Correct peak intensity. ✓
Correct separation of fringes. ✓



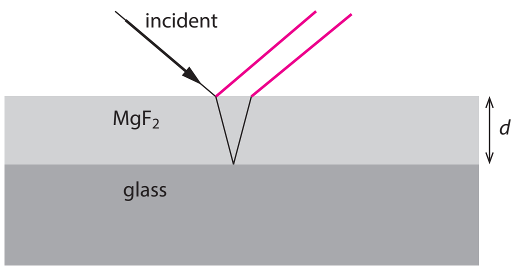
c i $d\sin\theta = n\lambda \Rightarrow d = \frac{n\lambda}{\sin\theta} = \frac{2\times6.2\times10^{-7}}{\sin58°} = 1.462\times10^{-6}\text{ m} = 1.462\times10^{-3}\text{ mm}$ ✓

Hence number of rulings per mm is $\frac{1}{1.462\times10^{-3}}$=684 ✓

ii We must have that $1.462\times10^{-6}\times\sin58° = n\lambda$ so that $n\lambda = 1.2398\times10^{-6}$ m. ✓
n = 1 does not lead to a visible wavelength. ✓

We cannot have n = 2 so we try n = 3 to find $\lambda = \frac{1.2398\times10^{-6}}{3} = 4.13\times10^{-7}$ m which fits the visible spectrum. ✓
No other value of n gives a visible wavelength. ✓

13 a Parallel reflected rays in red. ✓
Correct refraction of one of the rays. ✓



b At reflection point between air and magnesium fluoride. ✓
At reflection point between magnesium fluoride and glass. ✓

c At normal incidence the path difference is $2d$ and the phase difference due to reflection is zero. ✓

Hence for destructive interference $2dn = (m+\frac{1}{2})\lambda$. ✓

Giving for the least thickness (m = 0) $d = \frac{\lambda}{4n} = \frac{5.0\times10^{-7}}{4\times1.38} = 9.1\times10^{-8}$ m. ✓

14 **a** The number of secondary maximum is 2 less than the number of slits. ✓

And we have 2 secondary maxima. ✓

b **i** The secondary maxima becomes less pronounced. ✓

The primary maxima become brighter. ✓

The primary maxima become narrower. ✓

ii The separation between the primary maxima increases. ✓

c The average wavelength is $\dfrac{656.45+656.27}{2}=656.36$ nm. ✓

From $\dfrac{\lambda}{\Delta\lambda}=mN$ we have that $\dfrac{656.36}{656.45-656.27}=2\times N$. ✓

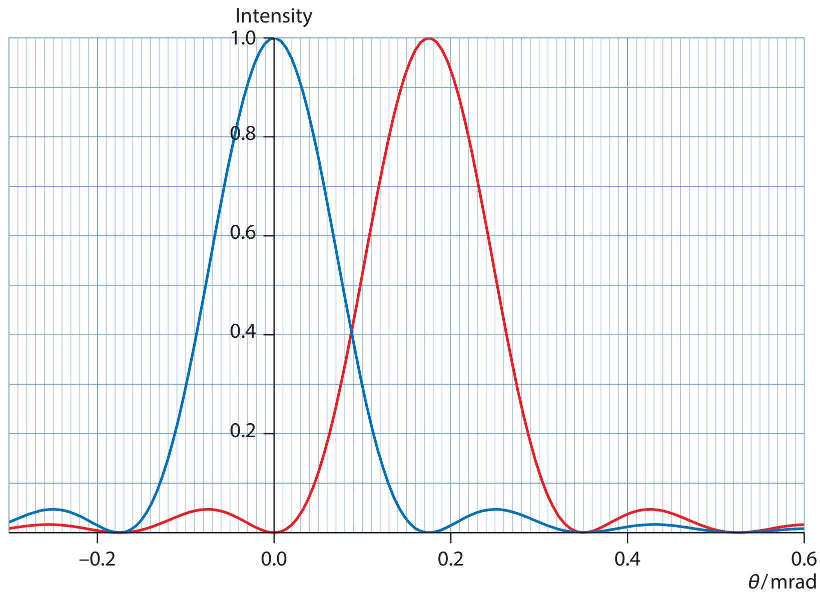
$N=1823$ ✓

15 **a** The first minimum is at 0.175 mrad. ✓

And so from $\theta=1.22\dfrac{\lambda}{b}$ we find $b=1.22\dfrac{\lambda}{\theta}=1.22\times\dfrac{5.0\times10^{-7}}{0.175\times10^{-3}}=3.49\times10^{-3}$ m. ✓

b **i** Same shape. ✓

With maximum coinciding with first minimum of the other pattern. ✓



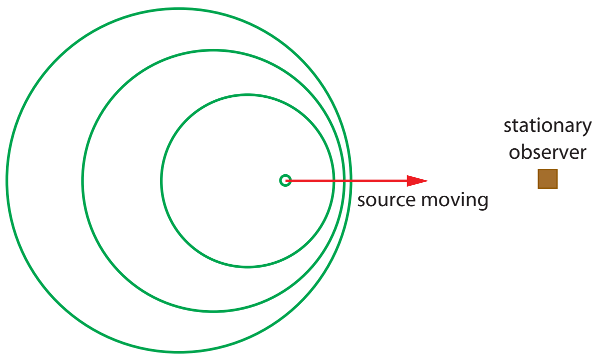
ii The angular separation of the two sources is $\dfrac{3.0\times10^{-2}}{D}$ where D is their distance from the slit. ✓

According to Rayleigh, $\dfrac{3.0\times10^{-2}}{D}=0.175\times10^{-3}$ giving $D=\dfrac{3.0\times10^{-2}}{0.175\times10^{-3}}=171\approx170$ m. ✓

16 **a** The change in observed frequency when there is relative motion between the source and the observer. ✓

b Circular wavefronts. ✓

Bunching in front of the source. ✓



c Ultrasound is directed at moving particles in the blood stream and the reflection is recorded. ✓
From the frequency shift it is possible to measure the speed of blood flow. ✓

d The speed of the point on the disc is $\frac{2\pi \times 0.20}{\frac{1}{8}} = 10.0\ \text{m s}^{-1}$. ✓

The frequencies received range from $\frac{340}{340+10} \times 2400\ \text{Hz} = 2331 \approx 2300\ \text{Hz}$ when source moves away from observer, ✓

to $\frac{340}{340-10} \times 2400\ \text{Hz} = 2473 \approx 2500\ \text{Hz}$ when source moves towards the observer. ✓

The wavelengths correspondingly vary from $\frac{340}{2473} = 0.137 \approx 0.14\ \text{m}$ to $\frac{340}{2331} = 0.146 \approx 0.15\ \text{m}$. ✓

Answers to exam-style questions

Topic 10

Where appropriate, 1 ✓ = 1 mark

1 C

2 C

3 C

4 C

5 C

6 C

7 D

8 B

9 C

10 A

11 **a** The potential at the surface is $V = -\frac{GM}{R} = -5.0 \times 10^{12}\ \text{J kg}^{-1}$. ✓

And so $M = -\frac{VR}{G} = \frac{5.0 \times 10^{12} \times 2.0 \times 10^{5}}{6.67 \times 10^{-11}} = 1.5 \times 10^{28}$ kg. ✓

b The potential energy at launch on the surface of the planet is mV. ✓

And so the total energy at launch is $\frac{1}{2}mv^2 + mV$. ✓

At the escape speed the total energy has to be zero. ✓

And the result follows.

c $v = \sqrt{-2V} = \sqrt{2 \times 5.0 \times 10^{12}}$ ✓

Which equals $v = 3.2 \times 10^{6}\ \text{m s}^{-1}$. ✓

d The work required is $W = m\Delta V$ with $\Delta V = (-1.2 \times 10^{12} - (-5.0 \times 10^{12}) = 3.8 \times 10^{12}\ \text{J kg}^{-1}$. ✓

And this is $W = 1500 \times 3.2 \times 10^{12} = 5.7 \times 10^{15}$ J. ✓

e The additional energy needed is the kinetic energy: from $\frac{mv^2}{r} = \frac{GMm}{r^2}$ we find $E_K = \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}mV$ where V is the potential at the position of the probe. ✓

And this is $E_K = -\frac{1}{2} \times 1500 \times (-1.2 \times 10^{12}) = 9.0 \times 10^{14}$ J. ✓

f The potential at the release point is $V_1 = -2.2 \times 10^{12}\ \text{J kg}^{-1}$ and from conservation of energy

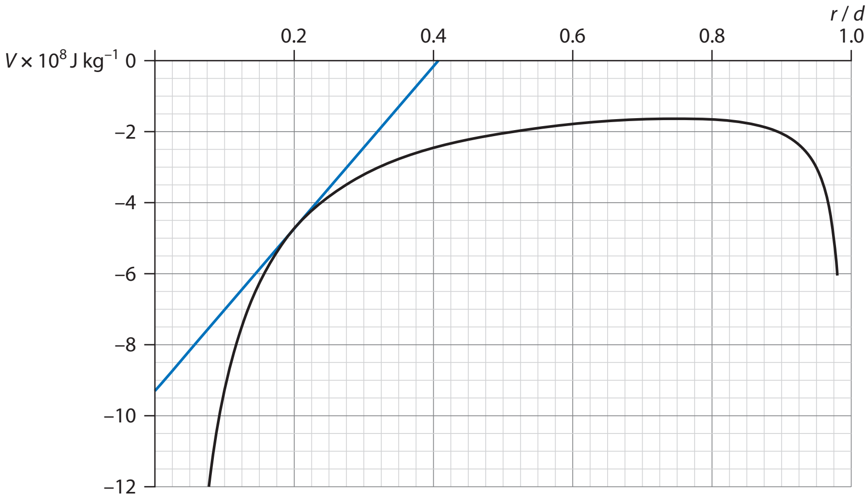
$mV_1 = mV_2 + \frac{1}{2}mv^2$ where is the potential at the surface. ✓

Hence $v = \sqrt{2(V_1 - V_2)} = \sqrt{2(-2.2 \times 10^{12} - (-5.0 \times 10^{12})} = 2.4 \times 10^{6}\ \text{m s}^{-1}$. ✓

12 a The slope of the tangent is gravitational field strength. ✓

Draw a tangent at the point with $\frac{r}{d} = 0.20$. ✓

Evaluate slope to be $g = \dfrac{0-(-9.2\times10^{8})}{(0.41-0)\times4.8\times10^{8}} \approx 4.7\ \text{N kg}^{-1}$. ✓



b The gravitational potential has zero slope there. ✓

Which implies that the gravitational field strength is zero at that point. ✓

c $g = \dfrac{GM}{r_1^2} - \dfrac{Gm}{r_2^2}$ ✓

$0 = \dfrac{GM}{0.75^2} - \dfrac{Gm}{0.25^2}$ ✓

Giving $\dfrac{M}{m} = \dfrac{0.75^2}{0.25^2} = 9.0$ ✓

13 a $qV_1 + \frac{1}{2}mv^2 = qV_2$ i.e. $q\dfrac{kQ}{r_1} + \frac{1}{2}mv^2 = q\dfrac{kQ}{r_2}$ ✓

$2.4\times10^{-6}\times\dfrac{8.99\times10^{9}\times8.8\times10^{-6}}{0.75} + \dfrac{1}{2}\times0.0075\times3.2^2 = 2.4\times10^{-6}\times\dfrac{8.99\times10^{9}\times8.8\times10^{-6}}{r_2}$ ✓

$0.2532 + 0.3840(= 0.6372) = \dfrac{0.1899}{r_2}$

Hence $r_2 = 0.2980 \approx 0.30$ m. ✓

b The pellet will move radially away from the sphere. ✓

With an increasing speed but a decreasing acceleration. ✓

c The total energy of the pellet is 0.6372 J and far away this will turn into kinetic energy. ✓

Hence $\frac{1}{2}\times0.075\times v^2 = 0.6372$ J leading to $4.1\ \text{m s}^{-1}$. ✓

14 a $qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2qV}{m}}$ ✓

Hence $v = \sqrt{\dfrac{2\times1.6\times10^{-19}\times29.1}{9.11\times10^{-31}}} = 3.197\times10^{6} \approx 3.2\times10^{6}\ \text{m s}^{-1}$. ✓

b The horizontal distance of 2.0 cm is covered at the constant speed found above. ✓

And so $x = vt \Rightarrow t = \dfrac{x}{v} = \dfrac{0.020}{3.197\times10^{6}} \approx 6.3\times10^{-9}$ s. ✓

c The vertical distance covered is $y = \frac{1}{2}at^2 \Rightarrow a = \frac{2y}{t^2} = \frac{2 \times 0.25 \times 10^{-2}}{(6.3 \times 10^{-9})^2} \approx 1.3 \times 10^{14}\ \text{m s}^{-2}$. ✓

And from $qE = ma$ we find $E = \frac{ma}{q} = \frac{9.11 \times 10^{-31} \times 1.3 \times 10^{14}}{1.6 \times 10^{-19}} \approx 740\ \text{N C}^{-1}$. ✓

d The vertical component of velocity at B is $v_y = at = 1.3 \times 10^{14} \times 6.3 \times 10^{-9} \approx 8.2 \times 10^5\ \text{m s}^{-1}$. ✓

Hence $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{8.2 \times 10^5}{3.2 \times 10^6} \approx 14°$. ✓

e The work done is the change in kinetic energy. ✓

Which is $\Delta E_K = \frac{1}{2}mv_y^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (8.2 \times 10^5)^2 = 6.3 \times 10^{-17}$ J. ✓

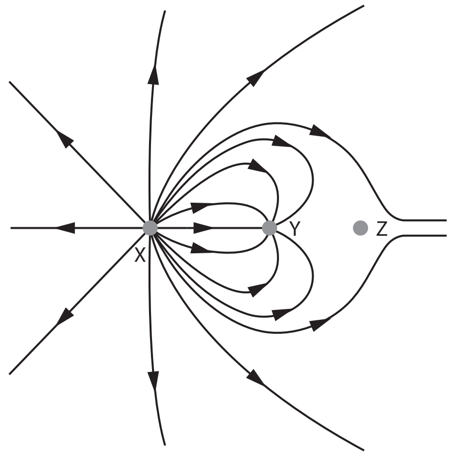
f The work done is also $W = q\Delta V$ and so $\Delta V = \frac{W}{q} = \frac{6.3 \times 10^{-17}}{1.6 \times 10^{-19}} = 394 \approx 390$ V. ✓

15 **a** Field lines are mathematical lines originating and ending in electric charges. ✓
Tangents to these lines give the direction of the electric field at a point. ✓

b They leave from positive charges (or infinity) and end in negative charges (or infinity). ✓
They cannot cross. ✓
Their density is proportional to the electric field strength. ✓

c X is positive and Y is negative. ✓

d **i** The field is zero at a position that may be approximated by Z. ✓



ii The ratio of the distance of Z from X to the distance from Y is about 2.5. ✓

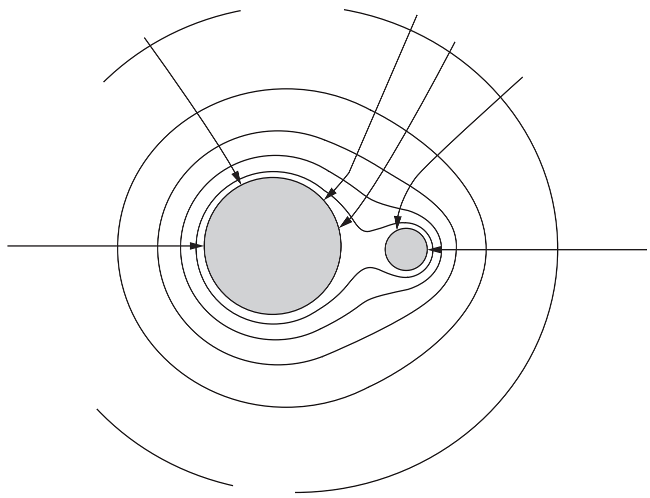
Hence from $0 = \frac{kQ_X}{r_1^2} - \frac{kQ_Y}{r_2^2}$ we find $\frac{Q_X}{Q_Y} = \frac{r_1^2}{r_2^2} = 2.5^2 \approx 6$. ✓

16 a i An equipotential surface is the set of all points that have the same potential. ✓

b i Field lines normal to equipotentials. ✓

And normal to spheres. ✓

(plus symmetrically paced lines one the lower side)



ii The potential difference between A and B is $\Delta V = 2.0 \times 10^6 \text{ J kg}^{-1}$. ✓

And so the work done is $m\Delta V = 1500 \times 2.0 \times 10^6 = 3.0 \times 10^9$ J. ✓

iii $g \approx \dfrac{\Delta V}{\Delta r}$ ✓

$g \approx \dfrac{10^6}{4.0 \times 10^6} = 0.25 \text{ N kg}^{-1}$ ✓

iv From a very large distance away the two bodies look like one point particle. ✓

And the equipotential surfaces of a single particle are spherical. ✓

c The potential; lines shown correspond to two masses so they are defined by $-\dfrac{GM_1}{r_1} - \dfrac{GM_2}{r_2} =$ constant, or just

$-\dfrac{M_1}{r_1} - \dfrac{M_2}{r_2} =$ constant. ✓

Two positive charges or two negative charges would give equipotential lines defined by

$-\dfrac{Q_1}{r_1} - \dfrac{Q_2}{r_2} =$ constant. ✓

And so would be the same as in the gravitational case. ✓

Answers to exam-style questions

Topic 11

Where appropriate, 1 ✓ = 1 mark

1 Constant and counter-clockwise. There is an error in the options for this question.

2 D

3 A

4 B

5 B

6 B

7 C

8 A

9 D

10 D

11 **a** As the magnet gets closer to the top of the coil the magnetic field at the coil increases. ✓
Hence the magnetic flux though the coil increases. ✓
By Faraday's law, a changing flux indices an emf. ✓

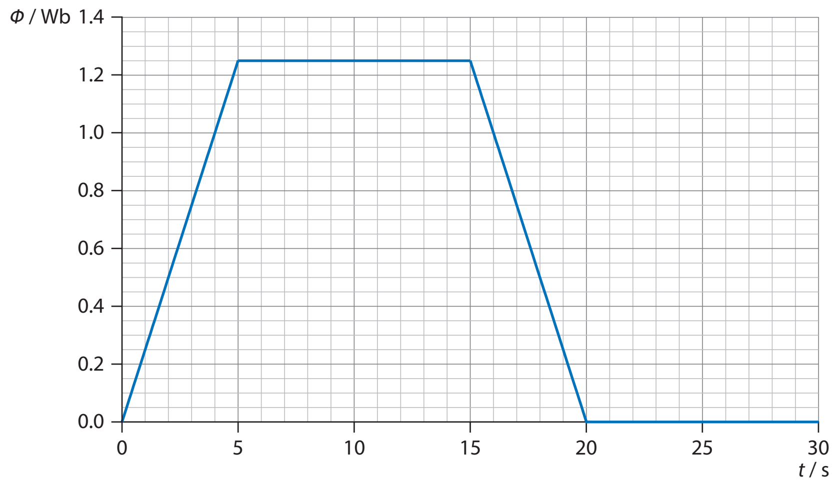
b **i** From C to D the magnet is moving faster than from A to B. ✓
Hence the rate of change of flux, and therefore emf, is higher. ✓

ii Since the magnet moves faster it takes less time to move past the magnet. ✓

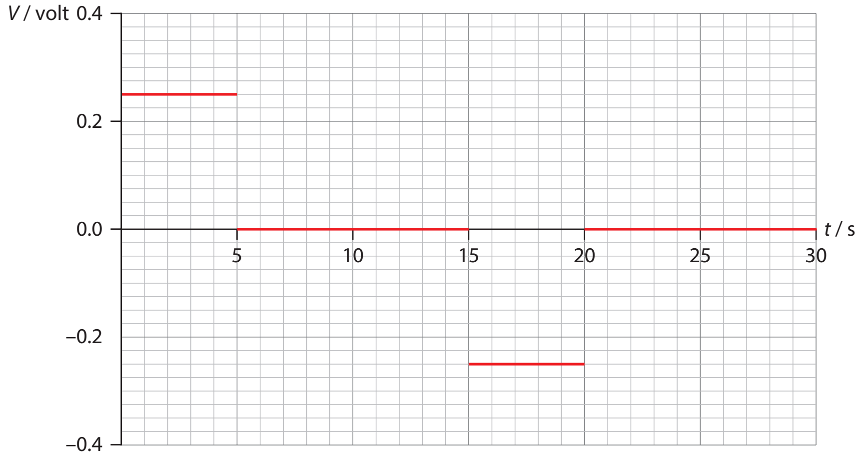
c **i** The graph is a graph of emf versus time, i.e. $\frac{d\Phi}{dt}$ versus time. ✓
So the area is the change in flux. ✓

ii The area from A to B is the change in flux from when the magnetic is very far away until it is essentially in the middle of the coil. ✓
The area from C to D is the exact opposite and so the areas are the same (in magnitude). ✓

12 **a** **i** Correct shape. ✓
Correct values of time. ✓
Correct values of flux. ✓



ii Correct shape. ✓
Correct values of time. ✓
Correct values of voltage. ✓



b **i** The induced current is $\frac{0.25}{0.75} = 0.333$ A. ✓

The magnetic force acting on the loop while entering or leaving the region of magnetic field is $F = NBIL = 50 \times 0.40 \times 0.333 \times 0.25 = 1.665$ N. ✓

Hence the power is pushing the loop through is $P = Fv = 1.665 \times 0.050 = 0.83$ W. ✓

ii This is power that is dissipated as thermal energy. ✓
In the cables of the coil. ✓

13 **a** **i** From $P = VI$ the current is he current is $I = \frac{P}{V} = \frac{120 \times 10^3}{240} = 500$ A. ✓

And so the power lost in the cables is $P = RI^2 = 0.80 \times 500^2 = 200$ kW. ✓

ii The power that must be supplied by the wind generator is 320 kW. ✓

And so the voltage is $V = \frac{P}{I} = \frac{320 \times 10^3}{500} = 640$ V. ✓

iii The efficiency is $e = \frac{\text{useful power}}{\text{input power}} = \frac{120}{320} = 0.375 \approx 0.38$. ✓

b The current would be 10 smaller. ✓
And so the power loss 100 times smaller i.e. 2.0 kW. ✓

c **i** The peak voltage is 340 V and so the rms voltage is $\frac{340}{\sqrt{2}} = 240.4 \approx 240$ V. ✓

ii $\bar{P} = V_{rms} I_{rms} = 18 \times 10^3$ W hence $I_{rms} = \frac{18 \times 10^3}{240} = 75$ A. ✓

Hence $I_{peak} = 75 \times \sqrt{2} = 106 \approx 110$ A. ✓

d **i** The alternating current in the primary coil produces an alternating magnetic field. ✓
The iron core confines the magnetic field lines within the core and hence into the secondary. ✓
Because the field is alternating the magnetic flux in the secondary coils varies with time. ✓
And hence by Faraday's law an emf is induced in the secondary coil. ✓

ii The magnetic field in the core creates small currents in the core by exerting magnetic forces on electrons. ✓
These currents dissipate energy as thermal energy in the core due to collisions with the core atoms. ✓

14 a i Capacitance is the charge per unit voltage that can be stored on one of the capacitor plates. ✓

ii Capacitors definitely store energy (which, for example, can be used to light up a light bulb connected to the capacitor as it discharges through the bulb). ✓

Whether it can store charge is a question of definition: the net charge is zero since the plates have equal and opposite charge so in that sense it does not store charge but it does store equal and opposite charges on each plate. ✓

b X and Y are in parallel so they correspond to a total capacitance of 360 pF. ✓

This and Z are in series so they correspond t an overall total of $\frac{1}{360}+\frac{1}{180}=\frac{3}{360}=\frac{1}{120}$, i.e. 120 pF. ✓

c i The charge on a plate of the total capacitor is $Q = C_{\text{total}}V = 120\times10^{-12}\times12 = 1.44\times10^{-9}$ C. ✓

And this is the same as the charge on Z. ✓

ii $V = \frac{Q}{C_Z} = \frac{1.44\times10^{-9}}{180\times10^{-12}} = 8.0$ V ✓

iii The potential difference across X is 4.0 V. ✓

And the charge is then $Q = C_X V = 180\times10^{-12}\times4.0 = 7.2\times10^{-10}$ C. ✓

15 a An ideal voltmeter has infinite resistance. ✓

And so no charge can move through it, hence no current. ✓

b i $C = \varepsilon_0\frac{A}{d} = 8.85\times10^{-12}\times\frac{0.68}{4.0\times10^{-3}}$ ✓

$C = 1.5\times10^{-9}$ F ✓

ii $Q = CV = 1.5\times10^{-9}\times9.0$ ✓

$Q = 1.35\times10^{-8}$ C ✓

iii $E = \frac{1}{2}CV^2 = \frac{1}{2}\times1.5\times10^{-9}\times9.0^2$ ✓

$E = 6.1\times10^{-8}$ J ✓

c i The charge cannot change since the ideal voltmeter prevents any motion of charge in the circuit. ✓

ii The charge in the dielectric will separate. ✓

Creating a small electric field in the dielectric directed opposite to the original electric field. ✓

Since the net electric field in between the plates has decreased, the potential difference must also decrease. ✓

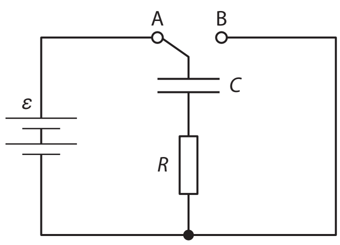
iii Since the potential difference decreased and the charge remained the same. ✓

The capacitance increased. ✓

16 a A circuit with 2 loops. ✓

C in series with R. ✓

Switch and battery in correct position. ✓



b 12 nC. ✓

c **i** $Q = CV \Rightarrow C = \frac{Q}{V}$ ✓

$C = \frac{12 \times 10^{-9}}{6.0} = 2.0 \times 10^{-9}$ F ✓

ii The work required to move deposit 12 nC on the capacitor plate is $W = qV = 12 \times 10^{-9} \times 3.0 = 3.6 \times 10^{-8}$ J. ✓

Since the average voltage is 3.0 V. ✓

iii $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.0 \times 10^{-9} \times 6.0^2$ ✓

$E = 3.6 \times 10^{-8}$ J ✓

iv The two energies are the same. ✓

As they must be by energy conservation. ✓

d The time constant for the circuit is $\tau = RC = 2.5 \times 10^6 \times 2.0 \times 10^{-9} = 5.0 \times 10^{-3}$ s. ✓

From $q = q_0 e^{-\frac{t}{t}}$ we find $e^{-\frac{t}{t}} = \frac{q}{q_0} = \frac{8.0}{12} = 0.667$. ✓

$I = -\frac{q_0}{\tau} e^{-\frac{t}{t}} = -\frac{12 \times 10^{-9}}{5.0 \times 10^{-3}} \times 0.667$ ✓

$I = (-)1.6 \times 10^{-6}$ A ✓

Answers to exam-style questions

Topic 12

Where appropriate, 1 ✓ = 1 mark

1 D

2 C

3 C

4 D

5 B

6 B

7 B

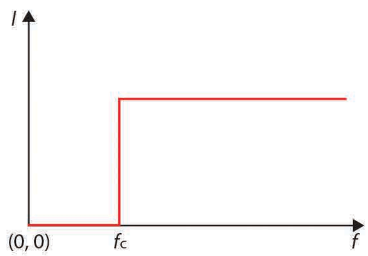
8 A

9 B

10 B

11 a i Photons are the massless particles of light. ✓
Whose energy is given by $E = hf$ where f is the frequency of light and h is Planck's constant. ✓

ii Straight line ✓
Horizontal ✓



iii The photocurrent is the rate of emission of electrons from the photosurface times the electron's charge, $I = eR$ as long as $f > f_c$. ✓
And this is independent of photon frequency or the electron speed. ✓

b i *One of:*
Emission without delay. ✓
Electron energy increases with photon frequency. ✓
Existence of a critical frequency below which no electrons are emitted. ✓

ii *Using the first feature:*
With very weak electromagnetic waves incident on the surface an electron would have to accumulate energy slowly and so would take a long time to leave the metal. ✓
In the photon model of light an electron absorbs all the energy of the photon at once and so there is no delay. ✓
Using the second feature:
The energy of electromagnetic waves does not depend on frequency. ✓
But the energy of a photon increases with frequency. ✓
Using the third feature:
The energy of electromagnetic waves does not depend on frequency. ✓
But the energy of a photon does and if the frequency is low the supplied energy cannot overcome the work function so no electrons are emitted. ✓

c **i** Extending the graph to the vertical intercept gives –3.4 V. ✓
So the work function is 3.4 eV. ✓

ii From $E = hf - \phi$ and $E = eV$ we have that $V = \frac{h}{e} f - \frac{\phi}{e}$ and so the gradient of the graph is the Planck constant divided by e. ✓

The gradient is $\frac{8.0 - 0}{3.0 \times 10^{15} - 0.90 \times 10^{15}} = 3.8 \times 10^{-15}\ \text{V Hz}^{-1}$. ✓

And so $h = 1.6 \times 10^{-19} \times 3.8 \times 10^{-15} = 6.1 \times 10^{-34}\ \text{C V Hz}^{-1} = 6.1 \times 10^{-34}\ \text{J s}$. ✓

iii The threshold frequency is 0.90×10^{15} Hz. ✓

And so the maximum wavelength is $\frac{3.0 \times 10^{8}}{0.90 \times 10^{15}} = 3.3 \times 10^{-7}$ m. ✓

d The energy of the emitted electrons does not depend on intensity. ✓
So the graph will not change. ✓

12 **a** The net force on the electron is the electric force of attraction between the electron and the proton i.e. $\frac{ke^2}{r^2}$. ✓

Equating this with the centripetal force $\frac{mv^2}{r}$ gives the answer. ✓

b The Bohr condition is that $mvr = n\frac{h}{2\pi}$. ✓

Squaring gives $m^2v^2r^2 = n^2\frac{h^2}{4\pi^2}$ and substituting the expression from the previous part leads to

$m^2\frac{ke^2}{mr}r^2 = n^2\frac{h^2}{4\pi^2}$. ✓

Simplifying gives $mke^2r = n^2\frac{h^2}{4\pi^2}$ and the answer. ✓

c The total energy of the electron is $E = \frac{1}{2}mv^2 - \frac{ke^2}{r}$. ✓

Substituting the value for the square of the speed in the first part again gives the answer. ✓

d It signifies that the electron is bound to the proton and cannot escape far away unless sufficient energy is provided to it. ✓

e From $\lambda = \frac{h}{p}$ we find $p = \frac{h}{\lambda}$ and so the Bohr condition becomes $\frac{h}{\lambda}r = n\frac{h}{2\pi}$. ✓

Simplifying gives the answer. ✓

f **i** An electron wave is a wave whose amplitude is related to the probability of finding the electron somewhere in space at a given time. ✓

ii The wave corresponds to $n = 4$. ✓

From **b**, $r = n^2\frac{h^2}{4\pi^2 mke^2} = 4 \times \frac{(6.63 \times 10^{-34})^2}{4\pi^2 \times 9.11 \times 10^{-31} \times 8.99 \times 10^{9} \times (1.6 \times 10^{-19})^2}$. ✓

$r = 2.1 \times 10^{-10}$ m. ✓

iii The total energy from **c** is $E = -\frac{ke^2}{2r} = \frac{8.99 \times 10^{9} \times (1.6 \times 10^{-19})^2}{2 \times 2.1 \times 10^{-10}} = 5.5 \times 10^{-19}$ J and this, or more, is what must be supplied. ✓

g The probability wave associated with the electron implies that the electron is not an object that is localised at a particular point at a given time, ✓
but can be thought to be spread out through space like waves do. ✓
The Bohr orbit only gives the average position of the electron. ✓

13 **a** To every particle there corresponds a wave of probability. ✓
With a wavelength that is given by the Planck constant divided by the momentum of the particle. ✓

b **i** $qV = E_K = \frac{p^2}{2m}$ ✓

Hence $p = \sqrt{2mqV}$ and the result follows from $\lambda = \frac{h}{p}$. ✓

ii $\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times120}} = 1.1\times10^{-10}$ m ✓

c In a Davisson–Germer type of experiment electrons that have been accelerated are directed at a crystal from which they diffract and interfere. ✓
From the interference pattern the wavelength may be determined. ✓
And this is consistent with the de Broglie formula. ✓

d The de Broglie wavelength of the bullet is $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{0.080\times420} \approx 2\times10^{-35}$ m. ✓

For diffraction effects to be seen the wavelength must be comparable to the size of the hole. ✓
But 2×10^{-35} m << 5.0 cm. ✓
And so no diffraction will be observed. ✓

14 **a** Tunnelling is a quantum mechanical phenomenon in which particles can be transmitted through energy barriers. ✓
That would classically be impossible due to energy conservation. ✓

b **i** The width is about $2.8\times10^{-10} - 1.3\times10^{-10} = 1.5\times10^{-10}$ m. ✓

ii From the graph the de Broglie wavelength before and after is the same. ✓
And hence the ratio is 1. ✓

iii The wavefunction squared is proportional to the probability of finding a particle somewhere. ✓

And so the transmitted ratio is $\left(\frac{3}{20}\right)^2 \approx 2\times10^{-2}$. ✓

c Protons have a higher mass so fewer of them would get transmitted. ✓

15 **a** **i** The electron antineutrino. ✓

ii Electrically neutral. ✓
Very small non-zero mass. ✓

b If no third particle were present in the products of the beta decay the electron would always carry away a fixed proportion of the total energy released. ✓
But experiments show that this is not the case which means a third particle must be sharing in the energy. ✓

c When the tree dies it will no longer absorb C-14 from its surroundings. ✓
The amount of C-14 present when the tree died will then diminish with time because C-14 is unstable and decays into N-14. ✓

d We may ignore C-14 in this part of the calculation since its concentration is so small. ✓

So 15 g correspond to $\frac{15}{12}\times6.02\times10^{23} = 7.525\times10^{23} \approx 7.5\times10^{23}$ atoms. ✓

e $A = \lambda\times N_{14}$ and so $N_{14} = \frac{A}{\lambda}$ with $\lambda = \frac{\ln 2}{5730\times365\times24\times3600} = 3.8359\times10^{-12}\ \text{s}^{-1}$. ✓

$N_{14} = \frac{A}{\lambda} = \frac{1.40}{3.8359\times10^{-12}} = 3.6498\times10^{11} \approx 3.6\times10^{11}$. ✓

Hence $\frac{N_{14}}{N_{12}} = \frac{3.6498\times10^{11}}{7.525\times10^{23}} = 4.85\times10^{-13}$. ✓

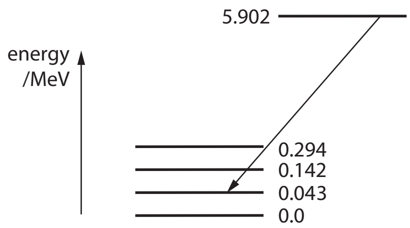
f $\dfrac{N_{14}}{N_{12}} = 4.85 \times 10^{-13} = 1.3 \times 10^{-12} e^{-\lambda t}$ so $e^{-\lambda t} = 0.3731$ ✓

$-\lambda t = \ln 0.3731 \Rightarrow t = -\dfrac{\ln 0.3731}{\lambda}$ ✓

$t = -\dfrac{\ln 0.3731}{3.8359 \times 10^{-12}} = 2.57 \times 10^{11}\ \text{s} \approx 8150$ year ✓

16 a Alpha particle and gamma ray energies in radioactive decay, ✓
are discrete. ✓

b i Correct transition selected. ✓



ii 5.902 – 0.043 = 5.86 MeV ✓

c i The nuclear force has a short range. ✓
And is practically zero for distances larger than the nuclear radii. ✓

ii It must overcome an energy barrier of height 30 MeV and its total energy is less than this. ✓
Leaving the nucleus would violate energy conservation. ✓

iii Like all particles alpha particles have wavelike properties and are described by quantum mechanical wavefunctions. ✓
Which allow for the tunneling phenomenon in which the wavefunction leaks out into the classically forbidden region. ✓

d The half-life has to do with the tunneling probability, i.e. how long an alpha particle takes to leave the nucleus on the average. ✓
And this tunneling probability is very sensitive to small changes in alpha particle energies. ✓

e The uncertainty in position is of order $\Delta x \approx 10^{-15}$ m. ✓

Hence the uncertainty in momentum is $\Delta p \approx \dfrac{h}{4\pi\Delta x} \approx \dfrac{h}{4\pi \times 10^{-15}} = 5.3 \times 10^{-20}$ N s. ✓

17 a i $\sin\theta = \dfrac{\lambda}{b} \Rightarrow b = \dfrac{\lambda}{\sin\theta}$ ✓

$E_K = \dfrac{p^2}{2m} \Rightarrow p = \sqrt{2E_K m} = \sqrt{2 \times 54 \times 10^6 \times 1.6 \times 10^{-19} \times 1.67 \times 10^{-27}} = 1.7 \times 10^{-19}$ N s ✓

Hence $\lambda = \dfrac{6.63 \times 10^{-34}}{1.7 \times 10^{-19}} = 3.9 \times 10^{-15}$ m and then $b = \dfrac{3.9 \times 10^{-15}}{\sin 15°} = 1.5 \times 10^{-14}$ m. ✓

ii $m \approx A \times 1.661 \times 10^{-27}$ kg. ✓

$V = \dfrac{4\pi}{3}\left(1.2 \times 10^{-15} \times A^{\frac{1}{3}}\right)^3 = 7.24 \times 10^{-45} \times A\ \text{m}^3$. ✓

$\rho = \dfrac{m}{V} = \dfrac{1.661 \times 10^{-27} \times A}{7.24 \times 10^{-45} \times A} = 2.29 \times 10^{17} \approx 2 \times 10^{17}\ \text{kg m}^{-3}$. ✓

b $E = \dfrac{kQq}{d}$ ✓

$d = \dfrac{8.99 \times 10^9 \times 2 \times 82 \times 1.6 \times 10^{-19}}{5.2 \times 10^6}$. ✓

$d = 4.54 \times 10^{-14} \approx 4.5 \times 10^{-14}$ m ✓

c i The only force acting on the alpha particle is the electric force. ✓

ii A sharp decrease in the number of scattered particles at high energies. ✓
As the energy increases the alpha particles approach closer to the nucleus and so the nuclear force acts on them, the nucleus absorbs some thus reducing the number that is being scattered. ✓

Answers to test yourself questions

Topic 1

1.1 Measurement in physics

1 Taking the diameter of a proton to be order 10^{-15} m we find $\frac{10^{-15}}{3\times10^{8}}=0.3\times10^{-23}=3\times10^{-24}\approx10^{-24}$ s.

2 The mass of the Earth is about $6\quad10^{24}$ kg and the mass of a hydrogen atom about 2×10^{-27} kg so we need $\frac{6\times10^{24}}{2\times10^{-27}}=3\times10^{51}\approx10^{51}$.

3 $\frac{10^{17}}{10^{-43}}=10^{60}$

4 A heartbeat lasts or 1 s so $\frac{75\times365\times24\times3600}{1}\approx8\times4\times2\times4\times10^{7}\approx2.6\times10^{9}\approx10^{9}$.

5 $\frac{10^{41}}{10^{30}}=10^{11}$

6 $\frac{10^{21}}{1.5\times10^{11}}\approx10^{10}$

7 There are 300 g of water in the glass and hence $\frac{300}{18}\approx\frac{300}{20}=15$ moles of water. Hence the number of molecules is $15\times6\times10^{23}=90\times10^{23}\approx10^{25}$.

8 There are $6\quad10^{4}$ g of water in the body and hence $\frac{6\times10^{4}}{18}\approx0.3\times10^{4}=3\times10^{3}$ moles of water. Hence the number of molecules is $3\times10^{3}\times6\times10^{23}=18\times10^{26}\approx10^{27}$.

9 The mass is about 1.7×10^{-27} kg and the radius about 10^{-15} m so the density is $\frac{1.7\times10^{-27}}{\frac{4\pi}{3}\times(10^{-15})^{3}}\approx\frac{1.7\times10^{-27}}{4\times10^{-45}}=0.5\times10^{18}=5\times10^{17}$ kg m^{-3}.

10 $\frac{10^{21}}{3\times10^{8}}\approx0.3\times10^{13}=3\times10^{12}$ s $\approx10^{5}$ yr

11 **a** $E=2.5\times1.6\times10^{-19}=4.0\times10^{-19}$ J

b $E=\frac{8.6\times10^{-18}}{1.6\times10^{-19}}=54$ eV

12 $V=(2.8\times10^{-2})^{3}=2.2\times10^{-5}$ m^{3}

13 $a=(588\times10^{-9})^{1/3}=8.38\times10^{-3}$ m

14 **a** 200 g

b 1 kg

c 400 g

15 The mass is about 10^{30} kg and the radius is $6.4 \quad 10^6$ m so the density is of about

$$\frac{10^{30}}{\frac{4\pi}{3}(6.4\times 10^6)^3} \approx 9\times 10^8 \approx 10^9 \text{ kg m}^{-3}.$$

16 In SI units the acceleration is $\frac{100\times\frac{10^3}{3600}}{4} = \frac{\frac{10^5}{4\times 10^3}}{4} = \frac{10^2}{16} \approx 6.25 \text{ m s}^{-2} \approx 0.7g$.

17 Assuming a mass of 70 kg made out of water we have $7 \quad 10^4$ g of water in the body and hence $\frac{7\times 10^4}{18} \approx 0.5\times 10^4 = 5\times 10^3$ moles of water. Hence the number of molecules is $5\times 10^3 \times 6\times 10^{23} = 30\times 10^{26} \approx 3\times 10^{27}$. Each molecule contains 2 electrons from hydrogen and 8 from oxygen for a total of $10\times 3\times 10^{27} \approx 10^{28}$ electrons.

18 The ratio is $\frac{F_e}{F_g} = \frac{ke^2}{Gm^2} = \frac{9\times 10^9 \times (1.6\times 10^{-19})^2}{6.7\times 10^{-11}\times(9.1\times 10^{-31})^2} \approx \frac{9\times 10^9\times 3\times 10^{-38}}{7\times 10^{-11}\times 81\times 10^{-62}} \approx \frac{3\times 10^{44}}{63} \approx \frac{10^{44}}{20} \approx 5\times 10^{42}$.

19 $f = cm^x k^y$. The units of m is kg i.e. M and those of k are $\frac{\text{N}}{\text{m}} = \frac{\text{kg m s}^{-2}}{\text{m}} = \text{kg s}^{-2} = \text{M T}^{-2}$. Hence

$$\text{T} = \text{M}^x(\text{M T}^{-2})^y = \text{M}^{x+y}\,\text{T}^{2y}.$$

From this we deduce that

$$x + y = 0$$

$$2y = 1 \Rightarrow y = \frac{1}{2} \Rightarrow x = -\frac{1}{2}$$

Thus, $f = c\sqrt{\frac{k}{m}}$.

20 $P = \frac{1.2 \quad 9.81 \quad 5.55}{2.450} = 2.6667 \quad 10^1$ W. The answer must be given to 2 s.f. and so

$$P = \frac{1.2 \quad 9.81 \quad 5.55}{2.450} = 2.7 \quad 10^1 \text{ W}.$$

21 $E_K = \frac{1}{2} \quad 5.00 \quad 12.5^2 = 3.9063 \quad 10^2$ J. The answer must be given to 3 s.f. and so $E_K = 3.91 \quad 10^2$ J.

22 **a** $\frac{243}{43} \approx \frac{250}{50} = 5$

b $2.80\times 1.90 \approx 3\times 2 = 6$

c $\frac{312\times 480}{160} \approx \frac{300\times 500}{150} = 1000$

d $\frac{8.99\times 10^9\times 7\times 10^{-16}\times 7\times 10^{-6}}{(8\times 10^2)^2} \approx \frac{10^{10}\times 50\times 10^{-22}}{60\times 10^4} \approx 10^{-16}$

e $\frac{6.6\times 10^{-11}\times 6\times 10^{24}}{(6.4\times 10^6)^2} \approx \frac{50\times 10^{13}}{40\times 10^{12}} \approx 10$

1.2 Uncertainties and errors

23 $sum = (180 \pm 8)\ \text{N} = (1.8 \pm 0.8) \quad 10^{2}\ \text{N}$

$dif = (60 \pm 8)\ \text{N} = (6.0 \pm 0.8) \quad 10^{1}\ \text{N}$

24 **a** $Q_0 = \frac{a}{b} = \frac{20}{10} = 2;\ \frac{\Delta Q}{Q_0} = \frac{\Delta a}{a} + \frac{\Delta b}{b} = \frac{1}{20} + \frac{1}{10} = 0.15 \Rightarrow \Delta Q = 2.0 \times 0.15 = 0.30$. Hence $Q = 2.0 \pm 0.3$.

b $Q_0 = 2 \quad 20 + 3 \quad 15 = 85;\ \Delta Q = 2 \times 2 + 3 \times 3 = 13$. Hence $Q = 85 \pm 13 \approx (8.5 \pm 0.1) \times 10^{1}$

c $Q_0 = 50 - 2 \times 24 = 2;\ \Delta Q = 1 + 2 \times 1 = 3$. Hence $Q = 2 \pm 3$

d $Q_0 = 1.00 \quad 10^{2};\ \frac{\Delta Q}{Q_0} = 2 \times \frac{\Delta a}{a} = 2 \times \frac{0.3}{10.0} = 6.00 \times 10^{-2} \Rightarrow \Delta Q = 100 \times 6.00 \times 10^{-2} = 0.06 \times 10^{-2}$.

Hence $Q = 1.00 \quad 10^{2} \pm 0.06 \quad 10^{2} = (1.00 \pm 0.06) \quad 10^{2}$

e $Q_0 = \frac{100^2}{20^2} = 25;\ \frac{\Delta Q}{Q_0} = 2 \times \frac{\Delta a}{a} + 2 \times \frac{\Delta b}{b} = 2 \times \frac{5}{100} + 2 \times \frac{2}{20} = 3.0 \times 10^{-1} \Rightarrow \Delta Q = 25 \times 3.0 \times 10^{-1} = 7.5 \approx 8$

Hence $Q = 25 \pm 8$

25 $F_0 = \frac{2.8 \quad 14^2}{8.0} = 68.6\ \text{N}$

$\frac{\Delta F}{F_0} = \frac{\Delta m}{m} + 2 \times \frac{\Delta v}{v} + \frac{\Delta r}{r} = \frac{0.1}{2.8} + 2 \times \frac{2}{14} + \frac{0.2}{8.0} = 0.3464 \Rightarrow \Delta F = 68.6 \times 0.3464 = 23.7 \approx 20\ \text{N}.$

Hence $F = (68.6 \pm 20)\ \text{N} \approx (7 \pm 2) \times 10^{1}\ \text{N}$

26 **a** $A_0 = \pi R^2 = 18.096\ \text{cm}^2.\ \frac{\Delta A}{A_0} = 2 \times \frac{\Delta R}{R} = 2 \times \frac{0.1}{2.4} = 0.0833 \Rightarrow \Delta A = 18.096 \times 0.0833 = 1.51 \approx 2\ \text{cm}^2.$

Hence $A = (18.096 \pm 2)\ \text{cm}^2 \approx (18 \pm 2)\ \text{cm}^2.$

b $S_0 = 2\pi R = 15.08\ \text{cm}.\ \frac{\Delta S}{S_0} = \frac{\Delta R}{R} = \frac{0.1}{2.4} = 0.04167 \Rightarrow \Delta S = 15.08 \times 0.04167 = 0.628\ \text{cm}^2.$

Hence $S_0 = (15.08 \pm 0.628)\ \text{cm} \approx (15 \pm 1)\ \text{cm}.$

27 $A_0 = ab = 37.4\ \text{cm}^2.\ \frac{\Delta A}{A_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0} = \frac{0.2}{4.4} + \frac{0.3}{8.5} = 0.080749 \Rightarrow \Delta A = 37.4 \times 0.080749 = 3.02 \approx 3\ \text{cm}^2.$

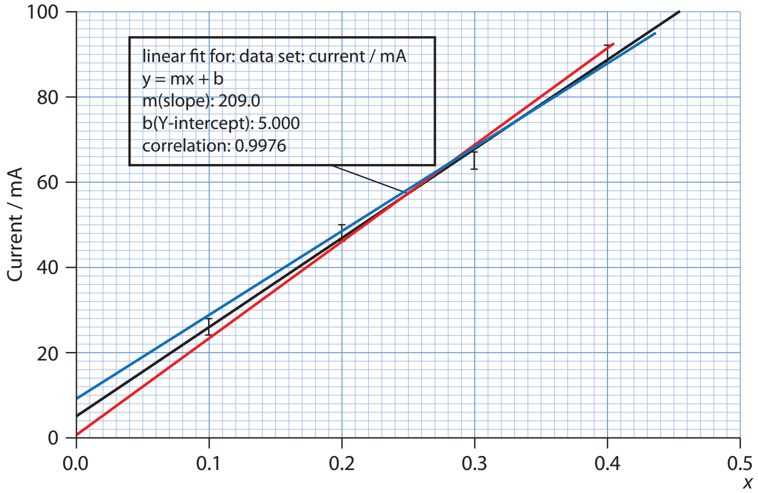
Hence $A = (37.4 \pm 3)\ \text{cm}^2 \approx (37 \pm 3)\ \text{cm}^2.$

$P_0 = 2(a + b) = 25.8\ \text{cm}.\ \Delta P = 2 \times \Delta a + 2 \times \Delta b = 2 \times 0.2 + 2 \times 0.3 = 1.0\ \text{cm}.$ Hence $P = (25.8 \pm 1)\ \text{cm} \approx (26 \pm 1)\ \text{cm}.$

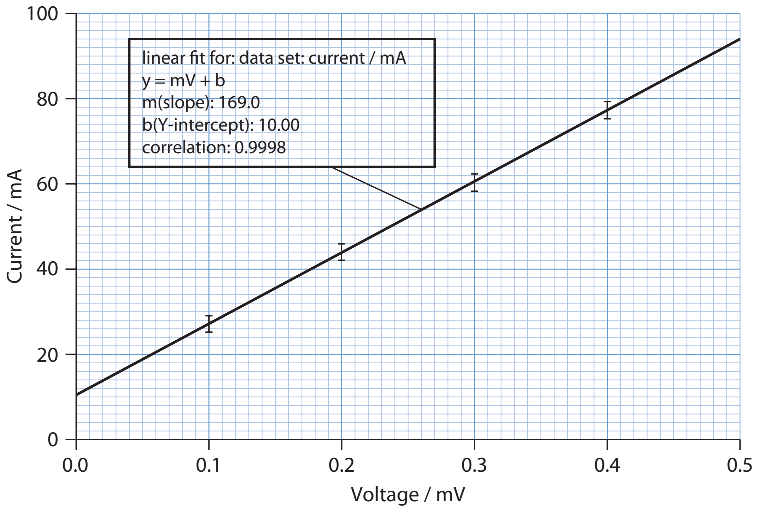
28 $\frac{\Delta T}{T_0} = \frac{1}{2}\frac{\Delta L}{L_0}$ (assuming g is accurately known). Hence $\frac{\Delta T}{T_0} = \frac{1}{2} \times 2\% = 1\%.$

29 $\frac{\Delta V}{V_0} = 2 \times \frac{\Delta R}{R_0} + \frac{\Delta h}{h_0} = 2 \times 4\% + 4\% = 12\%$

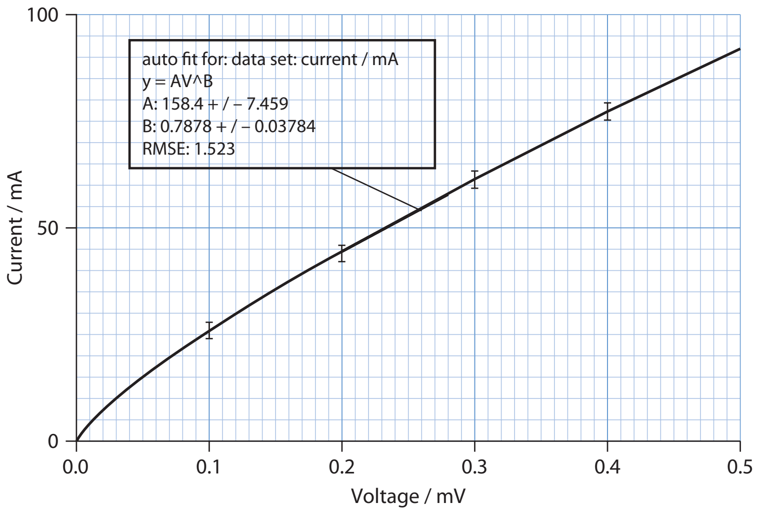
30 The line of best-fit does not go through the origin. There is a vertical intercept of about 4 mA. Lines of maximum and minimum slope give intercepts of about 0 and 9 mA implying an error in the intercept of about 4 mA. The intercept is thus (4 ± 4) mA. This just barely includes the origin so the conclusion has to be that they can be proportional.



31 The vertical intercept is about 10 mA. No straight line can be made to pass through the origin and the error bars unless a systematic error of about 10 mA in the current is invoked.



However, a line of best fit that is a curve can also be fitted through the data and that does go through the origin. (However, it may be objected that this particular functional form is chosen – at low voltages we might expect a straight line (Ohm's law). So a different functional form may have to be tried.)



32 Let P the common perimeter. Then the radius of the circle satisfies $2\pi R = P \Rightarrow R = \frac{P}{2\pi}$ and the side of the square $4a = P \Rightarrow a = \frac{P}{4}$. The circle area is then $A_c = \pi\left(\frac{P}{2\pi}\right)^2 = \frac{P^2}{4\pi}$. The square area is $A_s = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$ and is smaller.

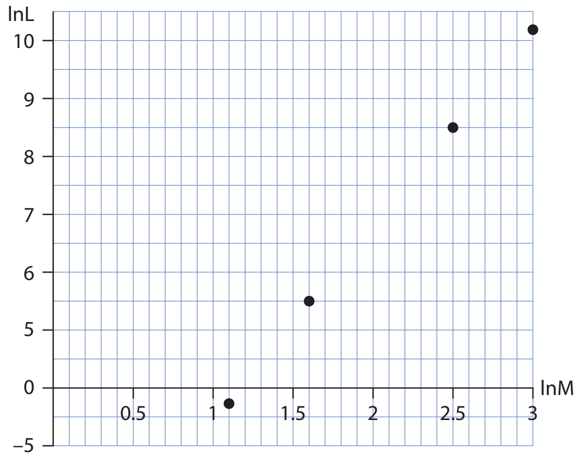
33 **a** The initial voltage V_0 is such that $\ln V_0 = 4 \Rightarrow V_0 = e^4 = 55\ \text{V}$.

b When $V = \frac{V_0}{2} \approx 27\ \text{V}$, $\ln V = \ln 27 \approx 3.29$. From the graph when $\ln V \approx 3.29$ we find $t \approx 7\ \text{s}$.

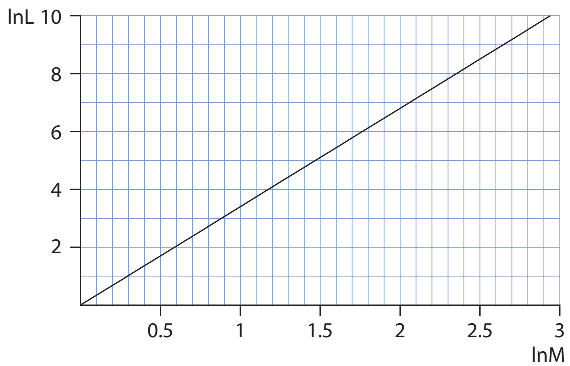
c Since $V = V_0 e^{-t/RC}$, taking logs, $\ln V = \ln V_0 - \frac{t}{RC}$ so a graph of $\ln V$ versus time gives a straight line with slope equal to $-\frac{1}{RC}$. The slope of the given graph is approximately $\frac{4-2}{0-20} = -0.10$. Hence

$$-\frac{1}{RC} = -0.10 \Rightarrow R = \frac{1}{0.10 \times C} = \frac{1}{0.10 \times 5 \times 10^{-6}} = 2 \times 10^6\ \Omega.$$

34 We expect $L = kM^{\alpha}$ and so $\ln L = \ln k + \alpha \ln M$. A graph of $\ln L$ versus $\ln M$ is shown below. The slope is α.



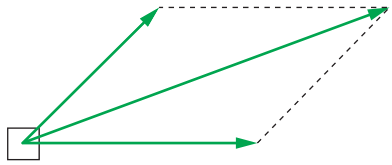
Drawing a best-fit line gives:

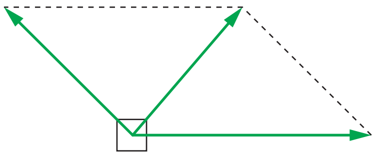


Measuring the slope gives $\alpha = 3.4$.

1.3 Vectors and scalars

35



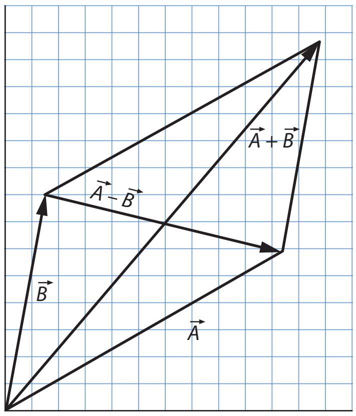


36 **a** $\vec{A} + \vec{B}$:

length 9 cm

$\Rightarrow$ F $\simeq$ 18 N

$\theta \simeq 49°$



b $\vec{A} - \vec{B}$:

length 4.5 cm

$\Rightarrow$ F $\simeq$ 9 N

$\Theta \simeq 14°$ below horizontal.

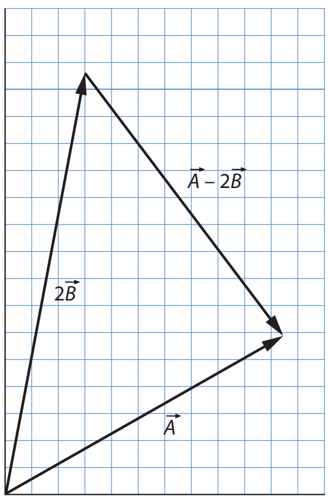
scale:
1 cm $\leftrightarrow$ 2 N

c $\vec{A} - 2\vec{B}$:

length 6.1 cm

$\Rightarrow$ F $\simeq$ 12.2 N

$\Theta \simeq 50°$ below horizontal.



37 The components are:

$A_x = 12\ \cos 30° = 10.39 \qquad B_x = 8.00\ \cos 80° = 1.389$

$A_y = 12\ \sin 30° = 6.00 \qquad A_y = 8.00\ \sin 80° = 7.878$

Hence

a $(A+B)_x = 10.39 + 1.389 = 11.799$

$(A+B)_y = 6.00 + 7.878 = 13.878$

The vector $\vec{A}+\vec{B}$ has magnitude $\sqrt{11.799^2 + 13.878^2} = 18.2$ and is directed at an angle

$\theta = \arctan\dfrac{13.878}{11.799} = 49.6°$ to the horizontal.

b $(A-B)_x = 10.39 - 1.389 = 9.001$

$(A-B)_y = 6.00 - 7.878 = -1.878$

The vector $\vec{A}-\vec{B}$ has magnitude $\sqrt{9.001^2 + 1.878^2} = 9.19$ and is directed at an angle

$\theta = \arctan -\dfrac{1.878}{9.001} = -11.8°$ (below) the horizontal.

c $(A-2B)_x = 10.39 - 2 \times 1.389 = 7.612$

$(A-2B)_y = 6.00 - 2 \times 7.878 = -9.756$

The vector $\vec{A}-2\vec{B}$ has magnitude $\sqrt{7.612^2 + 9.756^2} = 12.4$ and is directed at an angle

$\theta = \arctan -\dfrac{9.756}{7.612} = -52.0°$ (below) the horizontal.

38 **a** $\sqrt{4.0^2 + 4.0^2} = 5.66$ cm in a direction $\theta = 180° + \arctan\dfrac{4.0}{4.0} = 225°$.

b $\sqrt{124^2 + 158^2} = 201$ km in a direction $\theta = \arctan -\dfrac{158}{124} = -52°$.

c $\sqrt{0^2 + 5.0^2} = 5.0$ m at $\theta = 270°$ or $\theta = -90°$.

d $\sqrt{8.0^2 + 0^2} = 8.0$ N at $\theta = 0°$.

39 **a** $\sqrt{2.00^2 + 3.00^2} = 3.61$ at $\theta = \arctan\dfrac{3.00}{2.00} = 56.3°$

b $\sqrt{2.00^2 + 5.00^2} = 5.39$ at $\theta = 180° - \arctan\dfrac{5.00}{2.00} = 112°$

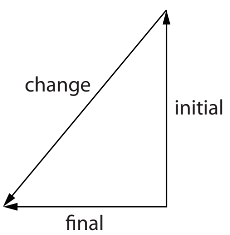
c $\sqrt{0^2 + 8.00^2} = 8.00$ at $\theta = 90°$

d $\sqrt{4.00^2 + 2.00^2} = 4.47$ at $\theta = \arctan -\dfrac{2.00}{4.00} = -26.6°$

e $\sqrt{6.00^2 + 1.00^2} = 6.08$ at $\theta = \arctan\dfrac{1.00}{6.00} = 9.46°$

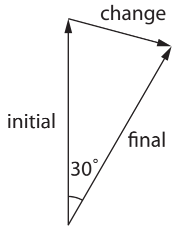
40 The displacement has components $\Delta r_x = 4 - 2 = 2$ and $\Delta r_y = 8 - 2 = 6$.

41 A diagram is:



The magnitude of the change n the velocity vector is $\sqrt{10^2+10^2}=14.1\text{ m s}^{-1}$. The vector makes an angle of 45° with the horizontal as shown in the diagram.

42 A diagram is:



The other two angles of the triangle are each $\frac{1}{2}(180°-30°)=75°$. Using the sine rule we find

$$\frac{\Delta p}{\sin 30°}=\frac{p}{\sin 75°}\Rightarrow \Delta p = p\times\frac{\sin 30°}{\sin 75°}=0.518p\approx 0.52p.$$

43 The components of the velocity vector at the various points are:

A: $v_{Ax}=-4.0\text{ m s}^{-1}$ and $v_{Ay}=0$

B: $v_{Bx}=+4.0\text{ m s}^{-1}$ and $v_{By}=0$

C: $v_{Cx}=0$ and $v_{Cy}=4.0\text{ m s}^{-1}$

Hence

a From A to B the change in the velocity vector has components $v_{Bx}-v_{Ax}=+4.0-(-4.0)=8.0\text{ m s}^{-1}$ and $v_{By}-v_{Ay}=0-0=0$.

b From B to C the change in the velocity vector has components $v_{Cx}-v_{Bx}=0-4.0=-4.0\text{ m s}^{-1}$ and $v_{Cy}-v_{By}=4.0-0=4.0\text{ m s}^{-1}$.

c From A to C the change in the velocity vector has components $v_{Cx}-v_{Ax}=0-(-4.0)=+4.0\text{ m s}^{-1}$ and $v_{Cy}-v_{Ay}=4.0-0=4.0\text{ m s}^{-1}$. The change in the vector from A to C is the sum of the change from A to B plus the change from B to C.

44 **A** $A_x=-10.0\cos 40°=-7.66$ and $A_y=-10.0\sin 40°=+6.43$

B $A_x=-10.0\cos 35°=-8.19$ and $A_y=-10.0\sin 35°=-5.74$

C $A_x=+10.0\cos 68°=+3.75$ and $A_y=-10.0\sin 68°=-9.27$

D $A_x=+10.0\cos(90°-48°)=+7.43$ and $A_y=-10.0\sin(90°-48°)=-6.69$

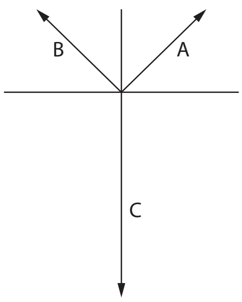
E $A_x=-10.0\cos(90°-30°)=-5.00$ and $A_y=-10.0\sin(90°-30°)=-8.66$

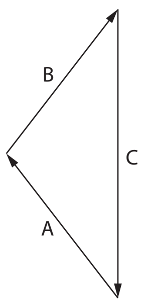
45 The vector we want is $\vec{C}=-(\vec{A}+\vec{B})$. The components of $\vec{A}$ and $\vec{B}$ are:

$A_x=6.0\cos 60°=+3.0$ and $A_y=6.0\sin 60°=+5.20$;

$B_x=6.0\cos 120°=-3.0$ and $A_y=6.0\sin 120°=+5.20$. Hence

$C_x=-(+3.0-3.0)=0$ and $C_y=-(+5.20+5.20)=-10.4$. The magnitude of the vector $\vec{C}$ therefore is 10.4 units and is directed along the negative $y-$ axis.





46 **a** $A_x = 12.0\cos 20° = +11.28$ and $A_y = 12.0\sin 20° = +4.10$;
$B_x = 14.0\cos 50° = +9.00$ and $A_y = 14.0\sin 50° = +10.72$. Hence the sum has components:
$S_x = +11.28 + 9.00 = 20.28$ and $S_y = +4.10 + 10.72 = 14.82$. The magnitude of the sum is thus
$\sqrt{20.28^2 + 14.82^2} = 25.1$. Its direction is $\theta = \arctan\frac{14.82}{20.28} = 36.2°$.

b $A_x = 15.0\cos 15° = +14.49$ and $A_y = 15.0\sin 15° = +3.88$;
$B_x = 18.0\cos 105° = -4.66$ and $B_y = 18.0\sin 105° = +17.39$. Hence the sum has components:
$S_x = 14.49 - 4.66 = 9.83$ and $S_y = +3.88 + 17.39 = 21.27$. The magnitude of the sum is thus
$\sqrt{9.83^2 + 21.27^2} = 23.4$. Its direction is $\theta = \arctan\frac{21.27}{9.83} = 65.2°$.

c $A_x = 20.0\cos 40° = +15.32$ and $A_y = 20.0\sin 40° = +12.86$;
$B_x = 15.0\cos 310° = +9.64$ and $B_y = 15.0\sin 310° = -11.49$. Hence the sum has components:
$S_x = 15.32 + 9.64 = +24.96$ and $S_y = +12.86 - 11.49 = +1.37$. The magnitude of the sum is thus
$\sqrt{24.96^2 + 1.37^2} = 25.0$. Its direction is $\theta = \arctan\frac{1.37}{24.96} = 3.14°$.

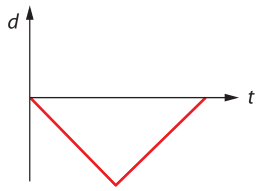
Answers to test yourself questions

Topic 2

2.1 Uniform motion

1 Distance traveled in first 1.5 h is $s = vt = 70 \times 1.5 = 105$ km. Remaining distance is 15 km and must be covered in 1.0 hr so average speed must be $v = \frac{15}{1.0}\frac{\text{km}}{\text{hr}} = 15\ \text{km hr}^{-1}$.

2 The velocity is initially constant and negative so displacement graph will be a straight line with negative slope. The velocity is then constant and positive so displacement graph is a straight line with positive slope.



3 The relative speed of the cyclists is $v = 35$ km hr^{-1}. They will then meet in a time of $v = \frac{70}{35} = 2.0\,\text{hr}$.

a The common displacement is $s = 15 \times 2.0 = 30$ km.

b The fly will travel a distance of $s = 30 \times 2.0 = 60$ km.

4 **a** The distance traveled is 80 m. The average speed is then $v = \frac{80}{20}\ \text{m s}^{-1} = 4.0\ \text{m s}^{-1}$.

b The displacement is zero and so the average velocity is zero.

5 Use $v = u + at$. So $8.0 = 2.0 + a \times 2.0 \Rightarrow a = \frac{8.0 - 2.0}{2.0} = 3.0\ \text{m s}^{-2}$.

6 From $v = u + at$, $28 = 0 + a \times 9.0 \Rightarrow a = \frac{28}{9.0} = 3.1\ \text{m s}^{-2}$, hence from $s = ut + \frac{1}{2}at^2$ we have that

$s = \frac{1}{2}3.1 \times 9.0^2 = 126\ \text{m} \approx 130\ \text{m}$.

7 From $v^2 = u^2 + 2as$ we find $0^2 = 12^2 + 2a \times 45 \Rightarrow a = -\frac{144}{90} = -1.6\ \text{m s}^{-2}$.

8 From $s = ut + \frac{1}{2}at^2$ we get $16 = -6.0t + \frac{1}{2} \times 2.0 \times t^2$. Solving the quadratic equation gives $t = 8.0$ s.

9 Use $s = ut + \frac{1}{2}at^2$. So $450 = \frac{1}{2}a \times 15.0^2 \Rightarrow a = \frac{900}{225} = 4.00\ \text{m s}^{-2}$. Then from $v = u + \text{at}$, $v = 4.00 \times 15.0 = 60.0\ \text{m s}^{-1}$.

10 **a** The distance traveled before the brakes are applied is $s = 40.0 \times 0.50 = -20$ m. Once the brakes are applied the distance is given from $v^2 = u^2 + 2as$, i.e. $0 = 40^2 + 2 \times (-4.0) \times s \Rightarrow s = 200$ m. The total distance is thus 220 m.

b 200 m as done in **a**.

c $s = 220 - 200 = 20$ m.

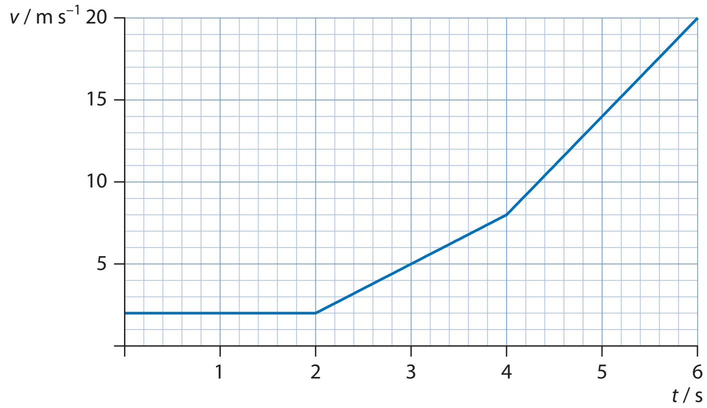
d It would be less since the speed is less.

11 **a** $s_1 = -\frac{1}{2} \times 10t^2 = -5t^2$ and $s_2 = -\frac{1}{2} \times 10(t-1)^2$. Two seconds after the second ball was dropped means that $t = 3.0$ s. Then, $s_1 = -45$ m and $s_2 = -20$ m. The separation is thus 25 m.

b $s_1 - s_2 = -5t^2 + \frac{1}{2} \times 10(t-1)^2 = -10t + 5$ so in magnitude this increases.

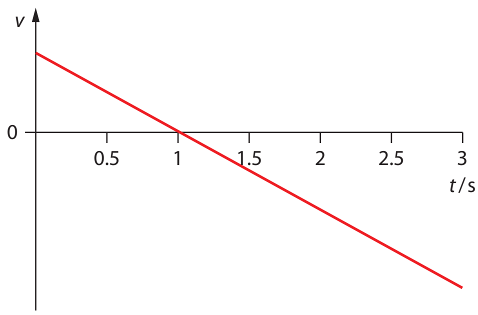
12 a The velocity at 2 s is $v_2 = 2 + 0 \times 2 = 2.00$ m s^{-1}. The velocity at 4 s is $v_4 = 2 + 3 \times 2 = 8.00$ m s^{-1}. The velocity at 6 s is $v_6 = 8 + 6 \times 2 = 20.0$ m s^{-1}. Alternatively, the area under the graph is 18.0 m s^{-1} and this gives the *change* in velocity. Since the initial velocity is 2.00 m s^{-1}, the final velocity is 20.0 m s^{-1}.

b

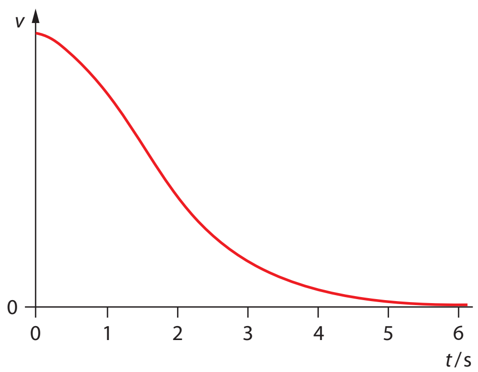


13 The acceleration is the slope of the velocity – time graph. Drawing a tangent to the curve at 2 s we find a slope of approximately $a = 2.0$ m s^{-2}.

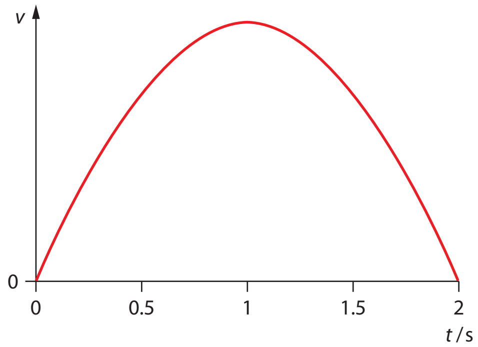
14 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially positive and begins to decrease. It becomes zero at 1 s and then becomes negative. The displacement graph is in fact a parabola and so the velocity is in fact a linear function. Of course we are not told that, so any shape showing the general features described above would be acceptable here.



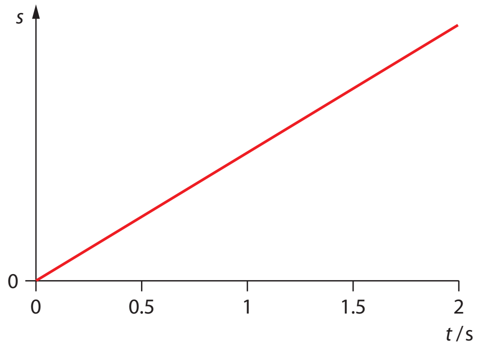
15 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially very large and continues to decrease all the time approaching a small value. The slope and hence the velocity are always positive. This explains the graph in the answers in the textbook.



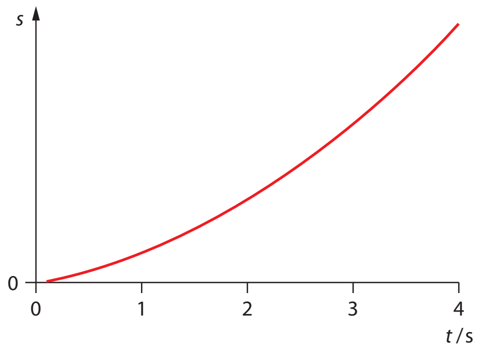
16 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially very small, becomes greatest at 1 s and starts decreasing thereafter becoming very small again. The slope and hence the velocity are always positive. This explains the graph in the answers in the textbook.



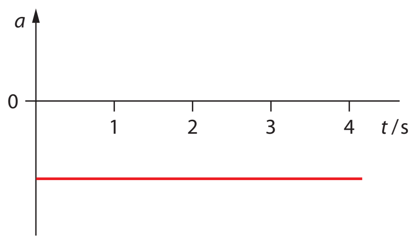
17 The acceleration is zero here so $x = vt$, i.e. The graph of displacement is a linear function with a positive slope (that may or may not go through the origin depending on the initial displacement).



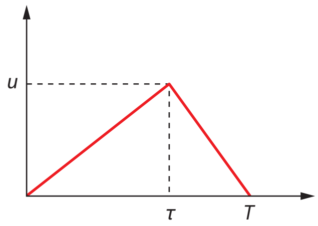
18 Here we have a constant positive acceleration and so $x = ut + \frac{1}{2}at^2$ which is the graph of a (concave down) parabola.



19 The acceleration is the slope of the velocity – time graph. The slope is large and positive initially and decreases to become zero at 1 s. It then becomes negative increasing in magnitude (i.e. becoming more negative).



20 You must push the car as hard as you can but then you must also pull back on it to stop it before it crashes on the garage. The velocity – time graph must be something like:



We know that: $u = 2\tau$. During pullback, we have that the velocity is given by $v = u - 3(t - \tau) = 2\tau - 3(t - \tau)$ $= 5\tau - 3t$. The velocity becomes zero at time T and so $0 = 5\tau - 3T$, i.e. $\tau = \frac{3T}{5}$. The area under the curve (triangle) is 15 m and is given by $\frac{1}{2}Tu = \frac{1}{2}T(2\tau) = \frac{1}{2}T\frac{6T}{5} = \frac{3T^2}{5}$.

Hence $\frac{3T^2}{5} = 15 \Rightarrow T^2 = 25 \Rightarrow T = 5$ s.

21 **a** The velocity is the slope of the displacement – time graph. Therefore the velocity is negative from A to B.

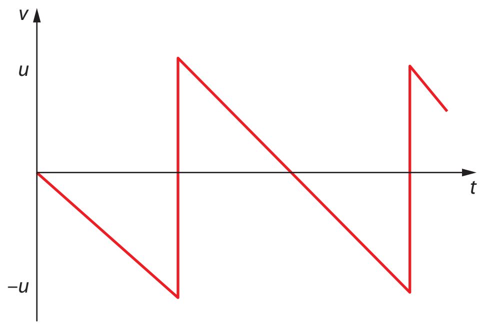
b Between B and C the slope and so the velocity is zero.

c From A to B the velocity is becoming less negative and so it is increasing. So the acceleration is positive.

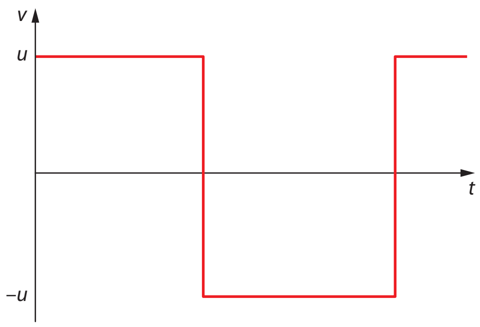
d From C to D the slope is increasing meaning the velocity is increasing. Hence the acceleration is positive.

22 See graphs below.

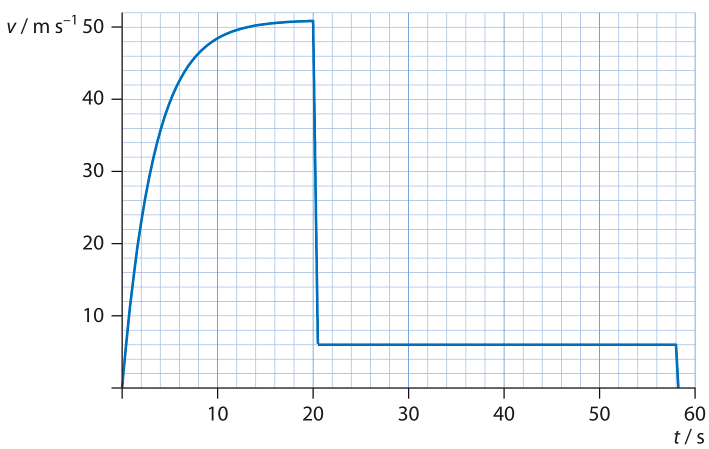
a



b



c



23 **a** Use $v^2 = u^2 + 2as$ to get $0 = 8^2 + 2\times(-10)\times s \Rightarrow s = 3.2$ m from the cliff.

b Use $s = ut + \frac{1}{2}at^2$ to get $-35 = 8\times t + \frac{1}{2}(-10)\times t^{22}$ and solve for time to get $t = 3.56$ s.

c $v = u + at = 8 - 10\times 3.56 = -27.6$ m s^{-1}.

d $3.2 + 3.2 + 35 = 41.4$ m.

e average speed is $\frac{41.4}{3.56} = 11.6$ m s^{-1} and average velocity is $\frac{-35}{3.56} = -9.83$ m s^{-1}.

24 **a** 60 m^2

b 40 m s^{-1}

25 The time to fall to the floor is given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 1.3}{10}} = 0.51$ s. The horizontal distance traveled is therefore $x = v_x t = 2.0\times 0.51 = 1.02 \approx 1.0$ m.

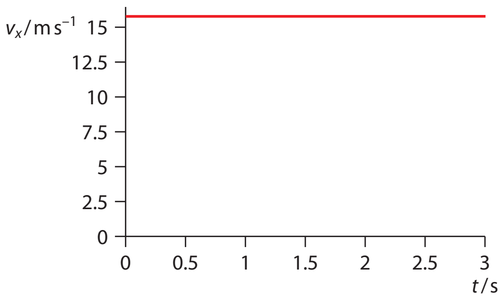
26 **a** The times to hit the ground are found from $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 4.0}{10}} = 0.894$ s and $\sqrt{\frac{2\quad 8.0}{10}} = 1.265$ s.

The objects are thus separated by $4.0\times(1.265 - 0.894) = 1.48 \approx 1.5$ m when they land.

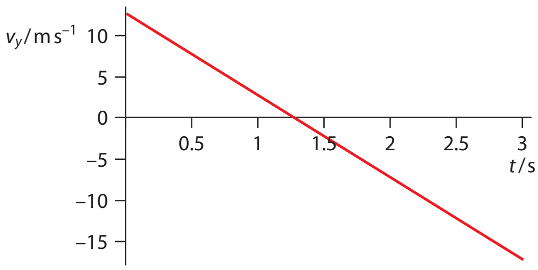
b The horizontal distance traveled by the object falling from 8.0 m is (see previous problem) $x = v_x t = 4.0\quad 1.265 = 5.06$ m. Thus the speed of the other object must be $v_x = \frac{5.06}{0.894} = 5.66 \approx 5.7$ m s^{-1}.

27 The components of velocity are:

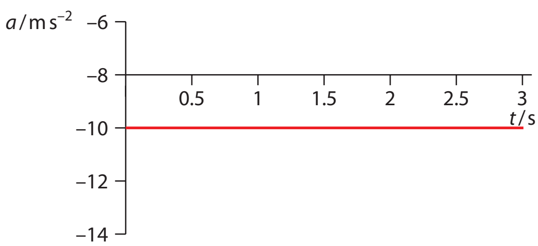
a $v_x = v\cos 40° = 20\cos 40° = 15.3$ m s^{-1} and so the graph is a horizontal straight line.



b $v_y = v\cos 40° - gt = (12.9 - 10t)$ m s^{-1} so that graph is a straight line with negative slope as shown in graph.



c The acceleration is constant so graph is a horizontal straight line.



28 $v_y = v\sin 40° - gt$. At the highest point this component is zero and so $t = \frac{v\sin 40°}{g} = \frac{24 \quad \sin 40°}{10} = 1.54$ s. Then from $y = vt\sin 40° - \frac{1}{2}gt^2$ we find $y = 24 \times 1.54 \times \sin 40° - \frac{1}{2} \times 10 \times 1.54^2 \approx 12$ m.

29 Graphs are shown in the answers to the textbook. They correspond to:

a The horizontal displacement is given by $x = 20t \quad \cos 50° = 12.6t$ whose graph is a straight line.

b The vertical displacement is $y = 20t \times \sin 50° - \frac{1}{2} \times 10t^2 = 15.3t - 5t^2$ whose graph is a concave down parabola.

30 In time t the monkey will fall a vertical distance $y = \frac{1}{2}gt^2$ but so will the bullet and hence the bullet will hit the monkey.

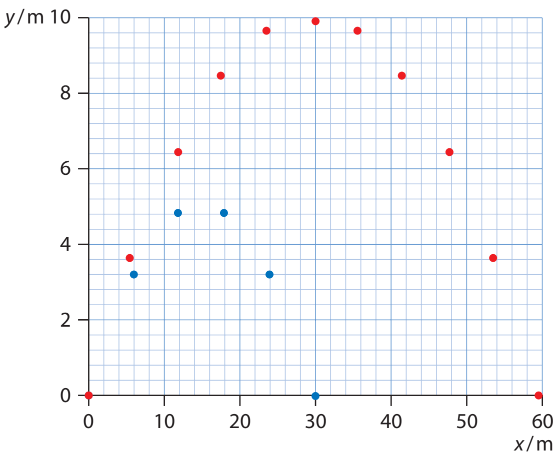
31 a i The ball covers a horizontal distance of 60 m in 2.0 s and so the horizontal velocity component is $u_x = \frac{60}{2.0} = 30$ m s^{-1}. The ball climbs to a height of 10 m in 1.0 s and so from $y = \frac{u_y + v_y}{2}t$ we have $10 = \frac{u_y + 0}{2} \times 1.0 \Rightarrow u_y = 20$ m s^{-1}.

ii The angle of launch is $\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\left(\frac{20}{30}\right) = 34°$.

iii The vertical component of velocity becomes zero at 1.0 s and so $v_y = u_y - gt \Rightarrow 0 = 20 - g \times 1.0 \Rightarrow g = 20$ m s^{-2}.

b The velocity is horizontal to the right and the acceleration is vertically down.

c With $g = 40$ m s^{-2}, the ball will stay in the air for half the time and so will have half the range. The maximum height is reached in 0.50 s and is $y = u_y t - \frac{1}{2}gt^2 = 20 \times 0.50 - \frac{1}{2} \times 40 \times 0.50^2 = 5.0$ m, i.e. half as great as before.



32 a The initial velocity components are: $u_x = 20.0\cos 48° = 13.38$ m s^{-1} and $u_y = 20.0\sin 48° = 14.86$ m s^{-1}.

The ball hits the sea when the vertical displacement is $y = -60.0$ m. Thus $y = u_y t - \frac{1}{2}gt^2 \Rightarrow$ $-60.0 = 14.86t - 4.90t^2$. Solving for the positive root we find $t = 5.33$ s. Hence $v_x = u_x = 13.38$ m s^{-1} and $v_y = u_y - gt = 14.86 - 9.81 \times 5.33 = 37.43$ m s^{-1}. The speed at impact is thus $v = \sqrt{13.38^2 + (-37.43)^2} = 39.7 \approx 40$ m s^{-1} at $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(-\frac{37.74}{13.38}\right) = -70°$ to the horizontal.

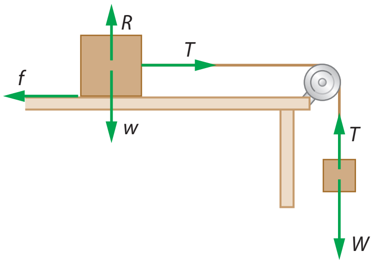
b Some of the kinetic energy of the ball will be converted into thermal energy and so the speed at impact will be less. The horizontal component of velocity will decrease in the course of the motion and will tend to go to zero but the vertical component will never become zero (after reaching the maximum height). This means that the angle of impact will be steeper.

33 **a** Terminal speed is the eventual constant speed reached by a projectile as a result of an air resistance force that increases with speed.

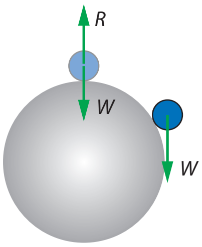
b Initially the net force on the particle is just the weight. As the speed increases so does the resistance force. Eventually the resistance force will equal the weight and from then on the particle will move with zero acceleration, i.e. with a constant terminal speed.

2.2 Forces

34

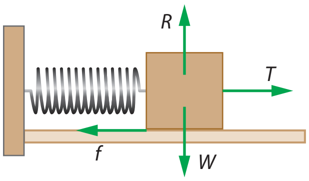


35 **a** and **b**

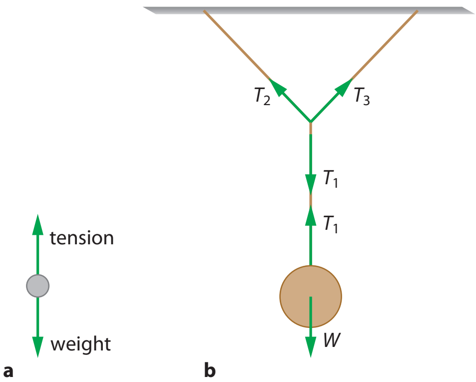


36 The tension is the same in both cases since the wall exerts a force of 50 N to the left on the string just as in the second diagram.

37



38



39 A 30 N to the right
B 6 N to the right
C 8 N to the left
D 15 N to the right
E 10 N down
F 20 N up

40 The horizontal components clearly cancel put leaving a net force of $2 \quad 20 \quad \sin 45° = 28$ N in the up direction.

41 Because there would be no vertical force to cancel the weight of the block.

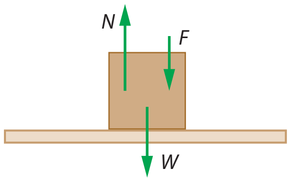
42 **a** Since the string is being pulled slowly we have equilibrium until one of the strings breaks. If the lower string is being pulled with force F then the tension in the lower string will be F and the tension in the upper string will be T where $T = mg + F$. The tension in the upper string is thus greater and will reach breaking point first.

b If the lower string is pulled down very abruptly, the inertia of the block will keep it momentarily motionless and so the tension in the lower string will reach a high value before the upper one does. Hence it will break.

43 The largest frictional force that can develop between the mass and the table is $f_{max} = \propto_s N =$ $0.60 \quad 2.00 \quad 9.8 = 11.8$ N. This is also the tension holding the hanging mass up. Hence $mg = 11.8 \Rightarrow m = 1.2$ kg.

44 Equilibrium demands that $W = F + N \Rightarrow N = W - F = 220 - 140 = 80$ N.

45



Equilibrium demands that $W + F = N \Rightarrow N = 150 + 50 = 200$ N. This is the force that the table exerts on the block. By Newton's third law this is also the force exerted on the table by the block.

46 The component of the weight down the plane is $Mg\sin\theta$ and for equilibrium this is also the tension in the string. To have equilibrium for the hanging mass its weight must equal the tension and so $Mg\sin\theta = mg$. Hence $\theta = \sin^{-1}\frac{m}{M}$.

47 **a** One possibility is to have the mass of the body decrease as in the case of a rocket where the fuel is being burned and ejected from the rocket.

b That happens when the mass increases as for example cart that is being filled with water or sand while being pulled with a constant force.

48 The maximum force can only be 575 N and so the maximum acceleration is $a = \frac{600}{1400} = 0.43 \text{ m s}^{-2}$.

49 The forces on the man are his weight, mg and the reaction force R from the floor.

a **i** The acceleration is zero and so $R - mg = 0$, i.e. $R = mg$.

ii The acceleration is zero and so $R - mg = 0$, i.e. $R = mg$.

iii The net force is in the downward direction and equals $mg - R$. Hence, $mg - R = ma$ and so $R = mg - ma$.

iv From iii we have that $R = mg - mg = 0$.

b The man will be hit by the ceiling of the elevator that is coming down faster than the man.

50 As the elevator goes up the force that must be supplied by the arm on the book upwards must increase. Because you are not aware that you have to do that the book "feels" heavier and so moves down a bit. As the elevator comes to a stop the force necessary to keep it up decreases and so the book "fells" less heavy and so moves up a bit. The same thing happens when you start going down. As the elevator comes to a stop on the way down, the force needed to keep it up is again greater than the weight so the book falls.

51 **a** The forces on the man are: the tension from the rope, T, on his hands upward (this is the same as the force with which he pulls down on the rope); his weight 700 N downward; the reaction, R, from the elevator floor upward.

b On the elevator they are: the weight downward; the reaction, R, from the man downward; the tension, T, in the rope upward.

c The forces on the man *and* the elevator **together** are $2T$ upwards (one T on the elevator at the top and one T on the man from the rope). The combined weight is 1000 N. Thus

$2T - 1000 = 100a = 100 \times 0.50 \Rightarrow T = 525$ N. The net force on the man is

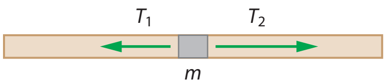
$R + T - 700 = 70a = 35 \Rightarrow R = 700 - 525 + 35 \Rightarrow R = 210$ N.

d We know that

$$\begin{aligned} 2T - 1000 &= 100a \\ R + T - 700 &= 70a \end{aligned} \quad \text{and so} \quad \begin{aligned} 2T - 1000 &= 100a \\ 2R + 2T - 1400 &= 140a \end{aligned}$$

Subtracting we get $2R - 400 = 40a$ and so $a = \dfrac{2R - 400}{40} = \dfrac{600 - 400}{40} = 5.0 \text{ m s}^{-2}$.

52 Suppose the tensions at some point were different.



The net force on the bit of string of mass m is $T_2 - T_1 = ma$. But the string is massless, $m = 0$ and so $T_2 - T_1 = 0$ meaning that the tensions are the same.

53 **a** Treat the two masses as one body. The net force is 60.0 N and so the acceleration is $a = \dfrac{60.0}{40.0} = 1.50 \text{ m s}^{-2}$. The net force on the back block is the tension in the string and so $T = ma = 10.0 \quad 1.50 = 15.0$ N.

b The tension would now be $T = Ma = 30.0 \quad 1.50 = 45.0$ N.

54 For three (planar) forces to be in equilibrium, any one force must have a magnitude that is in between the sum and the difference of the other two forces. This is the case here. Now, the resultant of the 4.0 N and the 6.0 N forces must have a magnitude of 9.0 N. This when the 9.0 N force is suddenly removed, the net force on the body is 9.0 N. The acceleration is therefore $a = \dfrac{9.0}{3.0} = 3.0 \text{ m s}^{-2}$.

2.3 Work, energy and power

55 The work done is $W = Fd\cos\theta = 24 \times 5.0 \times \cos 0° = 120$ J.

56 The work done is $W = Fd\cos\theta = 2.4 \times 3.2 \times \cos 180° = -7.7$ J.

57 The work done is $W = Fd\cos\theta = 25 \times 15 \times \cos 20° = 352 \approx 3.5 \times 10^2$ J.

58 The change in kinetic energy is $\Delta E_K = \dfrac{1}{2} \times 2.0(0 - 5.4^2) = -29.16$ J. This equal the work done by the resistive force i.e. $f \times 4.0 \times \cos 180° = -29.16 \Rightarrow f = 7.3$ N.

59 The work done has gone to increase the elastic potential energy of the spring i.e.

$W = \dfrac{1}{2} \times 200 \times (0.050^2 - 0.030^2) = 0.16$ J.

60 **a** **i** The minimum energy is required to just get the ball at A. Then,

$\dfrac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 4.0} = 8.9 \text{ m s}^{-1}$.

ii At position B, $\dfrac{1}{2} \times 1.0 \times 8.9^2 = 1.0 \times 9.8 \times 2.0 + \dfrac{1}{2} \times 1.0 \times v^2 \Rightarrow v = 6.3 \text{ m s}^{-1}$.

b At A: $\dfrac{1}{2} \times 1.0 \times 12.0^2 = \dfrac{1}{2} \times 1.0 \times v^2 + 1.0 \times 9.8 \times 4.0 \Rightarrow v = 8.1 \text{ m s}^{-1}$.

At B: $\dfrac{1}{2} \times 1.0 \times 12.0^2 = \dfrac{1}{2} \times 1.0 \times v^2 + 1.0 \times 9.8 \times 2.0 \Rightarrow v = 10 \text{ m s}^{-1}$.

61 The total energy at A is $E_A = 8.0 \quad 9.8 \quad 12 + \frac{1}{2} \quad 8.0 \quad 6.0^2 = 1085$ J. At B it is $E_B = \frac{1}{2} \quad 8.0 \quad 12^2 = 576$ J.

The total energy decreased by $1085 - 576 = 509$ J and this represents the work done by the resistive forces. The distance traveled down the plane is 24 m and so $f \times 24 = 509 \Rightarrow f = 21$ N.

62 **a** The work done is the area under the curve. This is a trapezoid and so $W = \frac{15 + 7.0}{2} \quad 8.0 = 88$ J.

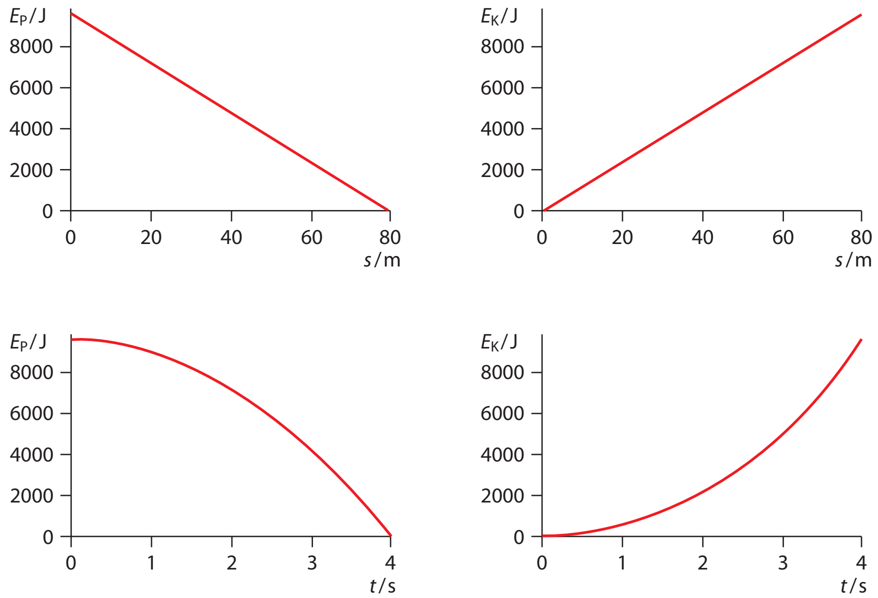
b The work done is the change in kinetic energy and so $\frac{1}{2}mv^2 = W$, giving $v = \sqrt{\frac{2 \times 88}{2.0}} \approx 9.4$ m s^{-1}.

63 **a** **i** The potential energy is given by $E_P = mgH - mgs = 12 \times 10 \times 80 - 12 \times 10s = 9600 - 120s$.

ii The kinetic energy is $E_K = E_{Total} - E_P = 9600 - (9600 - 120s) = 120s$.

b **i** Since the distance fallen s is given by $s = \frac{1}{2}gt^2$ the answers in **a** become $E_P = 9600 - 600t^2$

ii $E_P = 600t^2$. These four equations are in the graphs here.



c In the presence of a *constant* resistance force, the graph of potential energy against distance will not be affected. The graph of kinetic energy against distance will be a straight line with a smaller slope since the final kinetic energy will be less. The graph of potential energy against time will have the same shape but will reach zero in a longer time. Similarly, the kinetic energy – time graph will reach a smaller maximum value in a longer time.

64 From $P = Fv$, $F \times \frac{100 \times 10^3}{3600} = 90 \times 10^3 \Rightarrow F = 3240 = 3.24 \times 10^3$ N $\approx 3 \times 10^3$ N.

65 **a** From $P = Fv$, and $F = Mg = 1.2 \quad 10^4$ N we find $v = \frac{2.5 \times 10^3}{1.2 \times 10^4} = 0.21$ m s^{-1}.

b Most likely some of the power produced by the motor gets dissipated in the motor itself due to frictional forces and get converted into thermal energy and is not used to raise the block.

66 **a** The work done is $mgh = 50 \quad 9.8 \quad 15 = 7350$ J. The power is thus $\frac{7350}{125} = 59$ W.

b $e = \frac{59}{80} = 0.74$

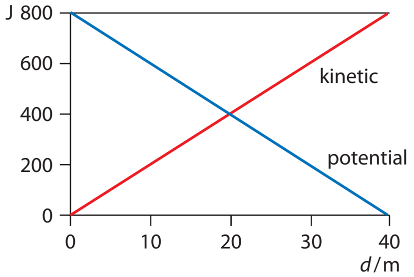
c The work required is double and the time is therefore also double, 250 s.

67 From $P = Fv$, $F \times \dfrac{240 \times 10^3}{3600} = 250 \times 10^3 \Rightarrow F = 3750 \approx 3.8 \times 10^3$ N.

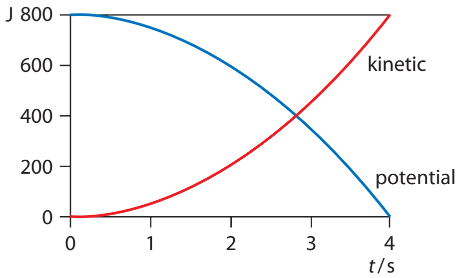
68 Electrical energy from the motor is converted to potential energy and thermal energy if the elevator is just pulled up. Normally a counterweight is being lowered as the elevator is being raised which means that the net change in gravitational potential energy is zero (assuming that the counterweight is equal in weight to the elevator). In this case all the electrical energy goes into thermal energy.

69 **a** The acceleration of the mass is $g \sin 30° = 5.0$ m s^{-2} and so the speed is $v = 5.0 \quad t$. Hence the kinetic energy (in joule) as a function of time is $E_K = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \quad 4.0 \quad (5.0 \quad t)^2 = 50t^2$. The distance s traveled down the plane is given by $s = \dfrac{1}{2}at^2 = \dfrac{1}{2} \quad 5.0t^2 = 2.5t^2$ and so the vertical distance h from the ground is given by $h = 20 - s \sin 30° = 20 - 1.25t^2$. Hence the potential energy is $E_P = mgh = 4.0 \times 10 \times (20 - 1.25t^2) = 800 - 50t^2$. These are the functions to be graphed with the results as shown in the answers in the textbook, page 532.

b



c



70 **a** The net force is zero since the velocity is constant and so $T = mg \sin \theta$.

b $W_T = Fd = mgd \sin \theta$

c $W_W = -mgh = -mgd \sin \theta$

d $W_N = 0$ since the angle is a right angle.

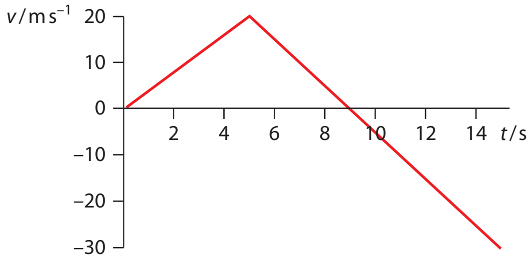
e Zero since the kinetic energy is constant (or zero because $W_W + W_W + W_N = mgd \sin \theta - mgd \sin \theta + 0 = 0$).

71 **a** From $s = \dfrac{1}{2}at^2$ we find $s = \dfrac{1}{2} \quad 4.0 \quad 5.0^2 = 50$ m.

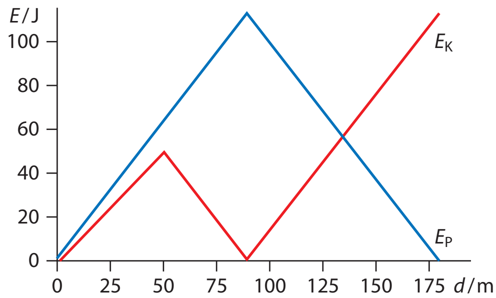
b At 5.0 s the speed acquired is $v = at = 4.0 \times 5.0 = 20$ m s^{-1}. From then on the acceleration becomes a deceleration of $a = g \sin \theta = 10 \times 0.5 = 5.0$ m s^{-2}. Then from $v^2 = u^2 - 2as$ we find $0 = 20^2 - 2 \times 5.0 \times s$ giving $s = \dfrac{20^2}{2 \quad 5.0} = 40$ m. The total distance up the plane is thus 90 m.

c The car will travel the distance of 90 m from rest with an acceleration of 5.0 m s^{-2} and so from $s = \dfrac{1}{2}at^2$ we get $90 = \dfrac{1}{2} \quad 5.0 \quad t^2$ giving $t^2 = \dfrac{180}{5.0} = 36 \Rightarrow t = 6.0$ s. (The car took 5.0 s to get to the 50 m up the hill. The remaining 40 m were covered in $0 = 20 - 5.0 \times t \Rightarrow t = 4.0$ s. The time from the start to get back down again is thus 15 s.)

d For the first 5 s the velocity is given by $v = 4.0t$. For the rest of the motion the velocity is $v = 20 - 5.0t$.



e Potential energy: For a distance d traveled up the plane the vertical distance is $h = \frac{d}{2}$ and so the potential energy is $E_P = mgh = 0.250 \quad 10 \quad \frac{d}{2} = 1.25d$. At 90 m, (the highest the car gets on the plane) we have $E_P = 1.25 \times 90 = 112.5 \approx 112$ J. For the last 90 m (the way down) the graph is decreasing symmetrically. These facts give the graph ion the answers in the textbook. Kinetic energy: For the first 50 m traveled we have that: the speed is given by $v^2 = u^2 + 2as = 0 + 2 \quad 4.0 \quad s = 8s$ and so the kinetic energy is $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \quad 0.250 \quad 8s = s$. (The kinetic energy attained at 50 m is thus 50 J.) In the next 40 m the speed is given by $v^2 = u^2 + 2as = 20^2 - 2 \times 5.0 \times s = 400 - 10s$ and so the kinetic energy decreases according to $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.250(400 - 10s) = 50 - 1.25s$. At 40 m the kinetic energy becomes zero. From then on it increases according to $E_K = \frac{1}{2} \quad 0.250 \quad 2 \quad 5.0 \quad s = 1.25s$. Putting all these together gives the graph below.



f The mechanical energy (kinetic plus potential) will be conserved when there are no external forces acting on the car (other than gravity) i.e. after the first 5.0 s.

g The motor was exerting a force F up the plane given by

$F - mg\sin\theta = ma \Rightarrow F = ma + mg\sin\theta = 0.250 \times 4.0 + 0.250 \times 10 \times \frac{1}{2} = 2.25$ N. The average speed up the plane was $\bar{v} = \frac{0 + 20}{2} = 10$ m s^{-1} and so the average power is $\bar{P} = F\bar{v} = 2.25 \quad 10 = 22.5$ W.

The maximum power was $P = Fv = 2.25 \quad 20 = 45$ W.

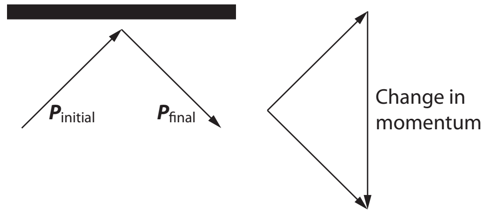
2.4 Momentum and impulse

72 $F_{ave} = \frac{\Delta p}{\Delta t} = \frac{12.0}{2.00} = 6.00$ N

73 **a** Impulse $= \Delta p = p_{final} - p_{initial} = 0.150 \times (-3.00) - 0.150 \times 3.00 = -0.900$ N s. (b) $|F_{ave}| = \left|\frac{\Delta p}{\Delta t}\right| = \frac{0.900}{0.125} = 7.20$ N.
This is the force exerted on the ball by the wall and so by Newton's third law this is also the force the ball exerted on the wall.

74 The total momentum before the collision is $m \times v + 2m \times \left(-\frac{v}{2}\right) = 0$. This is also the momentum after. If u is the speed after the collision then $3m \times u = 0 \Rightarrow u = 0$.

75 **a** The change in momentum is given by the following vector diagram. The angle between the vectors is a right angle.



The magnitude of the initial and of the final momentum is $p = 0.250 \quad 4.00 = 1.00$ N s. The direction of the change of momentum is given in the diagram. Its magnitude is $\sqrt{1.00^2 + 1.00^2} = \sqrt{2.00} = 1.41$ N s.

b It depends on what we take the system to be. If the system is just the ball then its momentum is not conserved since there is an external force acting on the ball (the force from the wall). If, on the other hand, we take the system to be the ball and the wall then the total momentum is conserved since there is now no external force acting on the system. The wall will acquire a momentum equal and opposite to the momentum change of the ball.

76 The initial total momentum is $4.0 \times 24 - 12.0 \times 2.0 = +72$ N s. The final total momentum is $-4.0 \times 3.0 + 12 \times v$. Hence $-12 + 12v = 72 \Rightarrow v = +7.0$ m s^{-1}. (The ball moves to the right.)

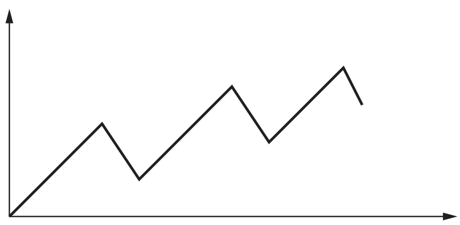
77 **a** The impulse is the area under the graph and so equals (we use the area of a trapezoid) $\frac{17+7}{2} \quad 8.0 = 96$ N s.

b Since the impulse is the change in momentum: $96 = mv - 0 \Rightarrow v = \frac{96}{3.00} = 32$ m s^{-1}.

c Now, $96 = 0 - mv \Rightarrow v = -\frac{96}{3.00} = -32$ m s^{-1}.

78 **a** The impulse supplied to the system is (area under curve) $3 \times 0.5 \times 100 - 25 \times 4 = 50$ N s. This is the change in momentum i.e. $25 \times \Delta v = 50$ N s $\Rightarrow \Delta v = 2.0$ m s^{-1}.

b (You can easily work out the numbers and slopes on the axes.)



79 **a** The ball is dropped from a height of h_1 so its speed right before impact will be given by (applying conservation of energy) $mgh_1 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gh_1}$. The ball will leave the floor on its way up with a speed found in the same way: $\frac{1}{2}mv_2^2 = mgh_2 \Rightarrow v_2 = \sqrt{2gh_2}$. The change in momentum is therefore $mv_2 - (-mv_1) = m(\sqrt{2gh_2} + \sqrt{2gh_1})$ and hence the net average force is $\frac{\Delta p}{\Delta t} = m\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\tau}$.

b $F = 0.250 \times \frac{\sqrt{2 \times 9.81 \times 6.0} + \sqrt{2 \times 9.81 \times 8.0}}{0.125} = 46.8 \approx 47$ N. This is the average net force on the ball. The forces on the ball are the reaction from the floor R and its weight so $R - mg = F \Rightarrow R = mg + F = 46.8 + 0.250 \times 9.81 = 49.2 \approx 49$ N. By Newton's third law this is also the force on the floor exerted by the ball.

80 **a** The question assumes the ball hits normally. From the previous problem the net force is

$$\frac{\Delta p}{\Delta t} = m\frac{v_2 - (-v_1)}{\tau} = m\frac{v_2 + v_1}{\tau}.$$

b This equals $R - mg$ where R is the average force exerted on the ball by the floor. Hence $R = m\dfrac{v_2 + v_1}{\tau} + mg$.

81 **a** From 0.5 s to 1.5 s i.e. for 1 s.

b A rough approximation would be to treat the area a triangle (of area $\frac{1}{2}$ 1.0 120 = 60 N s) but this too rough and would not be acceptable in a exam. There are roughly 120 rectangles in the area and each has area 0.1 4.0 = 0.4 N s so that the total area is $0.4 \times 120 = 48 \approx 50$ N s.

c From $F_{average}\Delta t = \Delta p$ we thus find $F_{average} = 50$ N.

82 The initial momentum is zero and will remain zero. Therefore the speed with which the 4.0 kg moves off is $2.0 \times 3.0 = 4.0 \times v \Rightarrow v = 1.5 \text{ m s}^{-1}$

The total kinetic energy of the two bodies is $\frac{1}{2} \times 2.0 \times 3.0^2 + \frac{1}{2} \times 4.0 \times 1.5^2 = 13.5 \approx 14$ J.

83 The initial momentum is zero and so must remain zero for the rocket-fuel system. $0 = (5000 - m)v - m \times (1500 - v)$.

In the first 1 second, $v = 15 \text{ m s}^{-1}$ and so $(5000 - m) \times 15 = m \times 1485 \Rightarrow m = \dfrac{75000}{1500} = 50.0$ kg in 1 second.

Answers to test yourself questions

Topic 3

3.1 Thermal concepts

1 **a** The thermal energy lost by one body must equal the thermal energy gained by the other because of energy conservation.

b The changes in temperature are not, however, necessarily equal because the masses and specific heat capacities may differ.

2 **a** From the definition, $Q = mc\Delta\theta \Rightarrow c = \frac{Q}{m\Delta\theta} = \frac{385}{0.150 \times 5.00} = 513\ \text{J kg}^{-1}\ \text{K}^{-1}$.

b It is the same.

3 The energy provided is $20 \times 3.0 \times 60 = 3600$ J. Hence
$0.090 \times 420 \times 4.0 + 0.310 \times c \times 4.0 = 3600 \Rightarrow c = 2.8 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$.

4 The energy provided is $40 \times 4.0 \times 60 = 9600$ J. Hence
$25 \times 15.8 + 0.140 \times c \times 15.8 = 9600 \Rightarrow c = 4.2 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$. The obvious assumptions are that the liquid and the calorimeter are heated uniformly and that none of the energy supplied gets lost to the surroundings.

5 The loss of potential energy is $mgh = 1360 \times 10 \times 86 = 1.17 \times 10^6$ J. Then,

$$C\Delta\theta = 1.17 \times 10^6 \Rightarrow \Delta\theta = \frac{1.17 \times 10^6}{16 \times 10^3} = 73\ \text{K}$$

6 **a** $C = m_1c_1 + m_2c_2 = 45.0 \times 450 + 23.0 \times 4200 = 1.17 \times 10^5\ \text{J K}^{-1}$.

b $\Delta Q = C\Delta\theta \Rightarrow \frac{\Delta Q}{\Delta t} = C\frac{\Delta\theta}{\Delta t}$. Hence $450 = 1.17 \times 10^5 \times \frac{\Delta\theta}{\Delta t} \Rightarrow \frac{\Delta\theta}{\Delta t} = 3.9 \times 10^{-3}\ \text{K s}^{-1}$. For a change of temperature of 20.0 K we then require a time of $\frac{20}{3.9 \times 10^{-3}} = 5.2 \times 10^3\ \text{s} = 87$ min.

7 The energy transferred from the water and the aluminum container is $Q = 0.300 \times 4200 \times 10 + 0.150 \times 900 \times 10 = 13950$ J. This is used to (a) raise the temperature of ice to the melting point of 0 °C, (b) melt the ice at 0 °C and (c) raise the temperature of the melted ice (which is now water) to the final temperature of 0 °C. Thus $13950 = m \times 2200 \times 10 + m \times 334 \times 10^3 + m \times 4200 \times 10$. Hence, $m = 0.035$ kg.

8 The mass of ice is $m = 20 \times 0.06 \times 900 = 1080$ kg. So we need $Q = 1080 \times 2200 \times 5 + 1080 \times 334 \times 10^3 = 3.7 \times 10^8$ J.

9 **a** Let the surface area (in square meters) of the pond be A. Then in time t the energy falling on the surface will be $Q = 600 \times A \times t$. The volume of ice is $V = A \times 0.01$ and so its mass is $m = (A \times 0.01) \times 900$. Then $600 \times A \times t = (A \times 0.01) \times 900 \times 334 \times 10^3$. We see that the unknown surface of area cancels out and is not required. Then, $t = \frac{0.01 \times 900 \times 334 \times 10^3}{600} = 5010\ \text{s} \approx 84$ min.

b This assumes that none of the incident radiation is reflected from the ice and that all the ice is uniformly heated.

10 **a** $Q_1 = 1.0 \times 2200 \times 10 = 2.2 \times 10^4$ J

b $Q_2 = 1.0 \times 334 \times 10^3 = 3.34 \times 10^5$ J

c $Q_3 = 1.0 \times 4200 \times 10 = 4.2 \times 10^4$ J

d In the melting stage.

11 The water will lose an amount of thermal energy $1.00 \times 4200 \times 10 = 42000\,\text{J}$. This energy is used to melt the ice and then raise the temperature of the melted ice to 10°C. Thus $m \times 334 \times 10^3 + m \times 4200 \times 10 = 42000 \Rightarrow m = 0.112$ kg.

12 Since the specific latent heat of vaporisation of water is so much larger than the specific latent heat of fusion we expect that the final temperature will be greater than the initial 30 °C of the water. Then:
$0.150 \times 4200 \times (T - 30) + 0.100 \times 334 \times 10^3 + 0.100 \times 4200 \times T = 0.050 \times 2257 \times 10^3 + 0.050 \times 4200 \times (100 - T)$.
This long equation can be solved for T (preferably using the Solver of your calculator) to give $T = 95$ °C.

3.2 Modelling a gas

13 There are $\frac{28}{2} = 14$ moles of hydrogen and so $14 \times 6.02 \times 10^{23} \approx 8 \times 10^{24}$ molecules.

14 There are $\frac{6.0}{4.0} = 1.5$ moles.

15 $\frac{2.0 \times 10^{24}}{6.02 \times 10^{23}} \approx 3.3$ moles.

16 Krypton has $\frac{84}{21} = 4.0$ moles; 4.0 moles of carbon correspond to $\frac{12}{4.0} = 3.0$ g of carbon.

17 From $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ i.e. That $\frac{12.0 \times 10^5}{295} = \frac{P_2}{393}$ hence $P_2 = 16.0 \times 10^5$ Pa. (Notice the change to kelvin.)

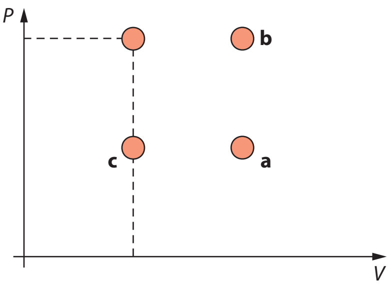
18 From $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ we deduce that $P_1V_1 = P_2V_2$ i.e. That $8.2 \times 10^6 \times 2.3 \times 10^{-3} = 4.5 \times 10^6 \times V_2$ hence $V_2 = 4.2 \times 10^{-3}\ \text{m}^3$.

19 A quantity of 12.0 kg of helium corresponds to $\frac{12 \times 10^3}{4} = 3.0 \times 10^3$ mol. Then from the gas law, $pV = nRT$ we get $P = \frac{nRT}{V} = \frac{3.00 \times 10^3 \times 8.31 \times 293}{5.00 \times 10^{-3}} = 1.46 \times 10^9$ Pa.

20 From the gas law, $pV = nRT$ we get $n = \frac{PV}{RT} = \frac{4.00 \times 1.013 \times 10^5 \times 12.0 \times 10^{-3}}{8.31 \times 293} = 1.998$ mol. Since the mass of one mole of carbon dioxide (CO_2) is 44 g, we need $44 \times 1.9998 = 87.9$ g.

21 We use $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ to get $\frac{P}{n_1} = \frac{\frac{P}{2}}{n_2}$ and hence $n_2 = \frac{n_1}{2}$. In other words to reduce the pressure to half its original value, half the molecules must leave the container. The original number of molecules can be found using $pV = nRT$ to get $n = \frac{PV}{RT} = \frac{5.00 \times 10^5 \times 300 \times 10^{-6}}{8.31 \times 300} = 0.0602$ and hence $N = 0.0602 \times 6.02 \times 10^{23} = 3.62 \times 10^{22}$.
So we will have to lose $\frac{N}{2} = 1.81 \times 10^{22}$ molecules. This will take $\frac{1.81 \times 10^{22}}{3.00 \times 10^{19}} = 603\ \text{s} \approx 10$ min.

22



23 Let there be n_1 moles of the gas in the left container and n_2 in the right. Then it must be true $\left(\text{using } n = \frac{PV}{RT}\right)$ that $n_1 = \frac{12 \times 10^5 \times 6.0 \times 10^{-3}}{RT}$ and $n_1 = \frac{6.0 \times 10^5 \times 3.0 \times 10^{-3}}{RT}$. When the gases mix we will have $n_1 + n_2$ moles in a volume of 9.0 dm^3 and so $n_1 + n_2 = \frac{P \times 9.0 \times 10^{-3}}{RT}$. Hence $\frac{12 \times 10^5 \times 6.0 \times 10^{-3}}{RT} + \frac{6.0 \times 10^5 \times 3.0 \times 10^{-3}}{RT} = \frac{P \times 9.0 \times 10^{-3}}{RT}$.

This means that $P = \frac{12 \times 10^5 \times 6.0 + 6.0 \times 10^5 \times 3.0}{9.0} = 10 \times 10^5 \text{ Pa} = 10 \text{ atm}$.

24 a The cross sectional area of the piston is $A = \frac{V}{h} = \frac{0.050}{0.500} = 0.10 \text{ m}^2$. The pressure in the gas is constant and equal to $P = \frac{F}{A} = \frac{10.0 \times 10}{0.010} = 1.0 \times 10^4 \text{ Pa}$.

b From the gas law, $n = \frac{PV}{RT} = \frac{1.0 \times 10^4 \times 0.050}{8.31 \times 292} = 0.206$. The number of molecules is then $N = 0.206 \times 6.02 \times 10^{23} = 1.24 \times 10^{23}$.

c From $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ we get $\frac{0.050}{292} = \frac{V_2}{425}$ hence $V_2 = 7.3 \times 10^{-2} \text{ m}^3$.

25 The mass is just $28 \times 2 = 56$ g. The volume is found from $V = \frac{nRT}{P} = \frac{2.0 \times 8.31 \times 273}{1.0 \times 10^5} = 0.045 \text{ m}^3$.

26 The molar mass of helium is 4.00 g per mole. A mass of 70.0 kg of helium corresponds to $\frac{70.0 \times 10^3}{4.00} = 1.75 \times 10^4$ mol. Thus $P = \frac{nRT}{V} = \frac{1.75 \times 10^4 \times 8.31 \times 290}{404} = 1.04 \times 10^5 \text{ Pa}$.

27 a $n = \frac{PV}{nRT} = \frac{150 \times 10^3 \times 5.0 \times 10^{-4}}{8.31 \times 300} = 3.01 \times 10^{-2}$ mol.

b $N = nN_A = 3.01 \times 10^{-2} \times 6.02 \times 10^{23} = 1.8 \times 10^{22}$

c $M = n\mu = 3.01 \times 10^{-2} \times 29 = 0.87$ g

28 a $V = \frac{nRT}{P} = \frac{1.0 \times 8.31 \times 273}{1.0 \times 10^5} = 2.27 \times 10^{-2} \text{ m}^3$

b We have 1 mole and so 4.00 g of helium. The density is thus $\rho = \frac{M}{V} = \frac{4.00 \times 10^{-3}}{2.27 \times 10^{-2}} = 0.176 \text{ kg m}^{-3}$.

c The change for oxygen is just the molar mass and so $\rho = \frac{32}{4} \times 0.176 = 1.41 \text{ kg m}^{-3}$.

29 Under the given changes the volume will stay the same and so the density will be unchanged.

30 We use $\frac{1}{2}mc^2 = \frac{3}{2}kT$. The mass of a molecule is $\frac{4.0}{6.02\times10^{23}} = 6.64\times10^{-23}$ g $= 6.64\times10^{-27}$ kg. Hence

$c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times850}{6.64\times10^{-27}}} \approx 2300\ \text{m s}^{-1}$.

31 From $\frac{1}{2}mc^2 = \frac{3}{2}kT$ we get $c = \sqrt{\frac{3kT}{m}}$. The mass of a molecule (in kg) is $\frac{M}{N_A}$

$\frac{4.0}{6.02\times10^{23}} = 6.64\times10^{-23}$ g $= 6.64\times10^{-27}$ kg. Hence $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kN_AT}{M}} = \sqrt{\frac{3RT}{M}}$ since $k = \frac{R}{N_A}$.

32 a $\frac{1}{2}mc^2 = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times300 = 6.2\times10^{-21}$ J

b $\frac{1}{2}m_1c_1^2 = \frac{1}{2}m_2c_2^2 \Rightarrow \frac{c_1}{c_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{\mu_2/N_A}{\mu_1/N_A}} = \sqrt{\frac{32}{4}} = \sqrt{8}$

Answers to test yourself questions

Topic 4

4.1 Oscillations

1 **a** An oscillation is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion i.e. the motion repeats in some way.

b In simple harmonic motion, the displacement from an equilibrium position and the acceleration are proportional and opposite each other.

2 It is an oscillation since we may define the displacement of the particle from the middle point and in that case the displacement changes direction and the motion repeats. The motion is not simple harmonic however since there is no acceleration that is proportional (and opposite) to the displacement.

3 It is an oscillation since the motion repeats. The motion is not simple harmonic however since the acceleration is constant and is not proportional (and opposite) to the displacement.

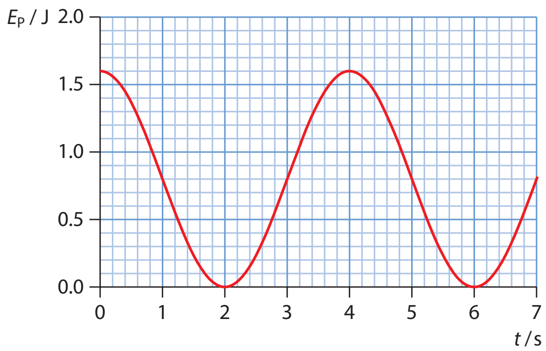
4 **a** The acceleration is opposite to the displacement so every time the particle is displaced there is a force towards the equilibrium position.

b The acceleration is not proportional to the displacement; if it were the graph would be a straight line through the origin.

5 **a** **i** It was not intended to ask about the mass – apologies!

ii The period is 8.0 s; the particle is at one extreme position at $t = 0$ and again at $t = 4.0$ s. This is half a period.

b



4.2 Travelling waves

6 The delay time between you seeing the person next to you stand up and you standing up and the number density of the people i.e. how many people per unit meter. For a fixed delay time, the closer the people are the faster the wave.

7 There is a disturbance that travels through the lie of dominoes just as a disturbance travels through a medium when a wave is present. You can increase the speed by placing them closer together. An experiment to investigate this might be to place a number of dominoes on a line of fixed length such that the dominoes are a fixed distance d apart. We must give the same initial push to the first domino (for example using a pendulum that is released from a fixed height and strikes the domino at the same place. We then measure time form when the first domino is hit until the last one is hit. Dividing the fixed distance by the time taken gives the speed of the pulse. We can then repeat with a different domino separation and see how the speed depends on the separation d.

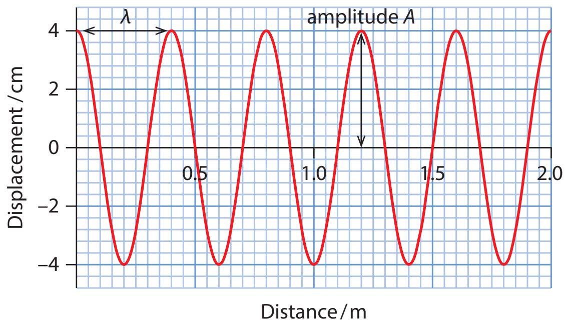
8 a Wavelength – the length of a full wave; the distance between two consecutive crests or troughs

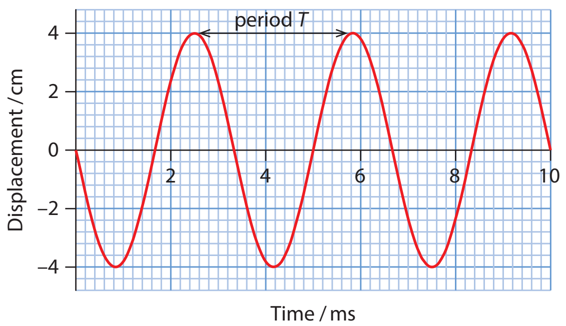
b Period – the time needed to produce one full oscillation or wave

c Amplitude – the largest value of the displacement from equilibrium of an oscillation

d Crest – a point on a wave of maximum displacement

e Trough – a point on a wave of minimum displacement





9 a In wave motion displacement refers to the difference in the value of a quantity such as position, pressure, density etc when the wave is present and when the wave is absent.

b In a transverse wave the displacement is at right angles to the direction of energy transfer, in a longitudinal it is parallel to the energy transfer direction.

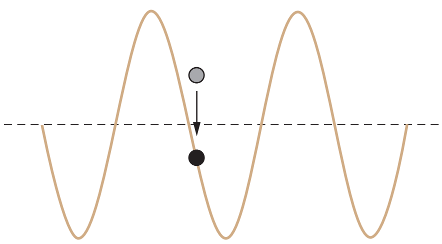
c The falling stone imparts kinetic energy to the water at the point of impact and so that water moves. It will continue moving (creating many ripples) until the energy is dissipated.

d We must recall that the intensity of a wave is proportional to the square of the amplitude. The amplitude will decrease for two reasons: first, some energy is bound to be dissipated as the wave moves away and so the amplitude has to decrease. Second, even in the absence of any energy losses, the amplitude will still decrease because the wavefronts get bigger as they move away from the point of impact of the ripple. The energy carried by the wave is now distributed on a longer wavefront and so the energy per unit wavefront length decreases. The amplitude must then decrease as well.

10 a From left to right: down, down, up.

b From left to right: up, up, down.

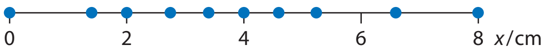
11



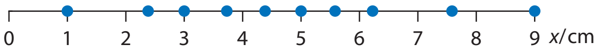
12 a $\lambda = \frac{v}{f} = \frac{330}{256} = 1.29$ m.

b $\lambda = \frac{v}{f} = \frac{330}{25 \times 10^3} = 1.32 \times 10^{-2}$ m.

13 a A wave in which the displacement is parallel to the direction of energy transferred by the wave.

b i 

ii At $x = 4.0$ cm

c i 

ii The compression is now at $x = 5.0$ cm.

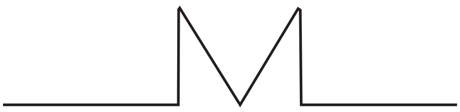
14 a $f = \frac{v}{\lambda} = \frac{340}{0.40} = 850$ Hz

b i A compression occurs at $x = 0.30$ m. Molecules just to the left of this point have positive displacement and so move to the right. Molecules just to the right move to the left creating the compression at $x = 0.30$ m.

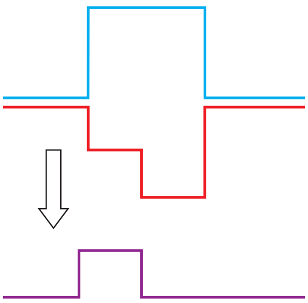
ii By similar reasoning $x = 0.10$ m is a point where a rarefaction occurs.

4.3 Wave characteristics

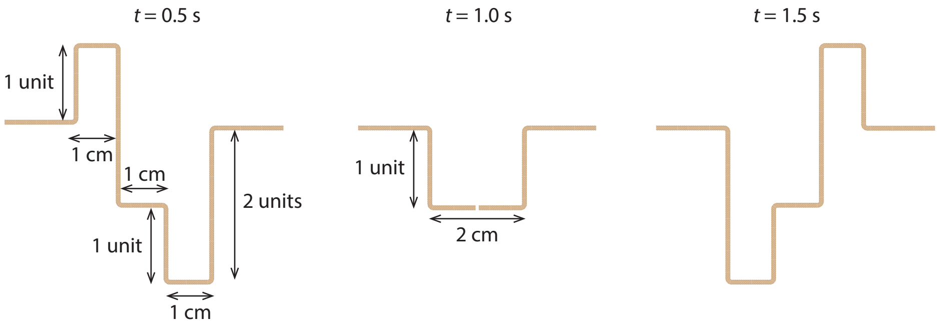
15 Adding the pulses point by point gives the following diagram.



16

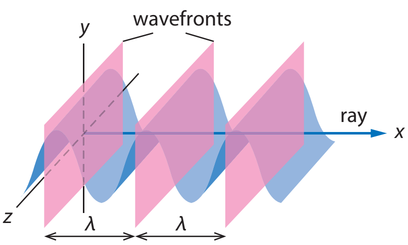


17 Adding the pulses point by point gives the following diagram.

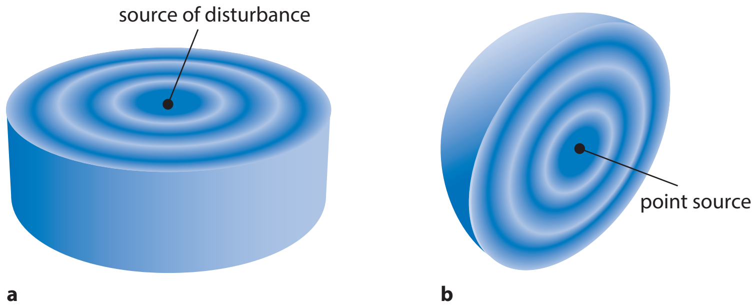


18 We add the pulses point by point. For example at $x = 0$ both waves have zero displacement and so we get zero displacement for the sum. At $x = 10$ cm, the blue pulse has $y = 0.50$ cm and the red pulse has $y = 0.75$ cm. The sum is 1.25 cm. At $x = 20$ cm, the blue pulse has $y = 0$ and the red pulse has $y = 1.0$ cm. The sum is 1.0 cm. At $x = 30$ cm, the blue pulse has $y = -0.50$ cm and the red pulse has $y = 0.70$ cm. The sum is 0.20 cm and so on.

19 a A wavefront is a surface on which all points have the same phase.



b A ray is the direction normal to wavefronts that corresponds to the direction of energy transfer.



20 **a** Polarised light is light in which the electric field oscillates on the same plane.

b Light can be polarised by passage through a polariser and by reflection off a n on-metallic surface.

21 In a polarised wave the displacement must be on the same plane. In a longitudinal wave the displacement is along the direction of energy transfer and so belongs to an infinity of planes at the same time. Hence it cannot be polarised.

22 **a** The light is not polarised. In the case of unpolarised light incident on an analyser, the intensity of the transmitted light would be half the incident intensity and so constant as required in the question.

b Since there is an orientation (call it X) of the analyser that makes the transmitted intensity zero, it follows that the incident light was polarised in a direction at right angles to the direction X.

c Since the intensity never becomes zero the light was not polarised. Since the intensity varies however, it follows that the incident light has unequal components in various directions so it is partially polarised.

23 **a** This relates the transmitted intensity I to the incident intensity I_0 when polarised light is incident and then transmitted through an analyser. The relation is $I = I_0 \cos^2 \theta$ where θ is the angle between the transmission axis and the direction of the incident electric field.

b $\frac{I}{I_0} = \cos^2 \theta = \cos^2 25° = 0.82$

24 **a** The light transmitted through the first polariser will be polarised in a given direction. The second polariser's axis is at right angles to this direction so the electric field has zero component along the axis of the second polariser. Hence no light gets transmitted.

b Light will be transmitted since now there will be a component of the electric field along the second polariser's axis.

c The situation is now identical to **a** and so no light goes through.

4.4 Wave behaviour

25 **a** From $1.00 \times \sin 38° = 1.583 \times \sin \theta_2$ we find $\sin \theta_2 = \frac{1.00 \times \sin 38°}{1.583} \Rightarrow \theta_2 = \sin^{-1} 0.3889 = 22.9°$.

b $n = \frac{c}{c_g} \Rightarrow c_g = \frac{c}{n} = \frac{3.0 \times 10^8}{1.583} = 1.9 \times 10^8 \text{ m s}^{-1}$

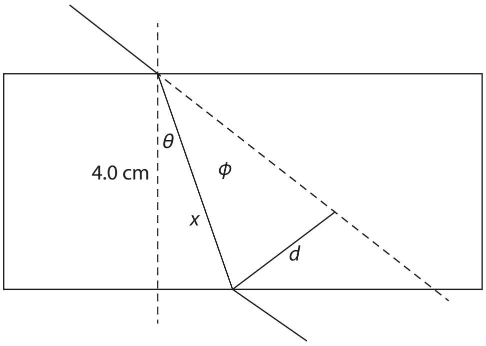
c The frequency in water is the same as that in air and so $\lambda_g = \frac{\lambda_a}{n} = \frac{6.8 \times 10^{-7}}{1.583} = 4.3 \times 10^{-7}$ m.

26 **a** $t = \frac{s}{c} = \frac{3.0}{3.0 \times 10^8} = 1.0 \times 10^{-8}$ s

b In this time, $1.0 \times 10^{-8} \times 6.0 \times 10^{14} = 6.0 \times 10^6$ full waves have been emitted. (Or, the wavelength is

$\lambda = \frac{3.0 \times 10^8}{6.0 \times 10^{14}} = 5.0 \times 10^{-7}$ m and in a length of 3.0 m we can fit $\frac{3.0}{5.0 \times 10^{-7}} = 6.0 \times 10^6$ full waves.)

27 First we find the angle of refraction (angle θ in the diagram).

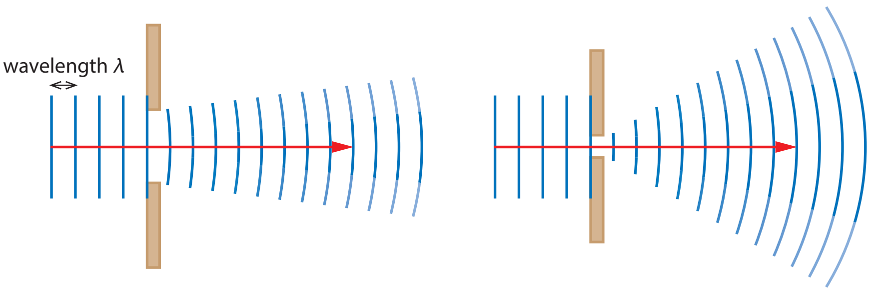


$1.00 \times \sin 40° = 1.450 \times \sin \theta$, hence $\theta = 26.3°$. This means that $x = \dfrac{4.0}{\cos 26.3°} = 4.46$ cm.

Now $\varphi = 40°2 - 26.3° = 13.7°$ and so $d = 4.46 \times \sin 13.7° = 1.06$ cm.

28 Let θ be the angle of incidence from air. The angle of refraction will be larger than θ and so as θ increases the angle of refraction will become $90°$ and so will not enter water. This happens when $\dfrac{\sin\theta}{340} = \dfrac{\sin 90°}{1500} \Rightarrow \theta = \sin^{-1}\dfrac{340}{1500} = 13.1°$.

29 The diagram must be similar to the one below.



30 There is no appreciable diffraction here; the wave continues straight through the opening.

31 There is poor reception because of destructive interference between the waves reaching the antenna directly and those reflecting off the mountain. The path difference is double the distance between the house and the mountain. The wave reflecting off the mountain will suffer a phase difference of π and so the condition for destructive interference is $2d = n\lambda$. The smallest d (other than zero) corresponds to $n = 1$ and so $d = 800$ m.

4.5 Standing waves

32 A standing wave is a special wave formed when two identical traveling waves moving in opposite directions meet and then superpose. This wave, unlike a traveling wave, has nodes i.e. points where the displacement is *always* zero. The antinodes, points where the displacement is the largest do not appear to be moving. A standing wave differs from a traveling wave in that it does not transfer energy and that the amplitude is variable. In a standing wave points in between consecutive nodes have the same phase whereas in a travelling wave the phase changes from zero to 2π after a distance of one wavelength.

33 A standing wave is formed when two identical traveling waves moving in opposite directions meet and then superpose.

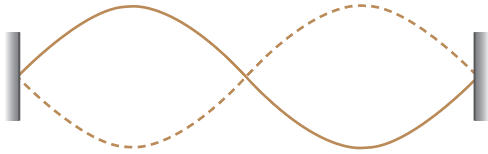
34 **a** A node is a point in the medium where the displacement is *always* zero.

b An antinode is a point in the medium where the displacement, at some instant, will assume its maximum value.

c Speed refers to the speed of the travelling waves whose superposition gives the standing wave.

35 **a** We must disturb the string with a frequency that is equal to the frequency of the second harmonic.

b



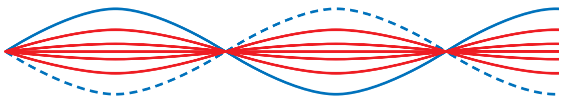
36 The wavelength of the wave will remain the same (and equal to twice the length of the string). Since the speed increases by $\sqrt{2}$ the frequency must do the same and so is 354 Hz.

37 The first harmonic has wavelength $2L$ (L is the length of the string) and the second a wavelength L. The ratio of the *frequencies* is then 2 since the speed is the same.

38 **a** The wavelength of the fundamental is $2L = 1.00$ m. The frequency is then $f = \frac{v}{2L} = 225$ Hz

b The sound produced by the vibrations of the string will have the same frequency i.e. 225 Hz and so the wavelength of sound will be $\lambda = \frac{c}{f} = \frac{340}{225} = 1.51$ m.

39



40 The wavelength of sound is $\lambda = \frac{c}{f} = \frac{340}{306} = 1.11$ m. Standing waves have wavelength given by $\lambda = \frac{4L}{n}$ with $n = 1, 3, 5, \ldots$. Therefore $\frac{4L}{n} = 1.11 \text{ m} \Rightarrow L = \frac{1.11 \times n}{4}$. This gives 0.28 m and 1.4m for $n = 3$ and $n = 5$.

41 **a** The wavelength is given by $\lambda = \frac{4L}{n} = \frac{0.800}{n}$ and also by $\lambda = \frac{c}{f} = \frac{c}{427}$. Hence

$\frac{c}{427} = \frac{0.800}{n} \Rightarrow c = \frac{427 \times 0.800}{n} = \frac{342}{n} \text{ m s}^{-1}$. The answer makes physical sense only if $n = 1$ (the first harmonic is established) in which case $c = 342 \text{ m s}^{-1}$.

b The next harmonic will have wavelength $\frac{4L'}{n} = 0.800 \Rightarrow L' = \frac{0.800n}{4} = 0.200n$. With $n = 3$ we get $L' = 0.600$ m.

42 **a** The wavelengths in the open tube are given by $\lambda = \frac{2L}{n}$. The frequencies of two *consecutive* harmonics are then $\left(f = \frac{c}{\lambda} = \frac{cn}{2L}\right)$, $300 = \frac{cn}{2L}$ and $360 = \frac{c(n+1)}{2L}$. This means that

$\frac{360}{300} = \frac{\frac{c(n+1)}{2L}}{\frac{cn}{2L}} \Rightarrow \frac{n+1}{n} = 1.2 \Rightarrow n + 1 = 1.2n \Rightarrow 0.2n = 1 \Rightarrow n = 5$; we have the fifth and sixth harmonics.

b We get $300 = \frac{340 \times 5}{2 \times L} \Rightarrow L = 2.833 \approx 2.8$ m.

43 The two harmonics have the same frequency and hence the same wavelength. The wavelength of the first harmonic in the open–open pipe is $\lambda = 2L_X$. The wavelength of the first harmonic in the closed–open pipe is $\lambda = 4L_Y$. Hence $2L_X = 4L_Y \Rightarrow \frac{L_X}{L_Y} = 2$.

44 With one step per second you shake the cup with a frequency of about 1 Hz. In the first harmonic mode the wavelength would be about twice the diameter of the cup i.e. 16 cm (we have antinodes at each end). This gives a speed of $v = 1 \times 16 = 16$ cm s^{-1}.

45 **a** A standing wave is made up of two traveling waves. The speed of energy transfer of the traveling waves is taken to be the speed of the standing wave.

b From $y = 5.0\cos(45\pi t)$ we deduce that the frequency of oscillation of point P and hence also of the wave is $\frac{45\pi}{2\pi} = 22.5$ Hz. The wavelength is then $\lambda = \frac{v}{f} = \frac{180}{22.5} = 8.0$ m. Since the diagram shows a second harmonic this is also the length of the string.

c The phase difference is π and so $y = 5.0\cos(45\pi t + \pi) = -5.0\cos(45\pi t)$.

46 **a** The hit creates a longitudinal wave that travels down the length of the rod and reflects of the end. The reflected waves pushes the hammer back.

b $v = \frac{s}{t} = \frac{2.4}{0.18 \times 10^{-3}} = 1.3 \times 10^{4}\ \text{m s}^{-1}$

c We assume free-free end points and so the wavelength is given by 2.4 m. The frequency is then

$$f = \frac{c}{\lambda} = \frac{1.3 \times 10^{4}}{2.4} = 5.6\ \text{kHz}.$$

Answers to test yourself questions

Topic 5

5.1 Electric fields

1 **a** $F = \frac{kQ_1Q_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{(5.0\times10^{-2})^2} = 28.8 \approx 29\text{ N}$

b The force would become 4 times as small.

i $F' = \frac{kQ_1Q_2}{(2r)^2} = \frac{F}{4}$

ii $F' = \frac{k2Q_1Q_2}{(2r)^2} = \frac{F}{2}$

iii $F' = \frac{k2Q_1\times2Q_2}{(2r)^2} = F$

2 The middle charge is attracted to the left by the charge on the left with a force of $F_1 = \frac{kqQ_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{(4.0\times10^{-2})^2} = 45\text{ N}$. It is attracted to the right by the charge on the right with a force of $F_2 = \frac{kqQ_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times3.0\times10^{-6}}{(2.0\times10^{-2})^2} = 135\text{ N}$. The net force is thus $135-45=90\text{ N}$ directed towards the right.

3 Suppose we call the distance (in cm) from the left charge x. Then we need

$$\frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{x^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times3.0\times10^{-6}}{(6-x)^2}$$

$$\frac{4.0}{x^2} = \frac{3.0}{(6-x)^2}$$

This means that

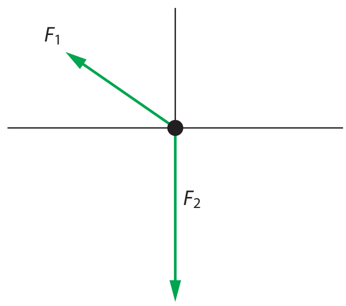
$4.0(6-x)^2 = 3.0x^2$

$4(36-12x+x^2) = 3x^2$

$x^2-48x+144=0$

The solution is $x = 3.22$ cm.

4 The forces are as shown. The distance between the charge Q and the charge 2Q is 5.0 cm.

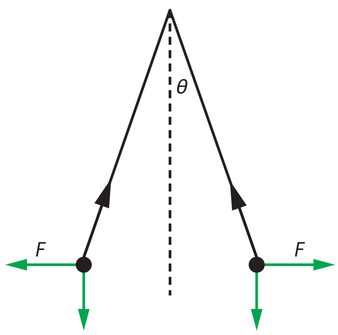


The magnitudes are $F_1 = \frac{kQ(2Q)}{d^2} = \frac{8.99\times10^9 \times 3.0\times10^{-6} \times 6.0\times10^{-6}}{(5.0\times10^{-2})^2} = 64.7$ N and

$F_2 = \frac{kQQ}{d^2} = \frac{8.99\times10^9 \times (3.0\times10^{-6})^2}{(3.0\times10^{-2})^2} = 89.9$ N. We need to find the components of F_1:

$F_{1x} = 64.7\cos\theta = 64.7\times\frac{4}{5} = 51.76$ N and $F_{1y} = 64.7\sin\theta = 64.7\times\frac{3}{5} = 38.82$ N. The components of the net force are: $F_x = -51.76$ N and $F_y = 38.82 - 89.9 = -51.08$ N. The net force has magnitude $F = \sqrt{51.76^2 + 51.08^2} = 72.7 \approx 73$ N and direction $180° + \arctan\frac{51.08}{51.76} = 224.6° \approx 225°$.

5 a A diagram is the following in which the angle θ of each string to the vertical is given by $\sin\theta = \frac{5}{85} \Rightarrow \theta = 3.37$.



We have that $T\cos\theta = mg$ and $T\sin\theta = F = \frac{kQ^2}{d^2}$ so that dividing side by side gives

$$\tan\theta = \frac{kQ^2}{mgd^2} \Rightarrow Q = \sqrt{\frac{mgd^2\tan\theta}{k}} = \sqrt{\frac{100\times10^{-6}\times9.8\times0.1^2\times\tan 3.37°}{8.99\times10^9}} = 8.0\times10^{-9}\text{ C}.$$

b This corresponds to $\frac{8.0\times10^{-9}}{1.6\times10^{-19}} = 5.0\times10^{10}$ electronic charges.

6 a Since the molar mass of water is 18 g per mole, a mass of 60 kg corresponds to $\frac{60\times10^3}{18} = 3333$ moles i.e. $3333\times6.02\times10^{23} = 2\times10^{27}$ molecules of water. A molecule of water contains 10 electrons (2 from hydrogen and 8 from oxygen) and so we have 2×10^{28} electrons in each person.

b The electric force is therefore $F_1 = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9\times(2.0\times10^{28}\times1.6\times10^{-19})^2}{(10)^2} = 9\times10^{26} \approx 10^{27}$ N, an enormous force.

c Assumptions include the use of Coulomb's law for objects that are not point charges, assuming the same distance between charges etc.

d We have neglected the existence of protons which gives each person a zero electric charge and hence zero electric force.

7 $E = \frac{F}{q} = \frac{3.0\times10^{-5}}{5.0\times10^{-6}} = 6.0\text{ N C}^{-1}$

8 The magnitude of each of the fields produced at P is: $E = \frac{kQ}{r^2} = \frac{9.0\times10^9\times2.00\times10^{-6}}{\left(\sqrt{0.05^2+0.30^2}\right)^2} = 1.95\times10^5\text{ N C}^{-1}$. The vertical components of the electric fields will cancel out leaving only the horizontal components. The horizontal component is $E_x = E\cos\theta = E\frac{d}{\sqrt{d^2+\frac{a^2}{4}}} = 1.95\times10^5\times\frac{0.30}{\sqrt{\frac{0.10^2}{4}+0.30^2}} = 1.92\times10^5\text{ N C}^{-1}$. The net field is then directed horizontally to the right and has magnitude $2\times1.92\times10^5 = 3.84\times10^5\text{ N C}^{-1}$.

9 The two electric fields are $E_1 = E_2 = 1.95 \times 10^5\,\mathrm{N\,C^{-1}}$. Adding vectorially by taking components gives $E_x = 0$ and

$$E_y = 2 \times 1.95 \times \times 10^5 \times \sin\theta = 2 \times 1.95 \times 10^5 \times \frac{0.05}{\sqrt{\frac{0.10^2}{4} + 0.30^2}} = 6.4 \times 10^5\,\mathrm{N\,C^{-1}}.$$

10 $I = nqAv = 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi(0.90 \times 10^{-3})^2 \times 3.6 \times 10^{-4} = 12.45 \approx 12\ \mathrm{A}$

11 **a** The current will be the same by conservation of charge.

b Since $I = nqAv$, we have that $A_1v_1 = A_2v_2$ and so $v_2 = \frac{A_1v_1}{A_2} = \frac{r_1^2v_1}{r_2^2} = \frac{1.0^2 \times 2.2 \times 10^{-4}}{2.0^2} = 5.5 \times 10^{-5}\ \mathrm{m\,s^{-1}}$.

12 From $I = nqAv$ we get $v = \frac{I}{nqA} = \frac{5.0}{5.8 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (2 \times 10^{-3})^2} = 4.3 \times 10^{-5} \approx 4 \times 10^{-5}\ \mathrm{m\,s^{-1}}$.

13 **a** One hour is $1 \times 60 \times 60 = 3600$ s and so $Q = It = 10 \times 3600 = 3.6 \times 10^4$ C.

b $N = \frac{Q}{e} = \frac{3.6 \times 10^4}{1.6 \times 10^{-19}} = 2.25 \times 10^{23} \approx 2.2 \times 10^{23}$

14 **a** and **b** These points are inside the conducting sphere so the electric field is zero there.

c $E = \frac{kQ}{R^2} = \frac{8.99 \times 10^9 \times 4.0 \times 10^{-6}}{(15.0 \times 10^{-2})^2} = 1.6 \times 10^6\ \mathrm{N\,C^{-1}}$

d At 20 cm, $E = 1.6 \times 10^6 \times \left(\frac{15}{20}\right)^2 = 9.0 \times 10^5\ \mathrm{N\,C^{-1}}$

5.2 Heating effect of electric currents

15 Electrons making up the current collide with lattice atoms and transfer some of their kinetic energy to these atoms. The average kinetic energy of the atoms increases and since temperature is proportional to the average kinetic energy of the atoms the temperature of the wire increases. The electric field keeps accelerating the electrons and so this process continues.

16 Doubling the length of the wire doubles the potential difference across its ends while the current stays the same. Since $R = \frac{V}{I}$ the resistance doubles.

17 **a** Yes since the graphs are straight lines through the origin.

b The resistance for wire A is lower and so this wire corresponds to the lower temperature.

18 Since the resistance is constant, $\frac{6.0}{1.5} = \frac{V}{3.5} \Rightarrow V = 14\ \mathrm{V}$.

19 It obeys Ohm's law so the resistance is the same, 12 Ω.

20 $R = \frac{V}{I} = \frac{220}{15} = 15\ \Omega$

21 **a** $V = IR = 2 \times 4 = 8$ V across the first and $V = IR = 2 \times 6 = 12$ V across the second.

b There is no potential difference between B and C since there is no resistance between these points.

22 **a** The resistance is $R = \frac{V^2}{P} = \frac{220^2}{120} = 403 \approx 4.0 \times 10^2\ \Omega$.

b $403 = \frac{2.0 \times 10^{-6} \times L}{\pi \times (0.03 \times 10^{-3})^2} \Rightarrow L = 0.57\ \mathrm{m}$

23 **a** The two 4.0 Ω resistors are in series and are equivalent to 8.0 Ω. The lower two 2.0 Ω are equivalent to 4.0 Ω. The 8.0 Ω and the 4.0 Ω resistors are in parallel and are equivalent to $\frac{1}{R}=\frac{1}{8.0}+\frac{1}{4.0}\Rightarrow R=2.7\ \Omega$.

b The 6.0 Ω and 4.0 Ω resistors are in parallel and are equivalent to $\frac{1}{R}=\frac{1}{6.0}+\frac{1}{4.0}\Rightarrow R=2.4\ \Omega$. This and the other two are in series for an total of $R=2.0+2.4+8.0=12.4\ \Omega$.

c All three are in parallel for a total of $\frac{1}{R}=\frac{1}{3.0}+\frac{1}{3.0}+\frac{1}{3.0}\Rightarrow R=1.0\ \Omega$.

24 We have that $12=I(R_1+R_2)$ and $\varepsilon=IR_2$ where R_1 is the resistance of wire AC and R_2 the resistance of wire BC. Thus $\frac{\varepsilon}{12}=\frac{R_2}{R_1+R_2}$. But the resistances are proportional to the lengths and so $\frac{\varepsilon}{12}=\frac{54}{100}\Rightarrow\varepsilon=6.48$ V.

25 **a** Applying Kirchoff's laws to the two loops gives: $3.0=20(x+y)+30(x+y)$ and

$2.0=20(x+y)+30(x+y)+10y$. These simplify to

$3.0=50x+50y$

$2.0=50x+60y$

These are solved to give $10y=-1.0\Rightarrow y=-0.10$ A, and $x=0.16$ A.

b The potential differences are

$20\ \Omega: V=20(x+y)=20\times(0.16-0.10)=1.2$ V

$30\ \Omega: V=30(x+y)=30\times(0.16-0.10)=1.8$ V

$10\ \Omega: V=10y=10\times(-0.10)=-1.0$ V

26 Applying Kitchhoff's law: $9.0+3.0=4.0x$ so right away $x=3.0$ A. $3.0=-3.0(x-y)-2.0(x-y)=-5x+5y$. Hence $y=3.6$ A.

27 Applying Kitchhoff's law: $9.0=2.0x+5.0\times1.0\Rightarrow x=2.0$ A. In second loop $\varepsilon=3.0\times1.67+5.0\times1.0=10$ V.

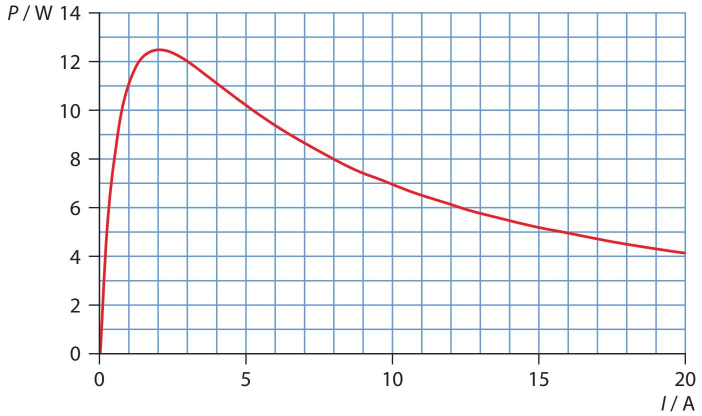
28 **a** From the graph, when the potential difference across each resistor is 1.5 V the current in X is about 2.68 A and in Y 1.55 A for a total current leaving the cell of 4.2 A.

b This has to be done by trial and error. The voltage across X plus that across Y must give 1.5 V. This is achieved for a current of about 1.1 A for which the voltages are 0.5 V and 1.0 V adding up to 1.5 V.

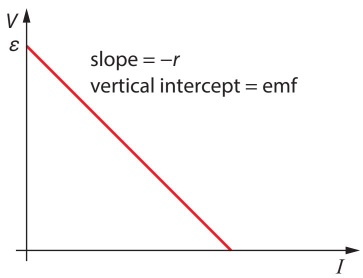
29 The top loop gives: $6.0=4.0x+3.2x\Rightarrow x=0.833$ A. The lower loop gives: $2.0=R\times0.8333x\Rightarrow R=2.4\ \Omega$. $\frac{L}{1}=\frac{2.4}{4}\Rightarrow L=0.60$ m.

5.2 Electric cells

30 Chemical energy in the top cell gets converted into thermal energy in the resistor, mechanical energy (and some thermal energy) in the motor (which in turn gets converted into gravitational potential energy as the load is being raised) and finally electrical energy that charges the lower battery.

31 

$R = 2.0\ \Omega$.

32 a 

b Since $V = \varepsilon - Ir$,

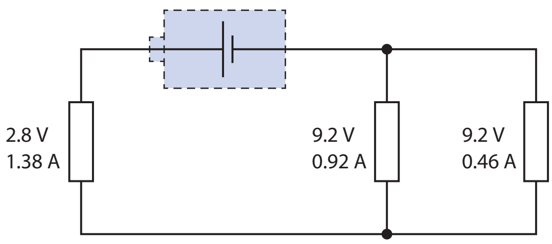
i the slope is the negative of the internal resistance of the battery

ii the vertical intercept is the emf of the battery

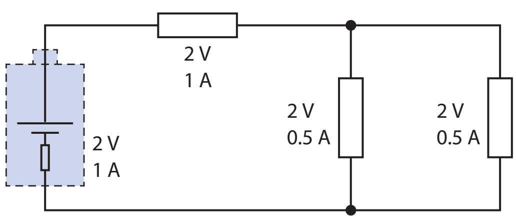
33 a The internal resistance is the slope and so equals $-r = \frac{2.4 - 9.6}{8.0 - 2.0} \Rightarrow r = 1.2\ \Omega$.

b Extending the line to find the vertical intercept gives an emf of about 12 V.

34 a The two parallel resistors are equivalent to $\frac{1}{R} = \frac{1}{10} + \frac{1}{20} \Rightarrow R = 6.67\ \Omega$. The total resistance of the circuit is then $R_T = 8.67\ \Omega$. The total current is then $I_T = \frac{12.0}{8.67} = 1.38$ A. The potential difference across the 2.0 Ω resistor is $V = 1.38 \times 2.0 = 2.77 \approx 2.8$ V. The potential difference across the parallel resistors is then $V = 12.0 - 2.77 = 9.2$ V. So each of the two resistors get a current of $\frac{9.2}{10} = 0.92$ A and $\frac{9.2}{20} = 0.46$ A.



b The two parallel resistors have a total of 2.0 Ω making a total circuit resistance of 6.0 Ω. The total current is then $I_T = \frac{6.0}{6.0} = 1.0$ A. The internal resistance and the 2.0 Ω resistor get 1.0 A of current and the potential difference across each is 2.0 V. The potential difference across the parallel combination is 2.0 V and so each gets 0.50 A of current.



35 **a** and **b** Let r be the internal resistance and ε the emf. The total resistance when in parallel is $2.0 + r$ and so $3.0 = \frac{\varepsilon}{2.0 + r}$. When in series the total resistance is $8.0 + r$ and so $1.4 = \frac{\varepsilon}{8.0 + r}$. We must solve the system of equations $3.0 = \frac{\varepsilon}{2.0 + r}$

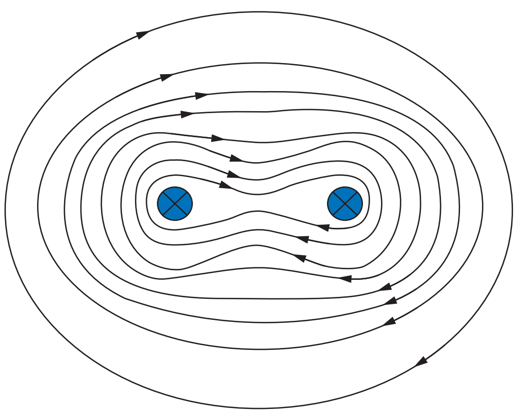
$$1.4 = \frac{\varepsilon}{8.0 + r}$$

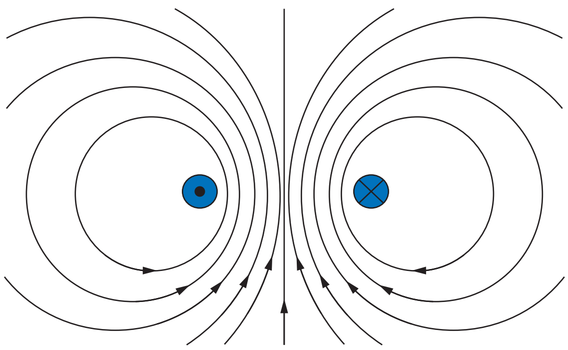
Dividing side by side $\frac{3.0}{1.4} = \frac{8.0 + r}{2.0 + r} \Rightarrow 2.14 = \frac{8.0 + r}{2.0 + r} \Rightarrow 4.28 + 2.14r = 8.0 + r \Rightarrow 1.14r = 3.17 \Rightarrow r = 3.25\ \Omega$ and so $\varepsilon = 5.25 \times 3.0 \approx 16$ V.

36 **a** The current everywhere is the same, call it x. Then $9.0 - 3.0 = 8x \Rightarrow x = 0.75$ A.
b The power in the top cell is $9.0 \times 0.75 \approx 6.8$ W and in the lower it is $-3.0 \times 0.75 \approx -2.2$ W.
c The power in the lower cell is negative implying that it is being charged.

5.4 Magnetic fields

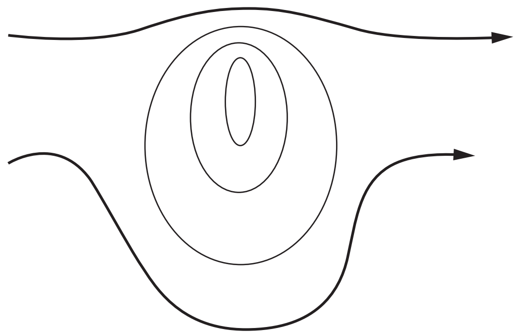
37





38 We must apply the right hand rule for force
a The magnetic field is into the page
b The force is into the page
c The magnetic field is out of the page
d The force is zero since the velocity is anti parallel to the field
e The force is zero since the velocity is parallel to the field

39



40 The magnetic field is directed into the page. In **a** the right hand rule (for a negative charge) gives a force downwards away from the wire. In **b** it gives a force to the right.

41 **a** the field is to the right and so the force is into the page
b the velocity is parallel to the field and the force is zero
c the force is towards the magnet (up the page)

42 **a** $eE = evB \Rightarrow B = \frac{E}{e} = \frac{2.4\times 10^{3}}{2.0\times 10^{5}} = 1.2\times 10^{-2}$ T. The electric force is upwards so the magnetic force is downwards. Therefore the field must be into the page.
b The condition in **a** is independent of charge and mass so the proton will be undeflected as well.
c The electric force will stay the same but the magnetic force will double. Therefore the electron will be deflected downwards.

43 **a** There are equal and opposite forces at the poles of the magnet giving a net force of zero.
b The forces are opposite so they will rotate the magnet counterclockwise.

44 The force is $F = BIL\sin\theta = 5.00\times 10^{-5}\times 3000\times 30.0\times \sin 30° = 2.25$ N.

45 **Note:** This requires knowledge of circular motion (Topic 6).
a We have that $evB = m\frac{v^2}{r} \Longrightarrow v = \frac{eBr}{m}$. But $v = 2\pi f$ and so $2\pi rf = \frac{eBr}{m} \Rightarrow f = \frac{eB}{2\pi m}$.
b The mass is different and so the answer changes.

46 **a** The combined magnetic field from the two wires at point R must point downwards so as to cancel the uniform field. Since R is closer to Q, the field of Q is larger than the field from P. Hence the current in Q must go out of the page.
b If the current increases, the net field from P and Q increases as well, so that the total field at R is no longer zero. If we move closer to Q the field from Q will be much larger than the field from P and so their combined field will be downwards and much larger than external field. Hence the point has to move to the left.

Answers to test yourself questions

Topic 6

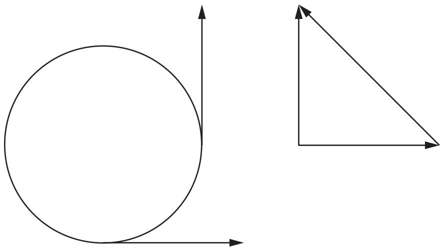
6.1 Circular motion

1 a The angular speed is just $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.24} = 5.07 \text{ rad s}^{-1}$. The linear speed is $v = \omega R = 5.07 \times 3.50 = 17.7 \text{ m s}^{-1}$.

b The frequency is $f = \frac{1}{T} = \frac{1}{1.24} = 0.806 \text{ s}^{-1}$.

2 $a = 4\pi^2 rf^2 = 4\pi^2 \times 2.45 \times (3.5)^2 = 1.2 \times 10^3 \text{ m s}^{-2}$.

3 a The average acceleration is defined as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. The velocity vectors at A and B and the change in the velocity $\Delta\vec{v}$ are shown below.



The magnitude of the velocity vector is 4.0 m s^{-1} and it takes a time of $\frac{2\pi \times 2.0}{4.0} = 3.14$ s to complete a full revolution. Hence a time of $\frac{3.14}{4} = 0.785$ s to complete a quarter of revolution from A to B. The magnitude of $\Delta\vec{v}$ is $\sqrt{4.0^2 + 4.0^2} = 5.66 \text{ m s}^{-1}$ and so the magnitude of the average acceleration is $\frac{5.66}{0.785} = 7.2 \text{ m s}^{-2}$. This is directed towards north-west and if this vector is made to start at the midpoint of the arc AB it is then directed towards the center of the circle.

b The centripetal acceleration has magnitude $\frac{v^2}{r} = \frac{16.0}{2.0} = 8.0 \text{ m s}^{-2}$ directed towards the center of the circle.

4 The centripetal acceleration is $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 rf^2$. Hence

$$f = \sqrt{\frac{a}{4\pi^2 r}} = \sqrt{\frac{50}{4\pi^2 \times 10}} = 0.356 \text{ s}^{-1} \approx 21 \text{ min}^{-1}.$$

5 a The centripetal acceleration is $\frac{v^2}{r} = \frac{4.00}{0.400} = 10.0 \text{ m s}^{-2}$. The tension is the force that provides the centripetal acceleration and so $T = ma = 1.00 \quad 10.0 = 10.0$ N.

b From $T = ma = 20.0$ N we have $a = \frac{v^2}{r} = 20.0 \text{ m s}^{-2}$ and so $v = \sqrt{20 \times 0.40} = 2.83 \text{ m s}^{-1}$.

c $20.0 = 1.00 \times \frac{4.00^2}{r} \Rightarrow r = \frac{16.0}{20.0} = 0.800$ m

6 With $a = 9.8 \text{ m s}^{-2}$ we have that $a = \frac{4\pi^2 r}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 \times 6.4 \times 10^6}{9.8}} = 5.08 \times 10^3 \text{ s} \approx 85$ min.

7 **a** $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 50.0 \times 10^3}{(25.0 \times 10^{-3})^2} = 3.2 \times 10^9 \text{ m s}^{-2}$

b The forces on the probe are (i) its weight, *mg*, and (ii) the normal reaction force *N* from the surface. Assuming the probe to stay on the surface the net force would be

$$mg - N = \frac{mv^2}{r} \Rightarrow N = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right) = m(8.0 \times 10^{10} - 3.2 \times 10^9) > 0.$$

This is positive so the probe can stay on the surface.

8 **a** $v = \frac{2\pi R}{T} = \frac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 2.99 \times 10^4 \approx 30 \text{ km s}^{-1}$

b $a = \frac{v^2}{r} = \frac{(2.99 \times 10^4)^2}{1.5 \times 10^{11}} = 5.95 \times 10^{-3} \approx 6.0 \times 10^{-3} \text{ m s}^{-2}$

c $F = ma = \frac{mv^2}{r} = 6.0 \times 10^{24} \times 5.95 \times 10^{-3} \approx 3.6 \times 10^{22} \text{ N}$

9 The components of *L* are:

$L_x = L\sin\theta, \quad L_y = L\cos\theta$

We have that

$L\sin\theta = m\frac{v^2}{R}$

$L\cos\theta = mg$

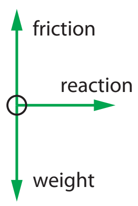
Dividing side by side:

$$\frac{L\sin\theta}{L\cos\theta} = \frac{m\frac{v^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{gR}$$

This gives $\Rightarrow R = \frac{v^2}{g\tan\theta} = \frac{180^2}{9.8 \times \tan 35°} = 4.7 \text{ km}$

10 **a**



b Let the normal reaction force from the wall be *N*. Then

$N = m\frac{v^2}{r}$

$mg = f_s$

For the minimum rotation speed the frictional force must be a maximum i.e. $f_s = \propto_s N$. I.e.

$N = m\frac{v^2}{r}$

$mg = \propto_s N$

Combining gives $mg = \mu_s m\frac{v^2}{r}$ i.e. $v = \sqrt{\frac{gr}{\mu_s}} = \sqrt{\frac{9.8\times 5.0}{0.60}} = 9.04\ \text{m s}^{-1}$. From $v = 2\pi rf$ we find

$f = \frac{v}{2\pi r} = \frac{9.04}{2\pi\times 5.0} = 0.288\ \text{rev s}^{-1} \approx 17\ \text{rev min}^{-1}$.

11 **a** Let v be the speed on the flat part of the road before the loop is entered. At the top the net force on the cart is its weight and the normal reaction force from the road, both directed vertically downwards. Then, $N + mg = \frac{mu^2}{R} \Rightarrow N = \frac{mu^2}{R} - mg$ where u is the speed at the top. For the cart not to fall off the road, we must have $N > 0$ i.e. $u^2 > gR$. From conservation of energy, $\frac{1}{2}mv^2 = mg(2R) + \frac{1}{2}mu^2$ and so $u^2 = v^2 - 4gR$.

Hence $v^2 - 4gR > gR$, i.e. $v > \sqrt{5gR} = 29.7 \approx 30\ \text{m s}^{-1}$.

b For just about equal to $\sqrt{5gR}$ we get $u = \sqrt{gR} = 13.3 \approx 13\ \text{m s}^{-1}$.

12 The tension in the string must equal the weight of the hanging mass i.e. $T = Mg$. The tension serves as the centripetal force on the smaller mass and so $T = m\frac{v^2}{r}$. Hence $m\frac{v^2}{r} = Mg \Rightarrow v = \sqrt{\frac{Mgr}{m}}$.

13 Let the tension in the upper string be T_U and T_L in the lower string. Both strings make an angle θ with the horizontal. We have that:

$$T_U \sin\theta = mg + T_L \sin\theta$$

$$T_U \cos\theta + T_L \cos\theta = m\frac{v^2}{r}$$

We may rewrite these as:

$$T_U \sin\theta - T_L \sin\theta = mg$$

$$T_U \cos\theta + T_L \cos\theta = m\frac{v^2}{r}$$

From trigonometry, $\sin\theta = \frac{0.50}{1.0} = 0.50 \Rightarrow \theta = 30°$. Further, $r = \sqrt{1.0^2 - 0.50^2} = 0.866\ \text{m}$. Therefore the equations simplify to

$$0.50\times(T_U - T_L) = 2.45 \qquad\qquad T_U - T_L = 4.90$$
$$0.866\times(T_U + T_L) = 18.48 \quad \text{or} \quad T_U + T_L = 21.33$$

Finally, $T_U = 13.1\ \text{N}$, $T_L = 8.22\ \text{N}$.

14 **a** By conservation of energy, $mgh = \frac{1}{2}mv^2$ and so $v = \sqrt{2gh} = \sqrt{2\times 9.81\times 120} = 48.9 \approx 49\ \text{m s}^{-1}$ (with this speed, this amusement park should not have a licence to operate!).

b The forces on a passenger are the weight and the reaction force R both in the vertically down direction. Thus $R + mg = m\frac{v^2}{r} \Rightarrow R = m\frac{v^2}{r} - mg$. The speed at the top is found from energy conservation as $mgH = \frac{1}{2}mv^2 + mg(2r) \Rightarrow v^2 = 9.81\times 240 - 2\times 9.81\times 60 = 1177$. Hence

$R = 60\times\frac{1177}{30} - 60\times 9.81 = 1765 \approx 1800\ \text{N}$.

c Using $v^2 = u^2 - 2as$ we get $0 = 49^2 - 2a\times 40$ and so $a = \frac{50^2}{2\times 40} = 30\ \text{m s}^{-2}$ (some passengers will be fainting now, assuming they are still alive!).

6.2 The law of gravitation

15 **a** $F = G\frac{Mm}{R^2} = 6.67 \times 10^{-11} \times \frac{5.98 \times 10^{24} \times 7.35 \times 10^{22}}{(3.84 \times 10^8)^2} = 1.99 \times 10^{20}\ \text{N}$

b $F = G\frac{Mm}{R^2} = 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30} \times 1.90 \times 10^{27}}{(7.78 \times 10^{11})^2} = 4.17 \times 10^{23}\ \text{N}$

c $F = G\frac{Mm}{R^2} = 6.67 \times 10^{-11} \times \frac{1.67 \times 10^{-27} \times 9.11 \times 10^{-31}}{(1.0 \times 10^{-10})^2} = 1.0 \times 10^{-47}\ \text{N}$

16 **a** Zero since it is being pulled equally from all directions.

b Zero, by Newton's third law.

c $F = G\frac{m^2}{4R^2}$, (d) $F = G\frac{m^2}{4R^2} + G\frac{Mm}{4R^2} = G\frac{m(m+M)}{4R^2}$

17 $\frac{g_A}{g_B} = \frac{\left(\frac{GM}{(9R)^2}\right)}{\left(\frac{GM}{R^2}\right)} = \frac{1}{81}$

18 $\frac{g_A}{g_B} = \frac{\left(\frac{G2M}{(2R)^2}\right)}{\left(\frac{GM}{R^2}\right)} = \frac{1}{2}$

19 Since star A is 27 times as massive and the density is the same the volume of A must be 27 times as large. Its radius must therefore be 3 times as large. Hence $\frac{g_A}{g_B} = \frac{\left(\frac{G27M}{(3R)^2}\right)}{\left(\frac{GM}{R^2}\right)} = 3$.

20 $\frac{g_{new}}{g_{old}} = \frac{\left(\frac{GM/2}{(R/2)^2}\right)}{\left(\frac{GM}{R^2}\right)} = 2$

21 Let this point be a distance x from the center of the Earth and let d be the center to center distance between the earth and the moon. Then

$$\frac{G81M}{x^2} = \frac{GM}{(d-x)^2}$$

$$81(d-x)^2 = x^2$$

$$9(d-x) = x$$

$$\frac{x}{d} = \frac{9}{10} = 0.9$$

22 **a** At point P the gravitational field strength is obviously zero.

b The gravitational field strength at Q from each of the masses is

$g = \frac{GM}{R^2} = 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{22}}{(\sqrt{2} \times 10^9)^2} = 1.0 \times 10^6\ \text{N kg}^{-1}$. The net field, taking components, is directed from Q to P and has magnitude $2g\cos 45° = 2 \times 1 \times 10^6 \cos 45° = 1.4 \times 10^6\ \text{N kg}^{-1}$.

23 We know that $\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. But $v = \frac{2\pi r}{T}$ and so we deduce that $T^2 = \frac{4\pi^2 r^3}{GM}$. Therefore

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11}\times6.0\times10^{24}\times(24\times60\times60)^2}{4\pi^2}} = 4.2\times10^7 \text{ m}.$$

24 **a** From $v^2 = \frac{GM}{r}$ we calculate $v = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{(6.4+0.560)\times10^6}} = 7.5828754\times10^3 \approx 7.6\times10^3 \text{ m s}^{-1}$.

b The shuttle speed is $v = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6.9595\times10^6}} = 7.5831478\times10^3 \text{ m s}^{-1}$. The relative speed of the shuttle and Hubble is 0.2724 m s^{-1} and so the distance of 10 km will be covered in $\frac{10^4}{0.2724} = 36711 \text{ s} \approx 10 \text{ hrs}$.

25 **a** $\frac{Gm_1m_2}{r^n} = m_2\frac{v^2}{r} \Rightarrow v^2 = \frac{Gm_1}{r^{n-1}}$. But $v = \frac{2\pi r}{T}$ and so $\left(\frac{2\pi r}{T}\right)^2 = \frac{Gm_1}{r^{n-1}}$ giving

$$\frac{4\pi^2r^2}{T^2} = \frac{Gm_1}{r^{n-1}}$$

$$T^2 = \frac{4\pi^2r^{n+1}}{Gm_1}$$

b For this to be consistent with Kepler's third law we need $n + 1 = 3 \Rightarrow n = 2$

Answers to test yourself questions

Topic 7

7.1 Discrete energy and radioactivity

1 **a** Discrete energy means that the atom cannot have any continuous value of energy but rather one out of many separate i.e. discrete values.

b The existence of emission atomic spectra is the best evidence for the discreteness of energy in atoms: the emission lines have specific wavelengths implying specific energy differences between levels.

2 The bright lines are formed when an electron makes a transition from a high energy state H to a lower energy state L. The photon emitted will have a wavelength determined from $\frac{hc}{\lambda} = \Delta E_{LH} \Rightarrow \lambda = \frac{hc}{\Delta E_{LH}}$ where ΔE_{LH} is the difference in energy between state H and L. The dark lines are formed when a photon is absorbed by an electron in a low energy state L which then makes a transition to a high energy state H. For the absorption to be possible the photon energy must equal the difference ΔE_{LH}. Hence this photon will have the same wavelength as the emission line wavelength.

3 The energy difference is 2.55 eV. Hence,

$$\frac{hc}{\lambda} = 2.55 \text{ eV} = 2.55 \times 1.6 \times 10^{-19} = 4.08 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{4.08 \times 10^{-19}}$$

$$\lambda = 4.875 \times 10^{-7} \approx 4.9 \times 10^{-7} \text{ m}$$

4 The energy differences between levels get smaller as n increases. Therefore transitions down to $n = 2$ (the visible light transitions) have wavelengths that are close to each other.

5 **a** Ground state is the energy level with the least possible energy.

b The energy difference between the ground state and the first excited state is 10.2 eV. That between the ground state and the second excited state is 12.1 eV. The incoming photons do not have exactly these amounts of energy so the hydrogen atoms will not absorb any of these photons.

c With incoming electrons it is possible that some will give 10.2 eV of their energy to hydrogen atoms so that the atoms make a transition to the first excited state. The electrons that do give this energy will bounce off the atoms wityh a kinetic energy of about 0.2 eV.

6 $2e$

7 **a** Isotopes are nuclei of the same element (hence have the same proton (atomic) number) that differ in the number of neutrons, i.e. they have different nucleon (mass) number.

b They have different mass and different radius.

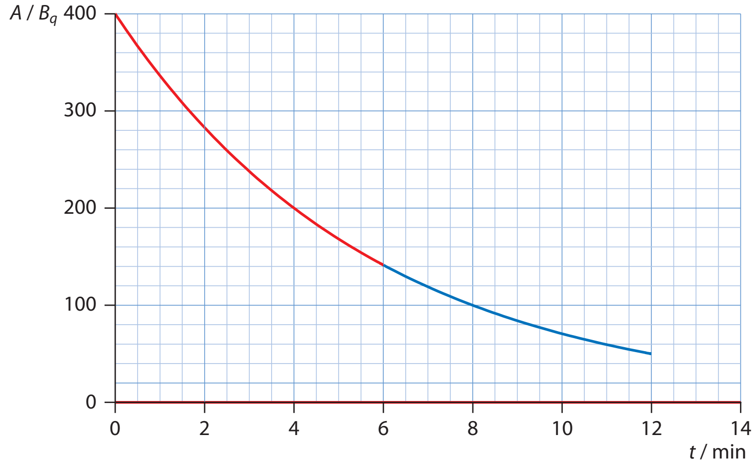
8 ${}^{210}_{83}\text{Bi} \rightarrow {}^{0}_{-1}e + \gamma + \bar{\nu} + {}^{210}_{84}\text{Po}$

9 ${}^{239}_{94}\text{Pu} \rightarrow {}^{4}_{2}\alpha + {}^{235}_{92}\text{U}$

10 18 min is 6 half-lives and so the sample will be reduced by $2^6 = 64$ times i.e. to 0.50 mg.

11 **a** Activity is the rate of decay.

b 4.0 min

c 

d We make the following table of numbers of X and Y nuclei:

Time/min	Number of X nuclei	Number fo Y nuclei	Ratio of Y to X
0	*N*	0	0
4	*N* /2	*N* /2	1
8	*N* /4	3*N*/4	3
12	*N* /8	7 *N*/8	7

So the required time is 12 min.

12 The equation is $C = \frac{k}{(d+d_0)^2}$, i.e. $d + d_0 = \sqrt{\frac{k}{C}}$. So a graph of d versus $\frac{1}{\sqrt{C}}$ would give a straight line with slope $\sqrt{k}$ and intercept $-d_0$.

13 From $I = I_0 e^{-\mu x}$ we deduce that $\ln I = \ln I_0 - \mu x$, so a graph of $\ln I$ versus x gives a straight line with slope $-\mu$.

14 **a** strong nuclear force

b electric force

15 As the nucleus gets heavier more protons and neutrons must be added to the nucleus. The neutrons contribute to nuclear binding through the nuclear force but the protons contribute to repulsion through the electrical force in addition to binding through the nuclear force that they also participate in. However, the electrical force has infinite range and all the protons in the nucleus repel each other whereas only the very near neighbors attract through the nuclear force. To make up for this imbalance it is necessary to have more neutrons i.e. particles that contribute to only binding.

7.2 Nuclear reactions

16 Note that the problem has given an atomic mass for nickel and we need the nuclear mass. Hence we must subtract the electron masses. The mass defect is

$$\begin{aligned}\delta &= 28m_p + (62-28)m_n - (M_{Ni} - Zm_e)\\ &= 28 \times 1.007\,276 + 34 \times 1.008\,665 - (61.928\,348 - 28 \times 0.000549)\\ &= 0.585\,362 \text{ u}\end{aligned}$$

and so the binding energy is

$$\begin{aligned}E &= \delta c^2 = 0.585\,362 \times 931.5c^2 \quad \text{MeV c}^{-2}\\ &= 545.26 \quad \text{MeV}\end{aligned}$$

Hence the binding energy per nucleon is $\frac{E}{A} = \frac{545.26}{62} = 8.79$ MeV. This is the highest binding energy per nucleon.

17 The mass defect is

$$\delta = 8m_p + 8m_n - (M_O - 8m_e)$$
$$= 8\times 1.007\,276 + 8\times 1.008\,665 - (15.994 - 8\times 0.000549)$$
$$= 0.137\,920 \text{ u}$$

and so the binding energy is

$$E = \delta c^2 = 0.137\,920 \times 931.5c^2 \text{ MeV c}^{-2}$$
$$= 128.47 \text{ MeV}$$

Hence the binding energy per nucleon is $\frac{E}{A} = \frac{128.47}{16} = 8.03\text{ MeV}$. Consider now the reaction ${}^{16}_{8}\text{O} \to {}^{1}_{1}\text{p} + {}^{15}_{7}\text{N}$.

The mass difference is $15.994 - 8\times 0.000549 - (1.007\,276 + 15.000 - 7\times 0.000549) = -0.012\,727$ u. The negative sign implies that the reaction can take place only when energy is supplied to the oxygen nucleus. This energy is $E = 0.012\,727 \times 931.5c^2 \text{ MeV c}^{-2} = 11.9\text{ MeV}$

18 **a** Using, $E = hf = \frac{hc}{\lambda}$ we find $\lambda = \frac{hc}{E} = \frac{6.63\times 10^{-34}\times 3.0\times 10^{8}}{0.051\times 10^{6}\times 1.6\times 10^{-19}} = 2.44\times 10^{-11}$ m.

b This is in the gamma ray area of the spectrum.

19 **a** ${}^{236}_{92}\text{U} \to {}^{117}_{46}\text{Pd} + {}^{117}_{46}\text{Pd} + 2\,{}^{1}_{0}\text{n}$

b Two neutrons are produced as well as photons.

c The mass difference is $236.045\,5561 - (2\times 116.917\,8 + 2\times 1.008\,665) = 0.192\,626$ u. The energy is $E = 0.192\,626\times 931.5c^2 \text{ MeV c}^{-2} = 179\text{ MeV}$. (Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

20 The mass difference is $235.043\,992 + 1.008\,665 - (97.912\,76 + 134.916\,5 + 3\times 1.008\,665) = 0.197\,402$ u. The energy released is $E = 0.197\,402\times 931.5c^2 \text{ MeV c}^{-2} = 184\text{ MeV}$.

21 The mass difference is $2.014\,102 + 3.016\,049 - (1.008\,665 + 4.002\,603) = 0.018\,883$ u. This energy is $E = 0.018\,883\times 931.5c^2 \text{ MeV c}^{-2} = 17.6 \approx 18\text{ MeV}$. (Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

22 The mass difference is $1.007\,276 + (7.016 - 3\times 0.000549) - 2\times(4.002\,603 - 2\times 0.000549) = 0.018\,619$ u. This corresponds to an energy $E = 0.018\,619\times 931.5c^2 \text{ MeV c}^{-2} = 17.3\text{ MeV}$ not including the kinetic energy of the accelerated proton.

23 The formula for the mass defect given in the textbook is $\delta = Zm_p + (A-Z)m_n - M_{nucleus}$. Now, $M_{nucleus} = M_{atom} - Zm_e$. Hence,

$$\delta = Zm_p + (A-Z)m_n - (M_{atom} - Zm_e)$$
$$= Z(m_p + m_e) + (A-Z)m_n - M_{atom}$$
$$= ZM_H + (A-Z)m_n - M_{atom}$$

where $M_H = m_p + m_e$ is the mass of the hydrogen atom.

24 **a** $Q_1 = \left(M_D + M_T - M_{He} - m_n\right)c^2$. Now let us look at the binding energy of each nucleus involved in the reaction.

$$E_D = (m_p + m_n - M_D)c^2 \Rightarrow M_D c^2 = (m_p + m_n)c^2 - E_D$$
$$E_T = (m_p + 2m_n - M_T)c^2 \Rightarrow M_T c^2 = (m_p + 2m_n)c^2 - E_T$$
$$E_{He} = (2m_p + 2m_n - M_{He})c^2 \Rightarrow M_{He} c^2 = (2m_p + 2m_n)c^2 - E_{He}$$

Hence replacing the masses in the equation for Q_1,

$$Q_1 = \left(M_D + M_T - M_{He} - m_n\right)c^2$$
$$= \left((m_p + m_n - E_D) + (m_p + 2m_n - E_T) - (2m_p + 2m_n - E_{He}) - m_n\right)c^2$$
$$= E_{He} - (E_D + E_T)$$

b $Q_2 = \left(M_U + M_{Zr} - M_{Te} - 2m_n\right)c^2$. Working as in **a**

$$E_U = (92m_p + 143m_n - M_U)c^2 \Rightarrow M_U c^2 = (92m_p + 143m_n)c^2 - E_U$$
$$E_{Zr} = (40m_p + 58m_n - M_{Zr})c^2 \Rightarrow M_{Zr}c^2 = (40m_p + 58m_n)c^2 - E_{Zr}$$
$$E_{Te} = (52m_p + 83m_n - M_{Te})c^2 \Rightarrow M_{Te}c^2 = (52m_p + 83m_n)c^2 - E_{Te}$$
$$Q_2 = \left(M_U - M_{Zr} - M_{Te} - 2m_n\right)c^2$$
$$= \left((92m_p + 143m_n - E_U) - (40m_p + 58m_n - E_{Zr}) - (52m_p + 83m_n - E_{Te}) - 2m_n\right)c^2$$
$$= E_{Zr} + E_{Te} - E_U$$

c The results in **a** and **b** show that, in general, the energy released can be found from the difference of the total binding energy **after** the reaction minus that **before** the reaction. Thus, to have energy released the binding energy after the reaction must be greater than that before. The peak of the binding energy curve is at nickel. Elements to the right and left of nickel have lower binding energy per nucleon. The issue here is how to use the binding energy curve to show that energy will be released for fission (involving elements heavier than nickel) and fusion (involving elements lighter than nickel). Notice that we cannot prove mathematically that this is the case without knowing the mathematical equation of the binding energy curve. However, the fact that the curve rises for light elements up to nickel and then drops for elements heavier than nickel is indicative that energy is released in both fusion and fission reactions.

7.3 The structure of matter

25 a In order to avoid absorption of alpha particles as well as avoid multiple scatterings.

b In order to avoid collisions of alpha particles with air molecules which would have deflected the alphas.

26 a The neutron is $d\,d\,u$ and so the antineutron must be $\bar{d}\,\bar{d}\,\bar{u}$. The electric charge is $\left(+\frac{1}{3}+\frac{1}{2}-\frac{2}{3}\right)e = 0$.

b The proton is $u\,u\,d$ and so the antiproton is $\bar{u}\,\bar{u}\,\bar{d}$ with electric charge $\left(-\frac{2}{3}-\frac{2}{3}+\frac{1}{3}\right)e = -e$, as expected.

27 The antiparticle of the K^+ has quark structure $\bar{u}\ s$.

28 It is -1 since this is an antibaryon.

29 a Violates: $-1 \rightarrow 0 + 0$

b Conserves: $-1 + 1 \rightarrow 0 + 0$

c Conserves: $-1 + 1 \rightarrow 0 + 0 + 1 - 1$

d Violates: $+1 \rightarrow 0 + 0$

30 Consider a decay such as $\Lambda^0 \rightarrow p^+ + \pi^-$ where the lambda baryon is uds. Notice that there is a strange quark on the left hand side of the decay but none on the right hand side. If this were a strong interaction process (or electromagnetic) the lifetime would be very short (less than about 10^{-20} s). However, the decay of the lambda has a much larger lifetime (of order 10^{-10} s). To explain this it was hypothesised that this long lifetime decay (and many others like it) were due to the weak interaction. The weak interaction being weaker than the strong would naturally lead to a long lifetime decay. To prevent this decay from happening via the strong or electromagnetic interactions, a new quantum number called strangeness was introduced that was assumed to be conserved in strong and electromagnetic interactions but not in weak interactions.

31 a The charge of $d\,\bar{s}$ is zero $\left(-\frac{1}{3}+\frac{1}{3}\right)$; the strangeness is + 1.

b No, they have different stangeness

32 a The charge of $c\,\bar{d}$ is $e\left(+\frac{2}{3}+\frac{1}{3}=1\right)$

b The strangeness is zero since it does not have strange quarks in it.

33 a Conserves: $0+0 \rightarrow +1-1$

b Conserves: $0+0 \rightarrow +1-1$

c Violates: $+1 \rightarrow 0+0$

d Violates: $0+0 \rightarrow 0-1$

34 a Electron neutrino

b Muon neutrino

c Tau antineutrino

d Electron antineutrino

e Electron antineutrino and tau neutrino

35 a Electron lepton number

b electron and muon lepton number

c electric charge

d baryon number

e Energy

f baryon number

36 a Yes because they have electric charge

b No because they do not have electric charge

37 Yes because it is made out of charged quarks

38 Since $\eta_c = c\,\bar{c}$ the antiparticle of the η_c is $\bar{c}\,c$ i.e. is the same as the η_c itself. However the antiparticle of the meson $K^0 = d\,\bar{s}$ would be $s\,\bar{d}$ and is different.

39 a Confinement means that color cannot be observed. This implies that one cannot find isolated quarks or gluons.

b The gluons will be very short lived and will produce hadrons along their path. The energy of the gluons will create quark antiquark pairs out of the vacuum and these will combine to make hadrons.

40 a We may deduce that $2m_u + m_d = 938$ and $m_u + 2m_d = 940$ (units of mass are MeV c^{-2}). We solve this system of equations to obtain the individual quark masses: from the first equation, $m_d = 938 - 2m_u$ and substituting this into the second gives

$$m_u + 2(938 - 2m_u) = 940$$

$$3m_u = 2 \times 938 - 940$$

$$m_u = 312\,\text{MeV c}^{-2}$$

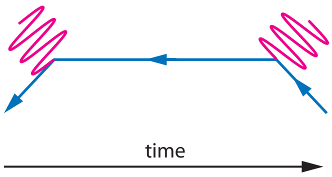
and so $m_d = 314\,\text{MeV c}^{-2}$.

b It follows that we can predict a mass of $312 + 314 = 626\,\text{MeV c}^{-2}$ for the mass of the π^+ meson, which is clearly incorrect.

c The reason for the disagreement is that both in the calculation of the masses in **a** as well as in the calculation of the mass of the pion in **b** we have neglected to take into account the sizable binding energy of the quarks. (There are also other technical reasons having to do with exactly what one means by the "mass" of the quarks.)

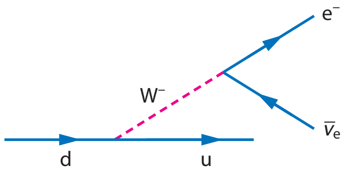
41 The Higgs particle is a crucial ingredient of the standard model of particles. Its interactions with other particles make those particles acquire mass.

42

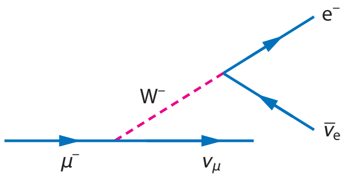


43 **a** d quark inside the neutron turns into a u quark, an electron and an electron antineutrino.

b



44 **a**



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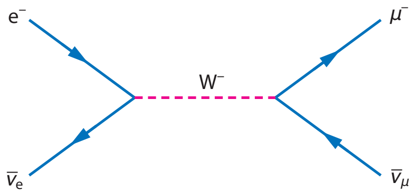

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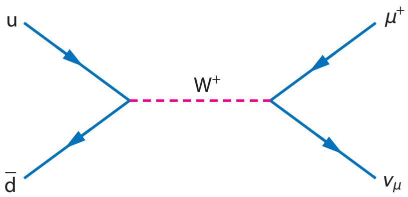

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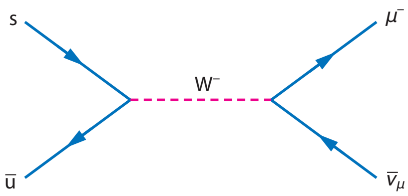
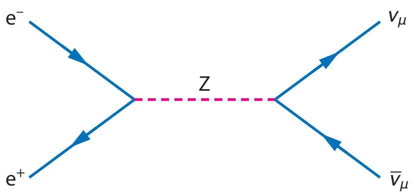


45 $W^- \rightarrow e^- + \overline{\nu}_e$

$W^- \rightarrow \mu^- + \overline{\nu}_\mu$

$W^- \rightarrow \tau^- + \overline{\nu}_\tau$

46 a



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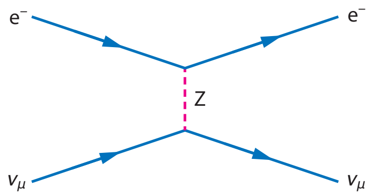

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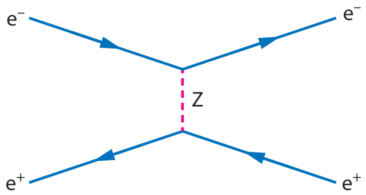


Answers to test yourself questions

Topic 8

8.1 Energy sources

1 **a** Specific energy is the energy that can be extracted from a unit mass of a fuel while energy density is the energy that can be extracted from a unit volume of fuel.

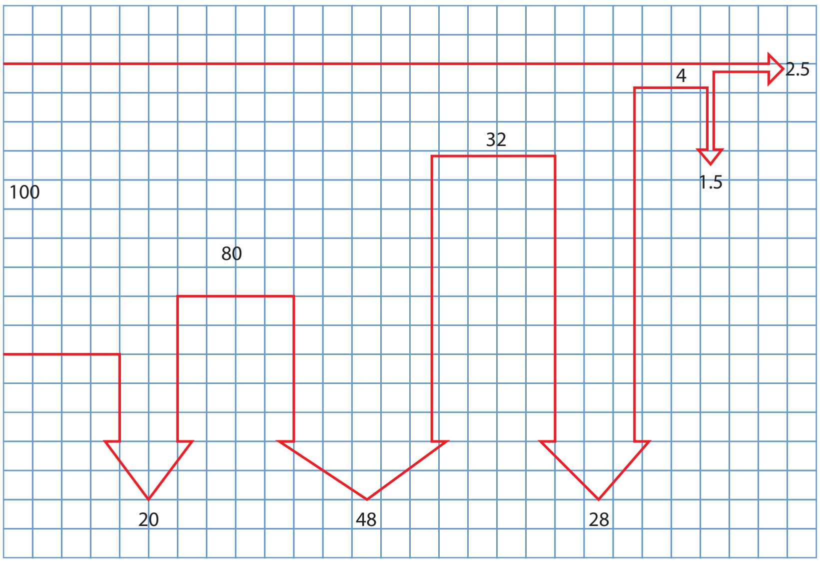
b The available energy is mgh. The energy density is the energy per unit volume that can be obtained and so $E_D = \frac{mgh}{V} = \frac{mgh}{\frac{m}{\rho}} = \rho gh = 10^3 \times 9.8 \times 75 = 7.4 \times 10^5\ \text{J m}^{-3}$

2 **a** In 1 s the energy is 500 MJ.

b In one year the energy is $5.0 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 1.6 \times 10^{16}$ J.

3 **a** The overall efficiency is $0.80 \times 0.40 \times 0.12 \times 0.65 = 0.025$.

b



4 **a** The energy produced by burning the fuel is $10^7 \times 30 \times 10^6 = 3.0 \times 10^{14}$ J. Of this 70% is converted to useful energy and so the power output is $P = \frac{0.30 \times 3.0 \times 10^{14}}{24 \times 60 \times 60} = 1.04 \times 10^9$ W.

b The rate at which energy is discarded is $P_{discard} = \frac{0.70 \times 3.0 \times 10^{14}}{24 \times 60 \times 60} = 2.43 \times 10^9$ W.

c Use $\frac{\Delta m}{\Delta t} c\Delta\theta = P_{discard}$ to get $\frac{\Delta m}{\Delta t} = \frac{P_{discard}}{c\Delta\theta} = \frac{2.43 \times 10^9}{4200 \times 5} = 1.2 \times 10^5\ \text{kg s}^{-1}$.

5 In time t the energy used by the engine is $20 \times 10^3 \times t$. The energy available is $0.40 \times 34 \times 10^6$ J and so $20 \times 10^3 \times t = 0.40 \times 34 \times 10^6 \Rightarrow t = 680$ s. The distance travelled is then $x = vt = 9.0 \times 680 = 6.1$ km.

6 The useful energy produced each day is $E = Pt = 1.0 \times 10^9 \times 24 \times 60 \times 60 = 8.64 \times 10^{13}$ J. The energy produced by burning the coal is therefore $0.40 = \frac{8.64 \times 10^{13}}{E_c} \Rightarrow E_c = 2.16 \times 10^{14}$ J. So the mass of coal to be burned is $m = \frac{2.16 \times 10^{14}}{30 \times 10^6} = 7.2 \times 10^6$ kg per day.

7 **a** The fissionable isotope of uranium is U – 235. This is found in very small concentrations in uranium ore which is mostly U – 238. Enrichment means increasing the concentration of U – 235 in a sample of uranium.

b The moderator is the part of the nuclear reactor where neutrons released from the fission reactions slow down as a result of collisions with the atoms of the moderator. The temperature of the moderator can be kept constant with a cooling system that removes the excess thermal energy generated in the moderator.

c Critical mass refers to the least mass of uranium that must be present for nuclear fission reactions to be sustained. If the mass of uranium is too small (i.e. below the critical mass) the neutrons may escape without causing fission reactions.

8 **a** The reaction is ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$. The mass difference is

$$\delta = (235.043\,992 - 92 \times 0.000549) + 1.008\,665$$
$$-\big((139.921\,636 - 54 \times 0.000549) + (93.915\,360 - 38 \times 0.000549) + 2 \times 1.008\,665)\big)$$
$$= 0.198\,331\ \text{u}$$

And so the energy released is $E = 0.198331 \times 931.5c^2\ \text{MeV}c^{-2} = 185\ \text{MeV}$.

b With N reactions per second the power output is $N \times 185\ \text{MeVs}^{-1}$. In other words, $N \times 185 \times 10^6 \times 1.6 \times 10^{-19} = 200 \times 10^6 \Rightarrow N = 6.8 \times 10^{18}\ \text{s}^{-1}$.

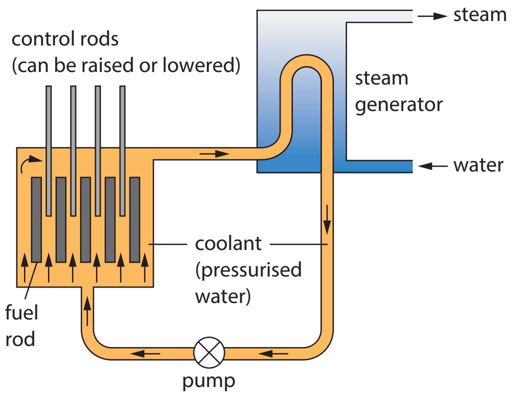
9 **a** One kilogram of uranium corresponds to $\frac{1000}{235} = 4.26$ moles and so $4.26 \times 6.02 \times 10^{23} = 2.57 \times 10^{24}$ nuclei. Each nucleus produces 200 MeV and so the energy produced by 1 kg (the energy density) is $2.57 \times 10^{24} \times 200 \times 10^6 \times 1.6 \times 10^{-19} \approx 8 \times 10^{13}\ \text{J kg}^{-1}$.

b $\frac{8 \times 10^{13}}{30 \times 10^6} = 2.7 \times 10^6\ \text{kg}$

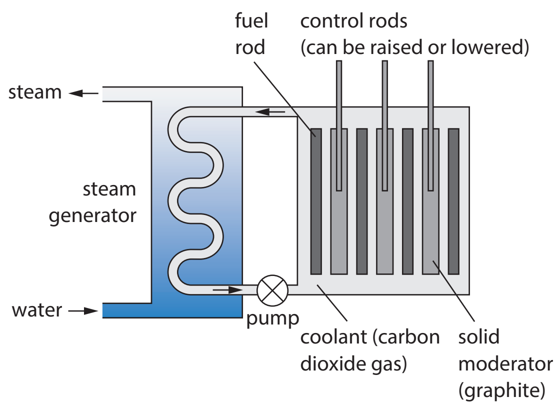
10 **a** The energy that must be produced in 1 s is $E = \frac{500 \times 10^6}{0.40} = 1.25 \times 10^9$ J. Hence the number of fission reactions per second is $\frac{1.25 \times 10^9}{200 \times 10^6 \times 1.6 \times 10^{-19}} = 3.9 \times 10^{19}$.

b The number of nuclei required to fission per second is 3.9×10^{19} which corresponds to $\frac{3.9 \times 10^{19}}{6.02 \times 10^{23}} = 6.5 \times 10^{-5}$ mol, i.e. a mass of $6.5 \times 10^{-5} \times 235 \times 10^{-3} = 1.5 \times 10^{-5}\ \text{kg s}^{-1}$.

11



pressurised water reactor (PWR)



gas-cooled reactor

a **i** fuel rods are pipes in which the fuel (i.e. uranium – 235) is kept.

ii control rods are rods that can absorb neutrons. These are lowered in or raised out of the moderator so that the rate of reactions is controlled. The rods are lowered in if the rate is too high – the rods absorb neutrons so that these neutrons do not cause additional reactions. They raised out if the rate is too small leaving the neutrons to cause further reactions.

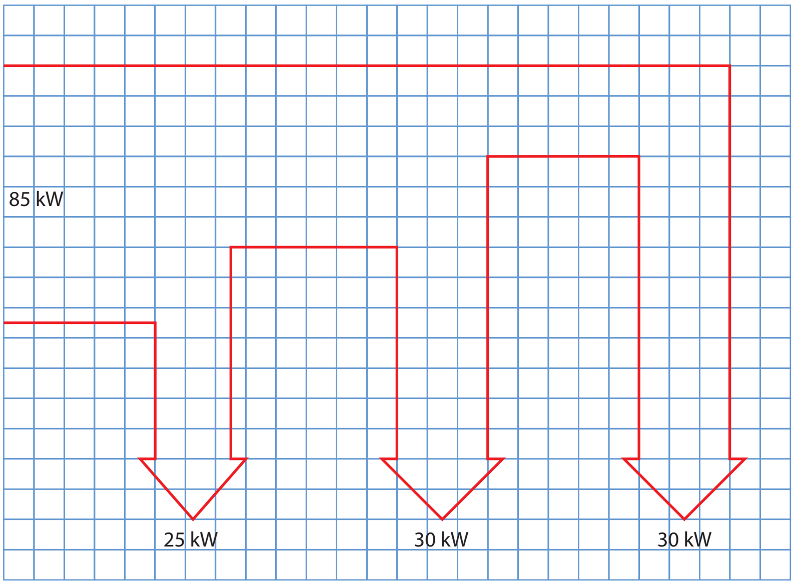
iii The moderator is the part of the nuclear reactor where neutrons released from the fission reactions slow down as a result of collisions with the atoms of the moderator. The temperature of the moderator can be kept constant with a cooling system that removes the excess thermal energy generated in the moderator.

b It is kinetic energy of the neutrons produced which is converted into thermal energy in the moderator as the neutrons collide with the moderator atoms.

12 A solar panel receives solar radiation incident on it and uses it to heat up water, i.e. it converts solar energy into thermal energy. The photovoltaic cell converts the solar incident on it to electrical energy.

13 **a** The power that must be supplied is 3.0 kW and this equals $700 \times A \times 0.70 \times 0.50$. Hence $700 \times A \times 0.70 \times 0.50 = 3.0 \times 10^3 \Rightarrow A = 12\ \text{m}^2$.

b



14 The power that is provided is $0.65 \times 240 \times A$ and this must equal $\frac{mc\Delta\theta}{\Delta t} = \frac{300 \times 4200 \times 35}{12 \times 60 \times 60}$.

Hence $0.65 \times 700 \times A = \frac{300 \times 4200 \times 35}{12 \times 60 \times 60} \Rightarrow A = 6.5\ \text{m}^2$.

15 The power incident on the panel is $600 \times 4.0 \times 0.60 = 1440\text{W}$. The energy needed to warm the water is $mc\Delta\theta = 150 \times 4200 \times 30 = 1.89 \times 10^7$ J and so $1440 \times t = 1.89 \times 10^7 \Rightarrow t = \frac{1.89 \times 10^7}{1440} = 1.31 \times 10^4\ \text{s} = 3.6\ \text{hr}$.

16 **a** From the graph this is about $T = 338\ \text{K}$

b $P = IA = 400 \times 2 = 800\text{W}$

c The useful power is the 320 W that is extracted. The efficiency is thus $\frac{320}{800} = 0.40$.

17 The power supplied at the given speed is (reading from the graph) 100 kW. Hence the energy supplied in 100 hrs is $100 \times 10^3 \times 1000 \times 60 \times 60 = 3.6 \times 10^{11}$ J.

18 We look at the power formula for windmills, $P = \frac{1}{2}\rho A v^3$ to deduce:

a **i** the area will increase by a factor of 4 if the length is doubled and so the power goes up by a factor of 4,

ii the power will increase by a factor of $2^3 = 8$

iii the combined effect is $4 \times 8 = 32$

b Not all the kinetic energy of the wind can be extracted because not all the wind is stopped by the windmill (as the formula has assumed). In addition, there will be frictional losses as the turbines turn as well as losses due to turbulence.

19 Assumptions include:

i no losses of energy due to frictional forces as the turbines turn

ii no turbulence in the air

iii that all the air stops at the turbines so that the speed of the air behind the turbines is zero (which is impossible).

20 The power in the wind before hitting the turbine is $\frac{1}{2}\rho_1 A v_1^3$ and right after passing through the turbine is $\frac{1}{2}\rho_2 A v_2^3$ so the extracted power is $\frac{1}{2}\rho_1 A v_1^3 - \frac{1}{2}\rho_2 A v_2^3 = \frac{1}{2} \times \pi \times 1.5^2 (1.2 \times 8.0^3 - 1.8 \times 3.0^3) \approx 2.0 \text{ kW}$.

21 From $P = \frac{1}{2}\rho A v^3$ we get $25 \times 10^3 = \frac{1}{2} \times 1.2 \times A \times 9.0^3 \Rightarrow A = 57.16 \text{ m}^2$ and so $\pi R^2 = 57.16 \Rightarrow R = 4.3 \text{ m}$. The assumptions made are the usual ones (see question 19)

i no losses of energy due to frictional forces as the turbines turn

ii no turbulence in the air, and

iii that all the air stops at the turbines so that the speed of the air behind the turbines is zero (which is impossible).

22 The potential energy of a mass Δm of water is Δmgh and so the power developed is the rate of change of this energy i.e. $\frac{\Delta m}{\Delta t} gh = 500 \times 9.8 \times 40 = 1.96 \times 10^5 \approx 2.0 \times 10^5 \text{ W}$.

23 The potential energy of a mass Δm of water is Δmgh and so the power developed is the rate of change of this energy i.e. $\frac{\Delta m}{\Delta t} gh = \rho \frac{\Delta V}{\Delta t} gh = \rho Q gh$.

24 The amount of electrical energy generated will always be less than the energy required to raise the water back to its original height. This is because the electrical energy generated is less than what theoretically could be provided by the water (because of various losses). So this claim cannot be correct.

25 **a** Coal power plant: chemical energy of coal → thermal energy → kinetic energy of steam → kinetic energy of turbine → electrical energy.

b hydroelectric power plant: potential energy of water → kinetic energy of water → kinetic energy of turbine → electrical energy.

c wind turbine: kinetic energy of wind → kinetic energy of turbine → electrical energy.

d nuclear power plant: nuclear energy of fuel → kinetic energy of neutrons → thermal energy in moderator → kinetic energy of steam → kinetic energy of turbine → electrical energy.

Chapter 8.2 Thermal energy transfer

26 **a** Energy has to be conserved so whatever energy enters the junction has to leave the junction and so the rates of energy transfer are the same.

b The temperature differences are not the same because X and Y have different thermal conductivity.

27 There is; in order to send the warm air that collects higher up in the room downwards.

28 Power is proportional to T^4 and so the ratio is $\left(\frac{900}{300}\right)^4 = 3^4 = 81$

29 **a** A black body is any body at absolute temperature T whose radiated power per unit area is given by σT^4. A black body appears black when its temperature is very low. It absorbs all the radiation incident on it and reflects none.

b A piece of charcoal is a good approximation to a black body as is the opening of a soft drink can.

c It increases by $\left(\frac{273+100}{273+50}\right)^4 \approx 1.8$

30 **a** The wavelength at the peak of the graph is determined by temperature and since the wavelength is the same so is the temperature.

b The ratio of the intensities at the peak is about $\frac{1.1}{1.9} \approx 0.6$.

31 We have that $e\sigma A T^4 = P \Rightarrow T = \sqrt[4]{\frac{P}{e\sigma A}}$ i.e. $T = \sqrt[4]{\frac{1.35 \times 10^9}{0.800 \times 5.67 \times 10^{-8} \times 5.00 \times 10^6}} = 278 \text{ K}$.

32 a We must have that $\sigma AT^4 \propto \frac{1}{d^2} \Rightarrow T \propto \frac{1}{\sqrt{d}}$.

b Using our knowledge of propagation of uncertainties, we deduce that $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta d}{d}$ and so $\frac{\Delta T}{T} = \frac{1}{2} \times 1.0\% = 0.005$. Hence, $\Delta T = 0.005T = 0.005 \times 288 = 1.4\,\text{K}$.

33 a Intensity is the power received per unit area from a source of radiation.

b $P = e\sigma AT^4$. $P = 0.90 \times 5.67 \times 10^{-8} \times 1.60 \times (273+37)^4 = 754\,\text{W}$. Assuming uniform radiation in all directions, the intensity is then $I = \frac{P}{4\pi d^2} = \frac{754}{4\pi(5.0)^2} = 2.4\ \text{W m}^{-2}$.

34 a Imagine a sphere centered at the source of radius d. The power P radiated by the source is distributed over the surface area A of this imaginary sphere. The power per unit area i.e. the intensity is thus $I = \frac{P}{A} = \frac{P}{4\pi d^2}$.

b We have assumed that the radiation is uniform in all directions.

35 a The peak wavelength is approximately $\lambda_0 = 0.65 \times 10^{-5}\,\text{m}$ and so from Wien's law: $\lambda_0 T = 2.9 \times 10^{-3}\,\text{K m}$ we find $T = \frac{2.9 \times 10^{-3}}{0.65 \times 10^{-5}} = 450\ \text{K}$.

b The curve would be similar in shape but taller and the peak would be shifted to the left.

36 a Albedo is the ratio of the reflected intensity to the incident intensity on a surface.

b The albedo of a planet depends on factors such as cloud cover in the atmosphere, amount of ice on the surface, amount of water on the surface and color and nature of the soil.

37 a The earth receives radiant energy from the sun and in turn radiates itself. The radiated energy is in the infrared region of the electromagnetic spectrum. Gases in the atmosphere absorb part of this radiated energy and reradiate it in all directions. Some of this radiation returns to the earth surface warming it further.

b The main greenhouse gases are water vapour, carbon dioxide and methane. See page 336 for sources.

38 a The energy flow diagram is similar to that in Fig. 8.10 on page 334.

b The reflected intensity is $350 - 250 = 100\,\text{W m}^{-2}$ and so the albedo is $\frac{100}{350} = 0.29$.

c It has to be equal to that absorbed i.e. $250\,\text{W m}^{-2}$.

d Use $e\sigma T^4 = I \Rightarrow T = \sqrt[4]{\frac{I}{e\sigma A}}$ i.e. (assuming a black body) $T = \sqrt[4]{\frac{250}{5.67 \times 10^{-8}}} = 258\ \text{K}$. You must be careful with these calculations in the exam. You must be sure as to whether the question wants you to assume a black body or not. Strictly speaking, in a model without an atmosphere the earth surface cannot be taken to be a black body – if it were no radiation would be reflected!

39 a The intensity radiated by the surface is I_1 and a fraction t of this escapes so we know that $I_3 = tI_1$. The intensity of radiation entering the surface is $(1-\alpha)\frac{S}{4} + \alpha I_1 + I_2$. This must equal the intensity of radiation leaving the surface which is I_1: $(1-\alpha)\frac{S}{4} + \alpha I_1 + I_2 = I_1$. The intensity of radiation entering the atmosphere is $(1-\alpha)I_1$ and that leaving it is $2I_2 + I_3$. Hence $2I_2 + I_3 = (1-\alpha)I_1$. So we have to solve the equations (we used $I_3 = tI_1$)

$$(1-\alpha)\frac{S}{4} + \alpha I_1 + I_2 = I_1$$

$$2I_2 + tI_1 = (1-\alpha)I_1$$

These simplify to

$$(1-\alpha)\frac{S}{4} + I_2 = (1-\alpha)I_1$$

$$2I_2 = (1-\alpha-t)I_1 \Rightarrow I_2 = \frac{(1-\alpha-t)}{2}I_1$$

Hence

$$(1-\alpha)\frac{S}{4}+\frac{(1-\alpha-t)}{2}I_1=(1-\alpha)I_1$$

$$(1-\alpha)\frac{S}{4}=(1-\alpha)I_1-\frac{(1-\alpha-t)}{2}I_1$$

$$(1-\alpha)\frac{S}{4}=\frac{(1-\alpha+t)}{2}I_1$$

$$I_1=\frac{2}{1-\alpha+t}(1-\alpha)\frac{S}{4}$$

Therefore, $I_2=\dfrac{1-\alpha-t}{1-\alpha+t}\times\dfrac{(1-\alpha)S}{4}$ and $I_3=\dfrac{2t}{1-\alpha+t}\times\dfrac{(1-\alpha)S}{4}$ as required.

b The intensity entering is $\frac{S}{4}$. The intensity leaving is $\alpha\frac{S}{4}+I_3+I_2$. The intensity leaving simplifies to

$$\alpha\frac{S}{4}+\frac{2t}{1-\alpha+t}\times\frac{(1-\alpha)S}{4}+\frac{1-\alpha-t}{1-\alpha+t}\times\frac{(1-\alpha)S}{4}=\frac{S}{4}\left(\alpha+(1-\alpha)\frac{2t}{1-\alpha+t}+(1-\alpha)\frac{1-\alpha-t}{1-\alpha+t}\right)$$

$$=\frac{S}{4}\left(\alpha+(1-\alpha)\frac{1-\alpha+t}{1-\alpha+t}\right)$$

$$=\frac{S}{4}\left(\alpha+(1-\alpha)\right)$$

$$=\frac{S}{4}$$

c The intensity radiated by the surface is $I_1=\dfrac{2}{1-\alpha+t}(1-\alpha)\dfrac{S}{4}$ and must equal σT^4, where T is the surface temperature. Hence

$$\frac{2}{1-\alpha+t}(1-\alpha)\frac{S}{4}=\sigma T^4$$

$$t=\frac{2(1-\alpha)\frac{S}{4}}{\sigma T^4}-1+\alpha$$

$$t=\frac{2(1-0.30)\times 350}{5.67\times 10^{-8}\times 288^4}-1+0.30$$

$$t\equiv 0.556$$

d i Let an intensity I be incident on the atmosphere of emissivity e. An amount eI will be absorbed and reradiated, an amount αI will be reflected and an amount tI will be transmitted. By energy conservation we have that $I = eI + \alpha I + tI$ from which $e=1-\alpha-t$ as required.

ii We equate $I_2=\dfrac{1-\alpha-t}{1-\alpha+t}\times\dfrac{(1-\alpha)S}{4}$ to $e\sigma T^4$ where T is the atmosphere temperature to get (recall that $e=1-\alpha-t$)

$$e\sigma T^4=\frac{1-\alpha-t}{1-\alpha+t}\times\frac{(1-\alpha)S}{4}$$

$$\sigma T^4=\frac{1}{1-\alpha+t}\times\frac{(1-\alpha)S}{4}$$

$$T=\sqrt[4]{\frac{1}{1-0.30+0.556}\times\frac{(1-0.30)\times 350}{5.67\times 10^{-8}}}$$

$$T=242\text{ K}$$

40 Radiation is a main mechanism to both the atmosphere and to space. In addition there is conduction to the atmosphere, as well as convection.

41 **a** Dry sub – tropical land has a high albedo, around 0.4 whereas a warm ocean has an albedo of less than 0.2.

b Radiation and convection currents are the main mechanisms.

c Replacing dry land by water reduces the albedo of the region. Reducing the albedo means that less radiation is reflected and more is absorbed and so an increase in temperature might be expected. The increase in temperature might involve additional evaporation and so more rain.

42 The rate of evaporation from water depends on the temperature of the water and the temperature of the surrounding air. These are both higher in the case of the tropical ocean water and evaporation will be more significant in that case.

43 There is more evaporation in region **b** implying that it is both warmer and wet. The fact that region **b** is warmer is further supported by the somewhat greater conduction which would be expected if the difference in temperature between the atmosphere and the land were larger.

44 We have that the original average albedo of the area was $0.6\times0.10+0.4\times0.3=0.18$ and the new one is $0.7\times0.10+0.3\times0.3=0.16$ for a reduction in albedo of 0.02. Hence the expected temperature change is estimated to be 2°C.

Answers to test yourself questions

Topic 9

9.1 Simple harmonic motion

1 They are not simple harmonic because as shown in the textbook the restoring force whereas opposite to, is not proportional to the displacement away from the equilibrium position. If however the amplitude of oscillations is small the force does become approximately proportional to the displacement and the oscillations are then approximately simple harmonic.

2 a We notice that $x_0 \cos\left(\omega t - \frac{\pi}{2}\right) = x_0 \sin \omega t$ and so the phase is $-\frac{\pi}{2}$.

b At $t = 0$ the equation says that $x = x_0 \cos \phi$. The next time x assumes this value is at a time given by $x_0 \cos(\omega T + \phi) = x_0 \cos \phi$. Thus we must solve the equation $\cos(\omega T + \phi) = \cos \phi$. This means that the angles $\omega T + \phi$ and ϕ differ by 2π and so solutions are $\omega T + \phi = \phi + 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

3 a At $t = 0$ we have $y = 5.0\cos(0) = 5.0$ mm.

b At $t = 1.2$ s we use the calculator (in **radian** mode) to find $y = 5.0\cos(2 \times 1.2) = -3.7$ mm.

c $-2.0 = 5.0\cos(2t) \Rightarrow 2t = \cos^{-1}\left(-\frac{2}{5}\right) = 1.98 \Rightarrow t = 0.99$ s.

d Use $v = \pm\omega\sqrt{x_0^2 - x^2}$. We know that $\omega = 2.0\ \text{s}^{-1}$. Therefore,
$6.00 = \pm 2.0\sqrt{25 - x^2} \Rightarrow 25 - x^2 = 9.0 \Rightarrow x = \pm 4.00$ mm.

4 a The equation is simply $y = 8.0\cos(2\pi \times 14t) = 8.0\cos(28\pi t)$.

b The velocity is therefore $v = -8.0 \times 28\pi \sin(28\pi t)$ and the acceleration is $a = -8.0 \times (28\pi)^2 \cos(28\pi t)$.
At $t = 0.025$ s we evaluate (in radian mode) $y = 8.0\cos(28\pi \times 0.025) = -4.7$ cm,
$v = -8.0 \times 28\pi \sin(28\pi \times 0.025) = -5.7\ \text{m s}^{-1}$ and $a = -8.0 \times (28\pi)^2 \cos(28\pi \times 0.025) = 3.6 \times 10^2\ \text{m s}^{-2}$.

5 The angular frequency is $\omega = 2\pi f = 2\pi \times 460 = 920\pi$. The maximum velocity is $\omega A = 920\pi \times 5.0 \times 10^{-3} = 14\ \text{m s}^{-1}$ and the maximum acceleration is $\omega^2 A = (920\pi)^2 \times 5.0 \times 10^{-3} = 4.2 \times 10^4\ \text{m s}^{-2}$.

6 a The equation of the string may be rewritten as $y = (6.0\sin(\pi x))\cos(2\pi \times 520t)$ from which we deduce that the frequency of all points is 520 Hz and that the phase of all points is zero.

b From **a** the amplitude is $A = 6.0\sin(\pi x)$ and so is different for different points on the string.

c The maximum amplitude is obtained when $\sin(\pi x) = 1$, i.e. the maximum amplitude is 6.0 mm.

d The displacement is always zero at the ends of the string, in particular at the right end where $x = L$, the length of the string. The displacement is zero *all the time* when $6.0\sin(\pi x) = 0$ i.e. when $\pi x = \pi \Rightarrow x = 1.0$ m.

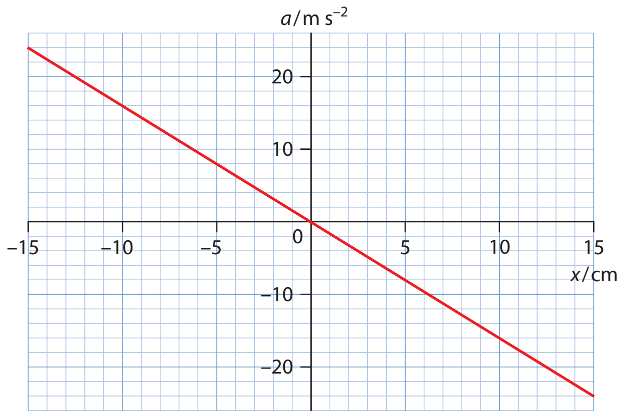
e When $x = \frac{3L}{4} = 0.75$ m the amplitude is $6.0\sin(\pi x) = 6.0\sin(0.75\pi) = 4.2$ mm.

7 a The area is approximately 0.50 cm (the exact value is 0.51 cm).

b This is the displacement from when the velocity is zero to when it is zero again i.e. from one extreme position until the other i.e. twice the amplitude.

c The period is 0.4 s and so the equation for displacement is $x = -0.25\sin\left(\frac{2\pi t}{0.4}\right) = -0.25\sin(5\pi t)$.

8 We need to graph the equation $a = -\omega^2 x$ where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.57\ \text{s}^{-1}$. The slope would be $\omega^2 = 158\ \text{s}^{-2}$ or just 1.58 since we are plotting cm on the horizontal axis.



9 **a** The defining relation for SHM is that $a = -\omega^2 x$ which implies that a graph of acceleration versus displacement is a straight line through the origin with negative slope just as the given graph.

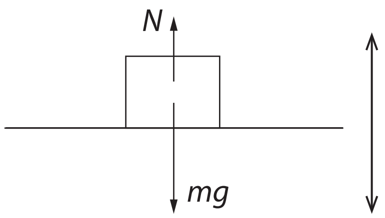
b The slope of the graph gives $-\omega^2$. The measured slope is $\frac{1.5}{0.10} = -15\ \text{s}^{-2}$ and so $\omega = \sqrt{15} = 3.873\ \text{s}^{-1}$. Thus the period is $T = \frac{2\pi}{3.873} = 1.6$ s.

c The maximum velocity is $\omega A = 3.873 \times 0.10 = 0.39\ \text{m s}^{-1}$.

d The maximum net force is $ma = m\omega^2 A = 0.150 \times 15 \times 0.10 = 0.225$ N.

e The total energy is $E_T = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} 0.150 \times 3.873^2 \times 0.10^2 = 0.012$ J.

10 **a** The forces on the mass when the plate is at the top are shown below:



The net force is $mg - N = ma$. Since we have simple harmonic motion $a = \omega^2 x = 4\pi^2 f^2 x$ in magnitude, and the largest acceleration is obtained when $x = A$, the amplitude of the oscillation. The frequency is 5.0 Hz. The critical point is when $N = 0$. I.e. $g = 4\pi^2 f^2 A$ and so $A = \frac{g}{4\pi^2 f^2} = \frac{9.8}{4\pi^2 \times 25} = 0.0099$ m. The amplitude must not exceed this value.

b At the lowest point:

$$N - mg = ma = m4\pi^2 f^2 A$$

$$\Rightarrow N = mg + m4\pi^2 f^2 A$$

$$N = 0.120 \times 9.8 + 0.120 \times 4 \times \pi^2 \times 25 \times 0.0099$$

$$N = 2.35\ \text{N}$$

11 **a** The volume within the sphere of radius x is $\frac{4\pi x^3}{3}$ and that of the entire sphere is $\frac{4\pi R^3}{3}$ therefore the mass enclosed is the fraction $M\frac{x^3}{R^3}$.

b $F = G\frac{\frac{Mx^3}{R^3}m}{x^2} = \frac{GMmx}{R^3}$

c The acceleration of the mass is given by $ma = -\frac{GMmx}{R^3} \Rightarrow a = -\frac{GM}{R^3}x$ which is the condition for SHM with $\omega^2 = \frac{GM}{R^3}$.

d $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$

e $T = 2\pi\sqrt{\frac{(6.4\times 10^6)^3}{6.67\times 10^{-11}\times 6.0\times 10^{24}}} = 5085\ s = 85$ min.

f From gravitation we know that $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$ as in **d.**

12 **a** When extended by an amount x the force pulling back on the body is $2kx$ and so

$ma = -2kx \Rightarrow a = -\frac{2k}{m}x$ and so $\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 120}{2.0}} = 10.95\ \text{s}^{-1}$ giving a period of

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.95} = 0.57$ s.

b With the springs connected this way, and the mass pulled to the side by small amount one spring will be compressed and the other extended. Hence the net force on the mass will still be $2kx$ so the period will not change.

13 **a** At the top the woman's total energy is gravitational potential energy equal to mgh where h is the height measured from the lowest position that we seek. At the lowest position all the gravitational potential energy has been converted into elastic energy $\frac{1}{2}kx^2$ and so $mgh = \frac{1}{2}kx^2$. Since $h = 15 + x$ we have that $mgh = \frac{1}{2}k(h-15)^2$.

We must now solve for the height h:

$$60\times 10\times h = \frac{1}{2}\times 220\times (h-15)^2$$

$$600h = 110(h^2 - 30h + 225)$$

$$110h^2 - 3900h + 24750 = 0$$

$$11h^2 - 390h + 2475 = 0$$

The physically meaningful solution is $h \approx 27$ m.

b The forces on the woman at the position in (a) are her weight vertically downwards and the tension in the spring upwards. Hence the net force is $F_{net} = T - mg = kx - mg = 220\times(27-15) - 600 = 2040$ N hence $a = \frac{F_{net}}{m} = \frac{2040}{60} = 34\ \text{m s}^{-2}$.

c Let x be the extension of the spring at some arbitrary position of the woman. Then the net force on her is $F_{net} = T - mg = kx - mg$ directed upwards i.e. opposite to the direction of x. So $ma = -(kx - mg)$. The acceleration is not proportional to the displacement so it looks we do not have SHM. But we must measure displacement from an equilibrium position. This is when the extension of the spring is x_0 and $kx_0 = mg$. In other words call the displacement to be $y = x - x_0$. Then

$ma = -(k(y + x_0) - mg) = -ky - kx_0 + mg = -ky$ since $kx_0 = mg$. Hence we do have the condition for SHM. And so $a = -\frac{k}{m}y$ so that $\omega^2 = \frac{k}{m} \Rightarrow \omega = 1.91\ \text{s}^{-1}$ and finally $T = \frac{2\pi}{\omega} = 3.28 \approx 3.3$ s.

d She will come to rest when the tension in the spring equals her weight i.e. when

$kx_0 = mg \Rightarrow x_0 = \frac{mg}{k} = \frac{60\times 10}{220} = 2.7$ m. Hence the distance from the top is $15 + 2.7 = 17.7 \approx 18$ m.

e It has been converted to other forms of energy mainly thermal energy in the air and at the point of support of the spring.

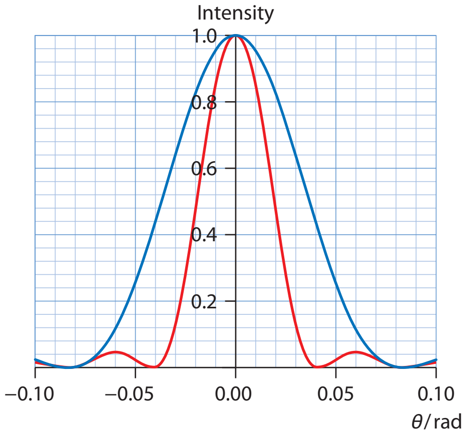
9.2 Single-slit diffraction

14 The diffraction angle is $\theta \approx \frac{\lambda}{b} = 0.333$ rad and so the angular width is double this i.e. 0666 rad or 38.2°.

15 The diffraction angle is $\theta \approx \frac{\lambda}{b} = \frac{6.00 \times 10^{-7}}{0.12 \times 10^{-3}} = 0.0050$ rad and so the angular width is double this i.e. 010 rad. The linear width is therefore $2d\theta = 2 \times 0.0050 \times 2.00 = 0.020$ m.

16 **a** The diffraction angle is about $\theta \approx \frac{\lambda}{b} = 0.0041$ rad and so $b \approx \frac{\lambda}{0.0041} \approx 24\lambda$.

b In **i** we have a smaller width and so a larger diffraction angle. In **ii** there will be no change since both wavelength and width halve. Notice however that if we were to pay attention to the vertical axis scale, with a smaller slit width less light would go through so in both cases the intensity would be less.



9.3 Interference

17 The separation is given by the booklet formula $s = \frac{\lambda D}{d} = \frac{680 \times 10^{-9} \times 1.50}{0.12 \times 10^{-3}} = 8.5$ mm.

18 The two flashlights are not coherent. This means that the phase difference between them keeps changing with time (very fast, on a time scale of nanoseconds). Thus, whatever interference pattern is produced at any moment in time, a different pattern will be produced a nanosecond later. Therefore all we can observe is an average of the rapidly changing patterns on the screen, i.e. no interference at all.

19 $d \sin\theta = n \times 680$ and $d \sin\theta = (n+1) \times 510$. Thus $n \times 680 = (n+1) \times 510 \Rightarrow 680n = 510n + 510 \Rightarrow n = 3$.

20 **a** The separation of the bright fringes is $s = \frac{3.1}{4} \times 10^{-3} = 0.775 \times 10^{-3}$ m. From $s = \frac{\lambda D}{d}$ we get

$$\lambda = \frac{sd}{D} = \frac{0.775 \times 10^{-3} \times 1.00 \times 10^{-3}}{1.2} = 6.46 \times 10^{-7} \approx 6.5 \times 10^{-7} \text{ m}$$

b The wavelength in water would be less (by a factor of 1.33) and so the distance would also be less.

21 **a** We must have $d \sin 20° = 1 \times \lambda$ and so $d = \frac{\lambda}{\sin 20°} = 2.92 \times \lambda$.

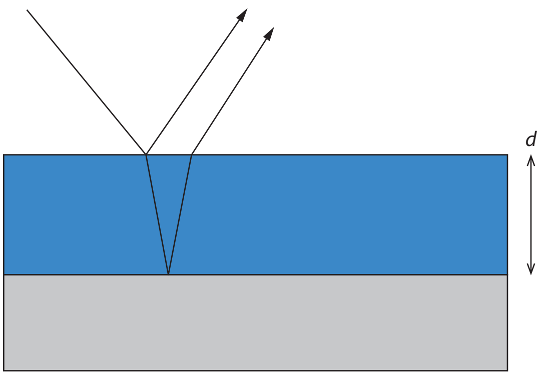
b From $s = \frac{\lambda D}{d}$, the distance between the bright fringes would double if d halves.

22 **a** Use $d \sin\theta = n\lambda$ with $d = \frac{1}{400}$ mm and $\lambda = 600.0$ nm to get:

n	θ
0	0.0°
1	13.89°
2	28.69°
3	46.05°
4	73.74°

b $n = 4$

23 There will be a phase change of π at both reflection points and so the condition for destructive interference (for normal incidence) is $2d = \left(k + \frac{1}{2}\right)\frac{\lambda}{n}$ where n is the refractive index of the coating.

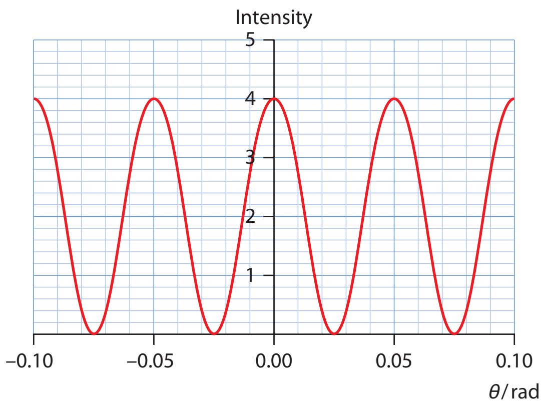


This gives $d = \left(k + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(k + \frac{1}{2}\right)\frac{680}{2 \times 1.38}$ nm. The least thickness d is obtained for $k = 0$ and is $d = 123$ nm.

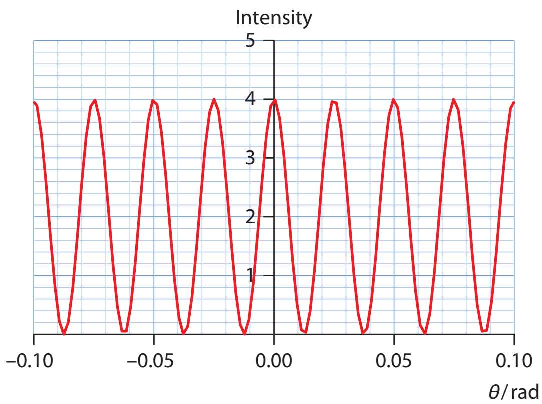
24 The reflected light must show constructive interference. There is a phase change only at the top reflection so the condition for constructive interference is $2d = \left(k + \frac{1}{2}\right)\frac{\lambda}{n}$ where n is the refractive index of the film. Then $d = \left(k + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(k + \frac{1}{2}\right)\frac{550}{2 \times 1.33} = \left(k + \frac{1}{2}\right) \times 206.8$ nm. Possible values of d are then $d = 103$ nm, $d = 310$ nm etc.

25 **a** Coherent light means light where the phase difference between any two points on the same cross section of the beam is constant. Monochromatic light means light of the same wavelength.

b The first maximum is observed at $d \sin\theta = \lambda \Rightarrow \theta = \sin^{-1}\frac{7.0 \times 10^{-7}}{1.4 \times 10^{-5}} = 0.05$ rad and this gives the following graph.



c The angle in **b** would now halve to 0.025 rad giving the following graph.



26 **a** See, for example, Figure 9.19 in the coursebook.

b **i** The intensity increases, the maxima become thinner and there are secondary maxima

ii the intensity of the maxima stays the same but their separation increases.

9.4 Resolution

27 The angular separation of the points is $\theta_A = \frac{1\times10^{-2}}{10\times10^{3}} = 10^{-6}$ rad. The diffraction angle is $\theta_D = \frac{1.22\times600\times10^{-9}}{20\times10^{-2}} = 4\times10^{-5}$ rad. The objects will not be resolved since $\theta_A < \theta_D$.

28 **a** The angular separation is $\theta_A = \frac{1.4}{d}$ and the diffraction angle is $\theta_D = 1.22\times\frac{500\times10^{-9}}{5.0\times10^{-2}} = 1.22\times10^{-5}$ rad. For resolution we need $\theta_A \geq \theta_D$, i.e. $\frac{1.4}{d} \geq 1.22\times10^{-5}$ and so $d \leq \frac{1.4}{1.22\times10^{-5}} = 115$ km.

b It would decrease since the diffraction angle would get smaller.

29 **a** The diffraction angle is $\theta_D = 1.22\times\frac{5.0\times10^{-7}}{4.0\times10^{-3}} = 1.52\times10^{-4}$ rad and this is the smallest angular separation that can be resolved.

b With $\theta_D = \theta_A = 1.52\times10^{-4}$ we get $1.52\times10^{-4} = \frac{s}{3.8\times10^{8}} \Rightarrow s \approx 58$ km.

30 **a** The diffraction angle is $\theta_D = 1.22\times\frac{21\times10^{-2}}{76} = 3.4\times10^{-3}$ rad and this is the smallest angular separation that can be resolved.

b The angular separation of the two stars is $\frac{3.6\times10^{11}}{8.8\times10^{16}} = 4.1\times10^{-6} < \theta_D$ so the stars cannot be resolved.

31 The diffraction angle is $\theta_D = 1.22\times\frac{8.0\times10^{-2}}{300} = 3.3\times10^{-4}$ rad. The angular separation of two points on a diameter of Andromeda is $\frac{2.2\times10^{5}}{2.5\times10^{6}} = 0.088 > \theta_D$ so the telescope sees Andromeda as an extended object.

32 The diffraction angle is $\theta_D = 1.22\times\frac{5.5\times10^{-7}}{4.5\times10^{-3}} = 1.5\times10^{-4}$ rad. When this is about equal to the angular separation of the earth and the moon, i.e. $\theta_A = \frac{3.8\times10^{8}}{d}$, the objects will be resolved. This means $\frac{3.8\times10^{8}}{d} = 1.5\times10^{-4} \Rightarrow d = 2.5\times10^{12}$ m.

33 **a** The diffraction angle is $\theta_D = 1.22\times\frac{5.5\times10^{-7}}{2.4} = 2.8\times10^{-7}$ rad.

b It is free from atmospheric disturbances such as light pollution, turbulence in the air etc.

34 **a** From $d\sin\theta = n\lambda$, we get using the average wavelength of the two lines: $d = \frac{3\times589.29\times10^{-9}}{\sin 12°} = 8.5\times10^{-6}$ m.

b For resolution: $mN = \frac{\bar{\lambda}}{\Delta\lambda} \Rightarrow N = \frac{\bar{\lambda}}{m\Delta\lambda} = \frac{589.29\times10^{-9}}{3\times0.597\times10^{-9}} = 329$.

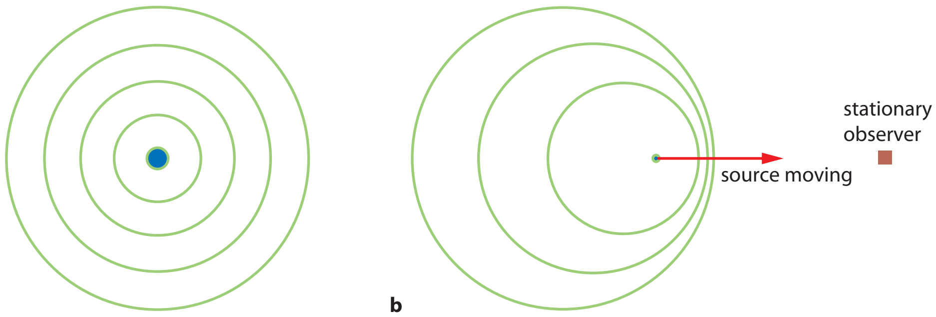
35 **a** From $\Delta\lambda = \frac{\bar{\lambda}}{mN} = \frac{550}{2\times3000} = 0.092$ nm.

b Increasing m and N both decrease $\Delta\lambda$ and so improve resolution. However the intensity of the light decreases with increasing m and so it is preferable to increase N instead.

9.5 The Doppler effect

Note: Take the speed of sound in still air to be 340 m s^{-1}.

36



37 **a** This is a case of a source moving towards the observer and so

$$f = f_0 \frac{c}{c-v} = 500\frac{340}{340-40} = 566.7 \approx 570 \text{ Hz}.$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{500} = 0.68 \text{ m}$

ii $\lambda' = \frac{c}{f} = \frac{340}{566.7} = 0.60 \text{ m}$

38 **a** This is a case of a source moving away from the observer and so

$$f = f_0 \frac{c}{c+v} = 480\frac{340}{340+32} = 438.7 \approx 440 \text{ Hz}.$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{480} = 0.71 \text{ m}$

ii $\lambda' = \frac{c}{f} = \frac{340}{438.7} \approx 0.78 \text{ m}$

39 **a** This is a case of an observer approaching a stationary source and so the relevant formula is and so. $f = f_0\left(1-\frac{v}{c}\right) = 512\left(1-\frac{12}{340}\right) = 493.9 \approx 490 \text{ Hz}.$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{512} = 0.66 \text{ m}$

ii $\lambda' = \frac{c}{f} = \frac{340-12}{493.9} \approx 0.66 \text{ m} = \lambda$

40 **a** This is a case of an observer moving away from a stationary source and so the relevant formula is and so

$$f = f_0\left(1+\frac{v}{c}\right) = 628\left(1+\frac{25}{340}\right) = 674.2 \approx 670 \text{ Hz}.$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{628} = 0.54 \text{ m}$

ii $\lambda' = \frac{c}{f} = \frac{340+25}{674.2} \approx 0.54 \text{ m} = \lambda$

41 An observer on the approaching car will measure a higher frequency (f_1) than that emitted (f_0) because we have a case of the Doppler effect with an approaching source. The wave will then be reflected with frequency f_1. The car is now acting as an approaching source. The frequency received back at the source (f_2) will be higher than that emitted from the car. This is the case of a double Doppler effect.

42 The frequency received by the receding observer is (observer moving away) $f = f_0\left(1-\frac{v}{c}\right)$. The wave is reflected backwards. The moving observer now acts as the source of the waves and the frequency emitted by this "source" is $f = f_0\left(1-\frac{v}{c}\right)$ therefore, the original source now acts as a stationary observer and so the frequency it receives is now $f\frac{1}{\left(1+\frac{v}{c}\right)} = f_0\frac{\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)}$. Hence $480 = 500\frac{\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)}$. Since $\frac{480}{500} = 0.96$ we have

$$0.96 = \frac{\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)}$$

$$0.96 + 0.96\frac{v}{c} = 1 - \frac{v}{c}$$

$$1.96\frac{v}{c} = 0.04$$

$$\frac{v}{c} = 0.0204$$

$$v = 0.0204 \times 340 = 6.9 \text{ m s}^{-1}$$

Hint: You can put the equation $480 = 500\dfrac{\left(1-\dfrac{v}{c}\right)}{\left(1+\dfrac{v}{c}\right)}$ directly into the SOLVER of your graphics calculator (with $x = \dfrac{v}{c}$) and get the answer immediately without any of the tedious algebra above.

43 The frequency received by the stationary observer is (source moving towards) $f = \dfrac{f_0}{\left(1-\dfrac{v}{c}\right)}$. The wave is reflected backwards. The stationary observer now acts as the source of the waves and the frequency emitted by this "source" is $f = \dfrac{f_0}{\left(1-\dfrac{v}{c}\right)}$. The original source now acts as a moving observer and so the frequency it receives is now $f\left(1+\dfrac{v}{c}\right) = f_0\dfrac{\left(1+\dfrac{v}{c}\right)}{\left(1-\dfrac{v}{c}\right)}$. Hence $512 = 500\dfrac{\left(1+\dfrac{v}{c}\right)}{\left(1-\dfrac{v}{c}\right)}$. Since $\dfrac{512}{500} = 1.024$ we have

$$1.024 = \frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)}$$

$$1.024 - 1.024\frac{v}{c} = 1 + \frac{v}{c}$$

$$2.024\frac{v}{c} = 0.024$$

$$\frac{v}{c} = 0.01186$$

$$v = 0.01186 \times 340 = 4.0\ \text{m s}^{-1}$$

44 As far as the observer is concerned the velocity of the source is $v_S + v_0$ and the speed of the wave is $v_0 + c$. So using the formula of the stationary observer and an approaching source we have

$$f_0 = \frac{f_S}{1-\left(\frac{v_S+v_0}{c+v_0}\right)} = \frac{f_S}{\left(\frac{c+v_0}{c+v_0}\right)-\left(\frac{v_S+v_0}{c+v_0}\right)} = f_S\frac{c+v_0}{c-v_S}.$$

45 **a** The frequency emitted is *f*. The observer is moving away so he receives a frequency $f_R = f\dfrac{c-v}{c}$. This frequency is reflected from the object which now acts as a receding source. The frequency received back at the original source is then $f' = f_R\dfrac{c}{c+v} = \left(f\dfrac{c-v}{c}\right)\dfrac{c}{c+v} = f\dfrac{c-v}{c+v} = f\dfrac{\left(1-\dfrac{v}{c}\right)}{\left(1+\dfrac{v}{c}\right)}$.

b If $\dfrac{v}{c}$ is small then $f' \approx f\left(1-\dfrac{v}{c}\right)\left(1+\dfrac{v}{c}\right) \approx f\left(1-2\dfrac{v}{c}\right)$. Hence $\dfrac{\Delta f}{f} = \dfrac{2v}{c}$.

c **i** $\dfrac{\Delta f}{f} = \dfrac{2v}{c} \Rightarrow v = \dfrac{c\Delta f}{2f} = \dfrac{1500 \times 2.4 \times 10^3}{2 \times 5.00 \times 10^6} = 0.36\ \text{m s}^{-1}$

ii Because there is a range of speeds for the blood cells and the ultrasound is not incident normally on the cells.

46 $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = c\dfrac{\Delta\lambda}{\lambda} = 3.0 \times 10^8 \times \dfrac{0.17 \times 10^{-7}}{5.48 \times 10^{-7}} = 9.3 \times 10^6\ \text{m s}^{-1}$

47 **a** $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = c\dfrac{\Delta\lambda}{\lambda} = 3.0 \times 10^{8} \times \dfrac{3.0 \times 10^{-9}}{657 \times 10^{-9}} = 1.4 \times 10^{6}\ \text{m s}^{-1}$

b The speed derived in **a** is just the component of velocity along the line of sight, not the total velocity.

48 The speed of a point on the Sun's equator is $v = \dfrac{2\pi R}{T} = \dfrac{2\pi \times 7.00 \times 10^{8}}{27 \times 24 \times 60 \times 60} = 1.89 \times 10^{3}\ \text{m s}^{-1}$.

The emitted frequency is $f = \dfrac{c}{\lambda} = \dfrac{3.0 \times 10^{8}}{5.00 \times 10^{-7}} = 6.00 \times 10^{14}$ Hz. The shifts are then

$$\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow \Delta f = f\frac{v}{c} = \frac{6.00 \times 10^{14} \times 1.89 \times 10^{3}}{3.00 \times 10^{8}} = 3.78 \times 10^{9}\ \text{Hz}.$$

49 **a** There is no shift since the velocity is at right angles to the direction of observation. The stars are neither approaching or moving away from the observer at that time.

b The speeds of the stars are $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = \dfrac{c\Delta\lambda}{\lambda} = \dfrac{3.00 \times 10^{8} \times 0.08 \times 10^{-7}}{6.58 \times 10^{-7}} = 3.65 \times 10^{6}\ \text{m s}^{-1}$ and

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow v = \frac{c\Delta\lambda}{\lambda} = \frac{3.00 \times 10^{8} \times 0.18 \times 10^{-7}}{6.58 \times 10^{-7}} = 8.21 \times 10^{6}\ \text{m s}^{-1}.$$

Answers to test yourself questions

Topic 10

10.1 Describing fields

1 a The net field at P is: $g = \frac{Gm}{(d/5)^2} - \frac{16Gm}{(4d/5)^2} = \frac{25Gm}{d^2} - \frac{16 \times 25Gm}{16d^2} = 0$

b The net potential at P is: $V = -\frac{Gm}{d/5} - \frac{16Gm}{4d/5} = -\frac{5Gm}{d} - \frac{16 \times 5Gm}{4d} = \frac{5Gm}{d} - \frac{20Gm}{d} = -\frac{25Gm}{d}$

2 a $V = -\frac{GM}{5R_e}$

$= -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{5 \times 6.38 \times 10^{6}}$

$= -1.249 \times 10^{7} \approx -1.25 \times 10^{7}\ \text{J kg}^{-1}$

b $E_P = -\frac{GMm}{5R_e}$

$= -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 500}{5 \times 6.38 \times 10^{6}}$

$= -6.252 \times 10^{9} \approx -6.25 \times 10^{9}\ \text{J}$

3 a $E_P = -\frac{GM_{earth}M_{moon}}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{3.84 \times 10^{8}} = -7.62 \times 10^{28}\ \text{J}$

b $V = -\frac{GM_{earth}}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{3.84 \times 10^{8}} = -1.04 \times 10^{6}\ \text{J kg}^{-1}$

c $v = \sqrt{\frac{GM_{earth}}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{3.84 \times 10^{8}}} = 1.02 \times 10^{3}\ \text{m s}^{-1}$

4 We must plot the function $E_P = -\frac{GM_{earth}m}{r} - \frac{GM_{moon}m}{d-r}$ giving the graph in the answers. Here m is the mass of the spacecraft and d the separation of the earth and the moon (center-to-center). Putting numbers in,

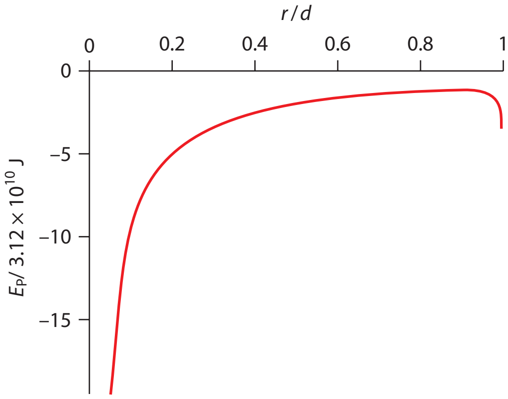
$E_P = -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 3.0 \times 10^{4}}{r} - \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 3.0 \times 10^{4}}{3.84 \times 10^{8} - r}$

$= \frac{1.2 \times 10^{19}}{r} - \frac{1.5 \times 10^{17}}{3.84 \times 10^{8} - r}$

$= \frac{1.2 \times 10^{19} / 3.84 \times 10^{8}}{r / 3.84 \times 10^{8}} - \frac{1.5 \times 10^{17} / 3.84 \times 10^{8}}{1 - r / 3.84 \times 10^{8}}$

$= \frac{3.1 \times 10^{10}}{x} - \frac{3.9 \times 10^{8}}{1 - x}$

where $x = \dfrac{r}{3.84 \quad 10^{8}}$. In this way the function can be plotted on a calculator to give the graph shown here.



5 **a** At r = 0.75, $g = \dfrac{GM_P}{(0.75d)^2} - \dfrac{GM_m}{(0.25d)^2} = 0$. Hence $\dfrac{M_P}{M_m} = \dfrac{(0.75d)^2}{(0.25d)^2} = 9$.

b The probe must have enough energy to get to the maximum of the graph. From then on the moon will pull it in. Then $W = \dfrac{1}{2}mv^2 = m\Delta V \Rightarrow v = \sqrt{2\Delta V} = \sqrt{2(-0.20\times 10^{12} - (-6.45\times 10^{12}))} = 3.5\times 10^{6}\ \text{m s}^{-1}$.

6 The tangential component at A is in the direction of velocity and so the planet increases its speed. At B it is opposite to the velocity and so the speed decreases. The normal component does zero work since the angle between force and displacement is a right angle and $\cos 90° = 0$.

7 The work done by an external agent in moving an object from $r = a$ to $r = b$ at a small constant speed.

8 **a** The pattern is not symmetrical and so the masses must be different. The spherical equipotential surfaces of the right mass are much less distorted and so this is the larger mass.

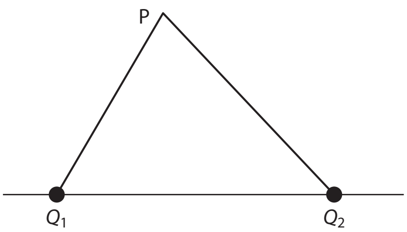
b The gravitational field lines are normal to the equipotential surfaces.

c From far away it looks like we have a single mass of magnitude equal to the sum of the two individual masses. The equipotential surfaces of a single point mass are spherical.

9 **a** $V = \dfrac{kq}{d/2} + \dfrac{kq}{d/2} = \dfrac{4kq}{d}$

b $V = \dfrac{kq}{d/2} - \dfrac{kq}{d/2} = 0$

10 A diagram is:



The potential at P is $V = \dfrac{8.99\times 10^{9}\times 2.0\times 10^{-6}}{0.4} - \dfrac{8.99\times 10^{9}\times 4.0\times 10^{-6}}{0.6} = -1.5\times 10^{4}$ V.

11 **a** $V = 4\ \dfrac{kq}{r}$ where $r = 0.050\sqrt{2}$ m. Hence $V = 4\times \dfrac{8.99\times 10^{9}\times 5.0\times 10^{-6}}{0.050\sqrt{2}} = 2.5\times 10^{6}$ V.

b $E = 0$

c The potential at the centre has a maximum value. At a maximum value the derivative is zero.

12 **a** The work done is

$$W = q\Delta V = q\left(\frac{kQ}{r_2} - \frac{kQ}{r_2}\right) = 1.0\times 10^{-3}\times\left(\frac{8.99\times 10^{9}\times 10}{2.0} - \frac{8.99\times 10^{9}\times 10}{10}\right) = 3.6\times 10^{7}\ \text{J}.$$

b No

13 The work done on the electron is

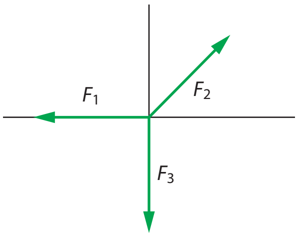
$$W = q\Delta V = q\left(\frac{kQ}{r} - 0\right) = (-1.6\times10^{-19})\times\frac{8.99\times10^{9}\times(-10)}{0.10} = +1.44\times10^{-7}\ \text{J}.$$

14 The work done ($W = q\Delta V$) is equal to the change in kinetic energy $\left(\frac{1}{2}mv^2\right)$. Hence

$$\frac{1}{2}\times 9.1\times10^{-31}\times v^2 = 1.6\times10^{-19}\times(200-100)$$

$$\Rightarrow v = \sqrt{\frac{2\times1.6\times10^{-19}\times100}{9.1\times10^{-31}}} = 5.9\times10^{6}\ \text{m s}^{-1}$$

15 a The forces are roughly as follows.



They have magnitudes:

$$F_1 = \frac{8.99\times10^{9}\times1\times10^{-6}\times2\times10^{-6}}{0.05^2} = 7.19\ \text{N}$$

$$F_2 = \frac{8.99\times10^{9}\times4\times10^{-6}\times2\times10^{-6}}{0.05^2+0.05^2} = 14.4\ \text{N}$$

$$F_3 = \frac{8.99\times10^{9}\times3\times10^{-6}\times2\times10^{-6}}{0.05^2} = 21.6\ \text{N}$$

We must find the components of F_2:

$F_{2x} = F_2\cos45° = 10.2$ N and $F_{2y} = F_2\sin45° = 10.2$ N. So the net force has components:

$F_x = 10.2 - 7.2 = 3.0$ N and $F_y = 10.2 - 21.6 = 11.4$ N. The net force is then $F = \sqrt{(11.4)^2 + (3.0)^2} = 11.8$ N.

The direction of the net force is $\arctan\left(\frac{-11.4}{3.0}\right) = -75°$.

b The distance of the center of the square from each of the vertices is

$a = \sqrt{0.025^2 + 0.025^2} = 0.0354$ cm. So the potential at the center is

$$V = \frac{kQ_1}{a} + \frac{kQ_2}{a} + \frac{kQ_3}{a} + \frac{kQ_4}{a} = \frac{8.99\times10^{9}}{0.0354}\times(-1\times10^{-6} + 2\times10^{-6} - 3\times10^{-6} + 4\times10^{-6})$$

$$V = 5.1\times10^{5}\ \text{V}$$

c The work done is $W = q\Delta V = q(V-0) = 1.0\times10^{-9}\times5.1\times10^{5} = 5.1\times10^{-4}$ J.

16 a Charge will move until both spheres are at the same potential. Then $\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$. By conservation of charge,

$q_1 + q_2 = Q$ where Q is the charge on the one sphere originally. Thus $\frac{q_1}{10} = \frac{q_2}{15} \Rightarrow 3q_1 = 2q_2$

and $q_1 + q_2 = 2.0$. Hence $q_1 = \frac{2}{5}\ 2.0 = 0.80$ ∝C and $q_2 = \frac{3}{5}\ 2.0 = 1.2$ ∝C.

b $\sigma_1 = \frac{0.80\times10^{-6}}{4\pi\times0.10^2} = 6.4\times10^{-6}\ \text{C m}^{-2}$ and $\sigma_2 = \frac{1.2\times10^{-6}}{4\pi\times0.15^2} = 4.2\times10^{-6}\ \text{C m}^{-2}$.

c $E_1 = \frac{kq_1}{r_1^2} = 4\pi k\sigma_1 = 4\pi\times8.99\times10^{9}\times6.4\times10^{-6} = 7.2\times10^{5}\ \text{N C}^{-1}$ and

$E_2 = 4\pi k\sigma_2 = 4\pi\times8.99\times10^{9}\times4.2\times10^{5} = 4.8\times10^{5}\ \text{N C}^{-1}$.

d The electric field is largest for the sphere with the larger charge density. The wire has to be long so that the charge of one sphere will not affect the charge distribution on the other so that both are uniformly charged.

17 You must draw lines that are normal to the equipotentials.

18 **a** The potential a distance x from the bottom plate is given by

$V = -250 + \frac{250-(-250)}{0.15}x = (-250 + 3.33\times 10^{3}x)$ V and so at $x = 3.00$ cm,

$V = (-250 + 3.33\times 10^{3} \times 0.0300) = -150$ V. Therefore the electric potential energy of the charge is

$E_P = qV = (-2.00\times 10^{-6})\times(-150) = 0.300$ mJ.

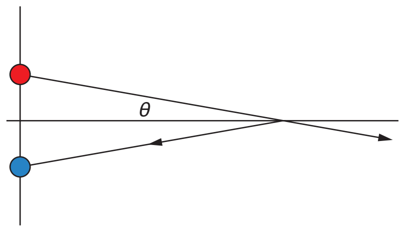
b The potential at $x = 12.0$ cm is $V = (-250 + 3.33\times 10^{3}\times 0.120) = 150$ V and hence $E_P = qV = (-2.00\times 10^{-6})\times 150 = -0.300$ mJ.

c The work done must be $W = q\Delta V = \Delta E_P = -0.300 - 0.300 = -0.600$ J.

19 **a** The kinetic energy of the electron $E_K = \frac{1}{2}mv^2 = \frac{1}{2}\times 9.1\times 10^{-31}\times(1.59\times 10^{6})^2 = 1.15\times 10^{-18}$ J gets converted to electric potential energy eV at the point where the electron stops. Hence the potential at P is $V = \frac{1.15\times 10^{-18}}{-1.6\times 10^{-19}} = -7.19$ V.

b $V = \frac{kQ}{r} \Rightarrow Q = \frac{Vr}{k} = \frac{(-7.19)\times 2.0\times 10^{-10}}{9\times 10^{9}} = -1.6\times 10^{-19}$ C.

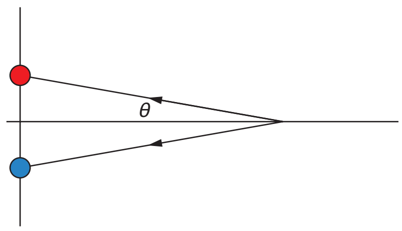
20 **a** The field due to each of the charges has the direction shown. It is clear that the net field will point in the negative y – direction.



The magnitude of the field due to one of the charges is $E = \frac{kQ}{r^2} = \frac{kQ}{a^2+d^2}$. The y – component is

$E_y = \frac{kQ}{a^2+d^2}\sin\theta = \frac{kQ}{a^2+d^2}\frac{a}{\sqrt{a^2+d^2}} = \frac{kQa}{(a^2+d^2)^{3/2}}$ and so the net field is $E_{net} = \frac{2kQa}{(a^2+d^2)^{3/2}}$.

b For two negative charges:



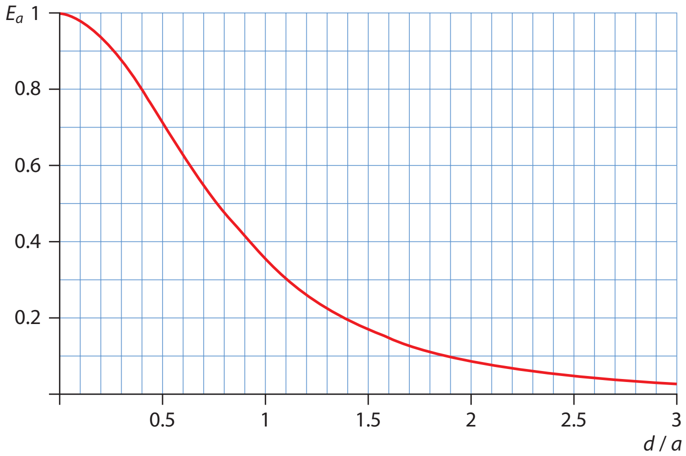
The net field is clearly directed to the left. It has magnitude

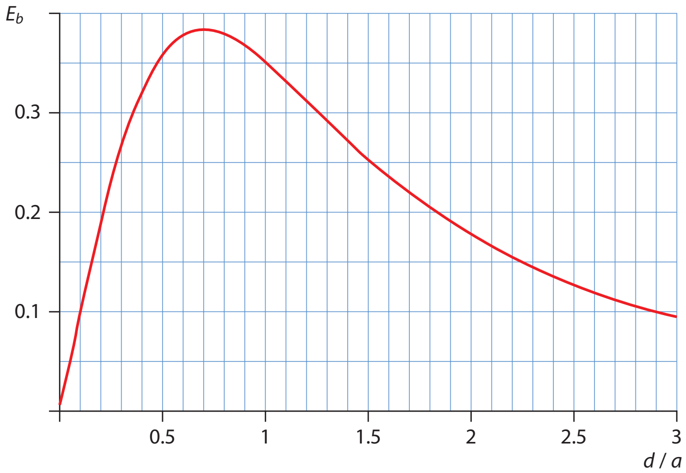
$E_{net} = 2E_x = \frac{2kQ}{a^2+d^2}\cos\theta = \frac{2kQ}{a^2+d^2}\frac{d}{\sqrt{a^2+d^2}} = \frac{2kQd}{(a^2+d^2)^{3/2}}$.

c We have $E_a = \frac{2kQa}{(a^2+d^2)^{3/2}} = \frac{2kQ}{a^2}\frac{1}{\left(1+\frac{d^2}{a^2}\right)^{3/2}}$

and $E_b = \frac{2kQd}{(a^2+d^2)^{3/2}} = \frac{2kQ}{a^3}\frac{d}{\left(1+\frac{d^2}{a^2}\right)^{3/2}} = \frac{2kQ}{a^2}\frac{d/a}{\left(1+\frac{d^2}{a^2}\right)^{3/2}}$.

The plots are (the vertical axis is in units of $\frac{2kQ}{a^2}$):





21 The initial potential energy of the three protons is zero. When at the vertices of the triangle of side a the potential energy is $E_P = 3\dfrac{k(e)(e)}{a} = \dfrac{3ke^2}{a}$ since there are three pairs of charges a distance a apart. This evaluates to $E_P = \dfrac{3\times 8.99\times 10^9\times(1.6\times 10^{-19})^2}{5.0\times 10^{-16}} = 1.4\times 10^{-12}\text{ J} \approx 8.6\text{ MeV}$. This is the energy that must be supplied.

10.2 Fields at work

22 a $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\dfrac{GM}{r}}$

Substituting values: $v = \sqrt{\dfrac{6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.38\times 10^6 + 500\times 10^3}} = 7.6\times 10^3\text{ m s}^{-1}$

b From $v = \dfrac{2\pi r}{T} \Rightarrow T = \dfrac{2\pi r}{v} = \dfrac{2\pi\times(6.88\times 10^6)}{7.6\times 10^3} = 5688\text{ s} = 94.8 \approx 95\text{ min}$

23 We know that $\dfrac{GMm}{R^2} = m\dfrac{v^2}{R} \Rightarrow v^2 = \dfrac{GM}{R}$. But $v = \dfrac{2\pi r}{T}$ and so we deduce that $T^2 = \dfrac{4\pi^2 R^3}{GM}$.

24 a From the previous problem, Therefore

$r = \sqrt[3]{\dfrac{GMT^2}{4\pi^2}} = \sqrt[3]{\dfrac{6.67\times 10^{-11}\times 6.0\times 10^{24}\times(24\times 60\times 60)^2}{4\pi^2}} = 4.2\times 10^7\text{ m}$. The distance from the surface is therefore $r = 4.2\times 10^7 - 6.38\times 10^6 = 3.6\times 10^4\text{ km}$.

b No, it has to be above the equator.

25 The net force is the gravitational force and this must point towards the center of the earth. This happens only for orbit 2.

26 As shown in the text the reaction force from the spacecraft floor is zero giving the impression of weightlessness. More simply, both spacecraft and astronaut are in free fall with the same acceleration.

27 **a** Apply energy conservation to get: total energy at the point the fuel runs out is

$E_T = \frac{1}{2}mv^2 - \frac{GMm}{2R} = \frac{1}{2}m\frac{GM}{2R} - \frac{GMm}{2R} = -\frac{GMm}{4R}$. At the highest point the kinetic energy is zero and so

$-\frac{GMm}{4R} = -\frac{GMm}{r}$ leading to $r = 4R$

b The total energy of the rocket at the point where the fuel runs out is negative so the rocket cannot escape, it will fall back down.

c Apply energy conservation again between the points where the fuel runs out and the crash point to get:

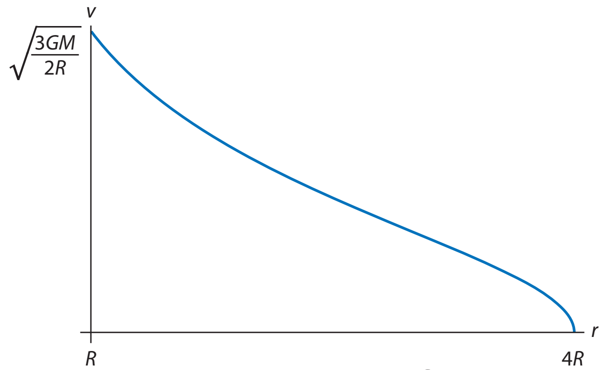
$-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$ leading to

$\frac{1}{2}v^2 = \frac{GM}{R} - \frac{GM}{4R} = \frac{3GM}{4R}$

$v = \sqrt{\frac{3GM}{2R}}$

d From energy conservation, when the rocket is a distance r from the centre of the planet:

$-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{r}$. This simplifies to $v = \sqrt{\frac{2GM}{r} - \frac{GM}{2R}}$ (where $R \le r \le 4R$). We need to plot this function. It is best to write the equivalent form: $v = \sqrt{\frac{GM}{2R}}\sqrt{\frac{4R}{r} - 1}$. The graph is then:



28 **a** We deduced many times that $v^2 = \frac{GM}{r}$ and so $E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$.

b $E_T = -\frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times 6.0 \times 10^{24}}{2 \times 1.5 \times 10^{11}} = -2.7 \times 10^{33}$ J

29 Using $E_K = \frac{GMm}{2r}$, $E_P = -\frac{GMm}{r}$ and $E_T = -\frac{GMm}{2r}$ we deduce that

a B has the larger kinetic energy

b A has the larger potential energy

c A has the larger total energy

30 **a** The total energy is negative so the satellite cannot escape.

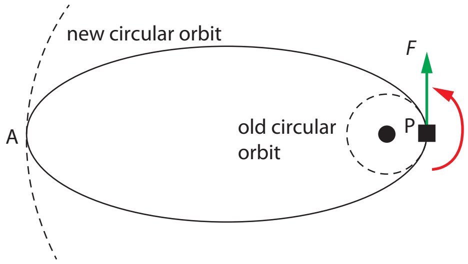
b From problem 30, $E_T = -\frac{GMm}{2r}$. Since we are told that $E_T = -\frac{GMm}{5R}$ and energy is conserved,

$-\frac{GMm}{2r} = -\frac{GMm}{5R} \Rightarrow r = \frac{5R}{2}$.

31 The engines do positive work increasing the total energy of the satellite. Since $E_T = -\frac{GMm}{2r}$ it follows that the orbit radius will increase.

A bit more: Since the kinetic energy is given by $E_K = \frac{GMm}{2r}$ and the orbit radius has increased the speed in the new circular orbit will decrease.

The firing of the rockets when the satellite is in the lower orbit makes the satellite move on an elliptical orbit. After half a revolution the satellite will be at A and further from the earth than in the original position at P. As the satellite gets to A its kinetic energy is reduced and the potential energy increases. At A the speed is too low for the new circular orbit and the engines must again be fired to increase the speed to that appropriate to the new orbit. (If the engines are *not* fired at A then the satellite will remain in the elliptical orbit and will return to P.)



32 The potential energy is given by $E_P = -\frac{GMm}{r}$. This is least when the distance to the sun, r, is smallest (remember, E_P is negative). Therefore since the total energy is conserved, the kinetic energy and hence the speed are greatest at P.

33 The escape speed is $v_{esc} = \sqrt{\frac{2GM}{R}}$. At the surface of the planet, $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$. Substituting:

$v_{esc} = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$.

34 **a** We have done this before.

b $T^2 = \frac{4\pi^2 r^3}{GM}$. Now $r \approx R$ and $\rho = \frac{M}{\frac{4\pi R^3}{3}} = \frac{3M}{4\pi R^3}$. Hence, $\frac{M}{R^3} = \frac{4\pi\rho}{3}$.

Substituting, $T = \sqrt{\frac{4\pi^2}{G}\frac{3}{4\pi\rho}} = \sqrt{\frac{3\pi}{G\rho}}$.

c $\frac{T_{planet}}{T_{earth}} = \sqrt{\frac{\rho_{earth}}{\rho_{planet}}} \Rightarrow \frac{\rho_{earth}}{\rho_{planet}} = \left(\frac{169}{85}\right)^2 = 3.95 \approx 4$

35 **a** We must use the formula $T^2 = \frac{4\pi^2 R^3}{GM}$ that we have derived many times already. Now $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$. Substituting, $T^2 = \frac{4\pi^2 R^3}{gR^2} = \frac{4\pi^2 R}{g}$. Hence $T = 2\pi\sqrt{\frac{R}{g}}$.

b $T = 2\pi\sqrt{\frac{3.4 \times 10^6}{4.5}} = 5.5 \times 10^3\ \text{s} = 91\ \text{min}$.

c From $T^2 = \frac{4\pi^2 R^3}{GM}$ we deduce that $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$ hence $\frac{91^2}{140^2} = \frac{(3.4 \quad 10^6)^3}{R_2^3}$ and so $R_2 = 4.5 \quad 10^6$ m. The height is therefore $h = 4.5 \times 10^6 - 3.4 \times 10^6 = 1.1 \times 10^6$ m.

36 **a** $F = \frac{GM^2}{4R^2}$

b $\frac{GM^2}{4R^2} = \frac{Mv^2}{R^2}$ and so $v^2 = \frac{GM}{4R}$. But $v^2 = \left(\frac{2\pi R}{T}\right)^2$ and so $\frac{GM}{4R} = \left(\frac{2\pi R}{T}\right)^2$. Hence $T^2 = \frac{16\pi^2 R^3}{GM}$

c $T = \sqrt{\frac{16\pi^2(1.0 \times 10^9)^3}{6.67 \times 10^{-11} \times 1.5 \times 2.0 \times 10^{30}}} = 2.8 \times 10^4\ \text{s} = 7.8\ \text{h}$

d $E_T = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 - \frac{GM^2}{2R}$. Since $v^2 = \frac{GM}{4R}$ we have that

$E_T = \frac{1}{2}M\frac{GM}{4R} \times 2 - \frac{GM^2}{2R} = \frac{GM^2}{4R} - \frac{GM^2}{2R} = -\frac{GM^2}{4R}$.

e Since energy is being lost the total energy will decrease. This implies that the distance R will decrease. (From the period formula in (b) the period will decrease as well.)

f **i** The total energy is $E_T = -\frac{GM^2}{4R}$ and the period is $T^2 = \frac{16\pi^2 R^3}{GM}$. Combining the two gives $E_T = -\frac{GM^2}{4\left(\frac{GMT^2}{16\pi^2}\right)^{3/2}}$ or $E_T = -cT^{-3/2}$ where c is a constant. Working as we do with propagation of uncertainties (or using calculus) we have that $\frac{\Delta E_T}{E_T} = \frac{3}{2}\frac{\Delta T}{T}$ or $\frac{\frac{\Delta E_T}{\Delta t}}{E_T} = \frac{3}{2}\frac{\frac{\Delta T}{\Delta t}}{T}$.

ii $\frac{\frac{\Delta E_T}{E_T}}{\Delta t} = \frac{3}{2}\frac{\frac{\Delta T}{T}}{\Delta t} = \frac{3}{2} \times \frac{72 \times 10^{-6}\ \text{s yr}^{-1}}{2.8 \times 10^4\ \text{s}} = 3.9 \times 10^{-9}\ \text{yr}^{-1}$

g The lifetime is therefore $\frac{1}{3.9 \times 10^{-9}\ \text{yr}^{-1}} = 2.6 \times 10^8$ yr.

37 **a** Force towards the centre of the circle.

b We equate the electric force to the centripetal forcer to get: $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = m\frac{v^2}{r}$. Solving for the speed gives the answer.

c The total energy is kinetic plus electric potential energy: $E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$. Using the previous result for speed: $v^2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{mr}$ gives $E = \frac{1}{2}m\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mr} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{r} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = -\frac{1}{8\pi\varepsilon_0}\frac{q^2}{r}$.

d The change in energy is an increase of $\Delta E = -\frac{1}{8\pi\varepsilon_0}\frac{q^2}{2r} - \left(-\frac{1}{8\pi\varepsilon_0}\frac{q^2}{r}\right) = +\frac{1}{16\pi\varepsilon_0}\frac{q^2}{r}$.

38 **a** As in the previous problem $v^2 = k\frac{e^2}{mr}$. Using also $v = \frac{2\pi r}{T}$ we get $\frac{4\pi^2 r^2}{T^2} = k\frac{e^2}{mr} \Rightarrow T^2 = \frac{4\pi^2 m}{ke^2}r^3$.

b $T = \sqrt{\frac{4\pi^2 \times 9.1 \times 10^{-31}}{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2} \times (0.5 \times 10^{-10})^3} = 1.397 \times 10^{-16} \approx 1.4 \times 10^{-16}$ s.

c The change in energy is $E = -\frac{ke^2}{2r}$. In the first orbit this evaluates to

$E_1 = \frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 0.5 \times 10^{-10}} \approx 2.30 \times 10^{-18}$ J. In the other orbit this becomes

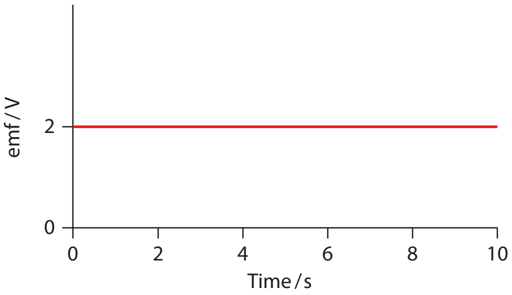
$E_2 = \frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 2.0 \times 10^{-10}} \approx 5.75 \times 10^{-19}$ J. The change is 1.7×10^{-18} J.

Answers to test yourself questions

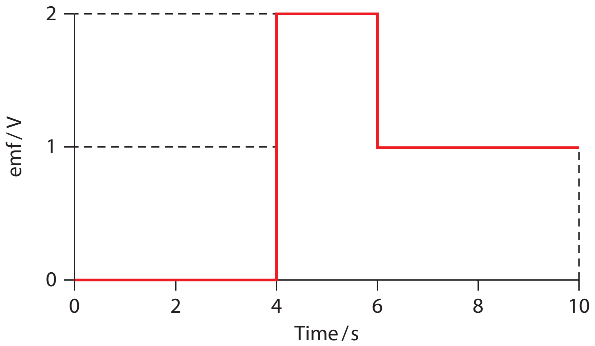
Topic 11

11.1 Electromagnetic induction

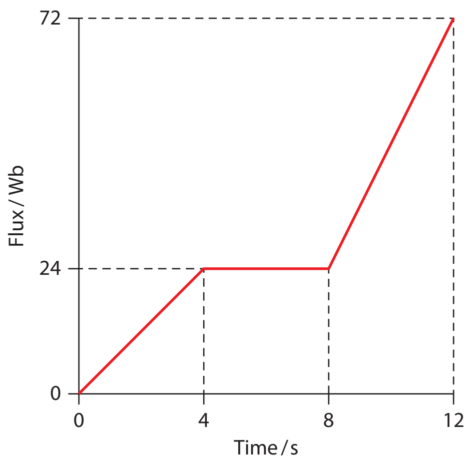
1 The flux is increasing at a constant rate so the induced emf is constant. It equals the slope which is 2.0 V giving the graph shown here.



2 The flux is not changing in the first 4 s so the induced emf is zero. In the next 2 s the slope and hence the emf is constant at 2 V. In the last 4 s the slope is 1 V. This gives the graph here.



3 **a** In the first 4 s the emf is constant at 6 V and so the flux is increasing at a constant rate. We have a straight line graph with slope 6. In the next 2 s the emf is zero which means that the flux is constant. Similarly, in the last 4 the emf is constant so the flux is increasing at a constant rate, i.e. the flux – time graph is a straight line with slope 12. A possible graph is shown here.



b The answer is not unique because there are many straight lines with the slopes given above – we just don't know the value of the flux, just its rate of change.

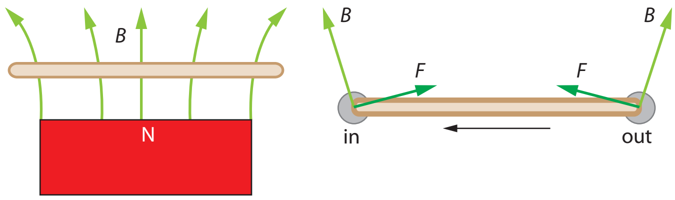
4 The magnetic field created by the outer solenoid is directed into the smaller coil. Since the current is increasing the flux is increasing. By Lenz's law the induced current must oppose the change i.e. it must decrease the flux. This can be done by having the induced current create a magnetic field directed out of the page. The current must then be counter-clockwise.

5 **a** Looking down from above the ring, we see that the magnetic field is directed towards us and the flux is increasing. So the induced current must produce a magnetic field directed away from us. By the right hand rule for the magnetic field direction, the current must be clockwise. As the ring moves away from the magnet we see magnetic field coming towards us and the flux is decreasing. So we must produce a current whose magnetic field comes towards us and so the current must be counter-clockwise. When the ring is half way down the length of the magnet the current must be zero.

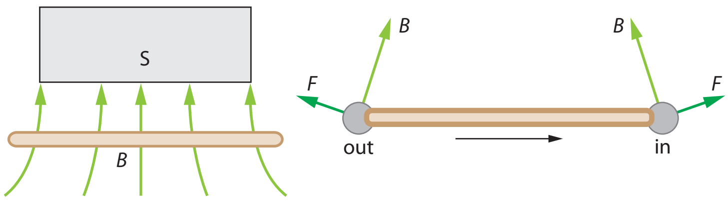
b The magnetic field we see now is directed away from us. So the induced current must create a magnetic field coming towards us and so the current is counter-clockwise. As the ring leaves, the flux is decreasing, the field is going away from us and the current must produce its own magnetic field away from us, i.e. the current is clockwise. Half way it must be zero.

6 This is answered in answer to question **5**.

7 **a** The diagram shows an edge – on view of the ring as it approaches the north pole of the magnet. The forces on two diametrically opposite points on the ring are as shown. The net force of these two forces is upwards. The same is true for any other two diametrically opposite points and so the net magnetic force on the ring is vertically upwards, making the ring fall slower than in free fall.



b As the ring moves away from the south pole the diagram is:



Therefore the net force is again upwards.

8 An electron in the rod is moving downwards and since the magnetic field is into the page the magnetic force on the electron will be directed to the left. Hence the left end will be negatively charged. The electrons that have moved to the left have left the right end of the rod positively charged.

9 **a** The magnetic field at the position of the loop is coming out of the page and is increasing. Hence the flux is increasing. To oppose this increase the induced current must produce a magnetic field into the page and so the current must be clockwise.

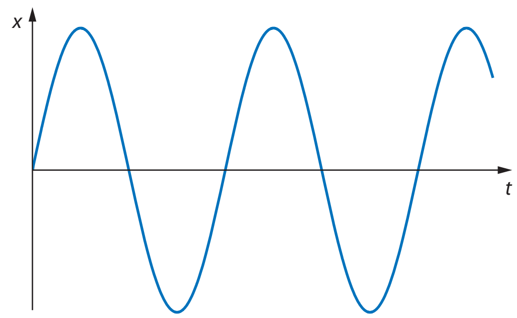
b The magnetic field at the position of the loop is coming out of the page and is decreasing. Hence the flux is decreasing. To oppose this decrease the induced current must produce a magnetic field out of the page and so the current must be counter-clockwise.

10 The induced emf is the rate of change of flux linkage i.e.

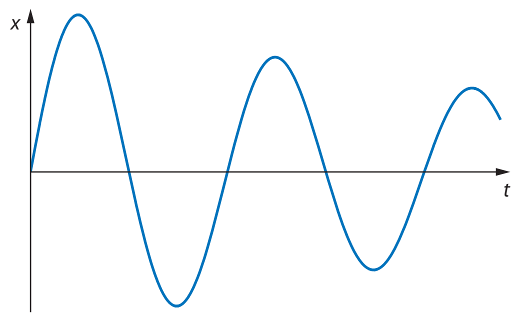
$$N\frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B}{\Delta t}A = N\frac{\Delta B}{\Delta t}\pi r^2 = 200 \times 0.45 \times \pi \times 0.01^2 \approx 28 \text{ mV}.$$

11 The flux in the loop is increasing and so there will be an emf and a current in the loop. By Lenz's law, the magnetic field of the induced current will be directed out of the page hence the current will be counter – clockwise. The force on the movable rod is thus directed to the right. (Note: this could have been guessed since by moving the rod to the right we decrease the are and hence the flux. This is what must happen to oppose the change in flux which is an increase since the field is increasing.)

12 **a** **i**



ii



b **i** There is an induced emf but no current and so no force on the magnet. The oscillations are simple harmonic.

ii Now there is a current. As the magnet moves downward the flux in the coil is increasing. Assume the lower pole is a north pole. The induced current will produce a magnetic field upward and so there will be a retarding force on the magnet that will make the oscillations die out. A similar argument applies when the magnet moves upwards.

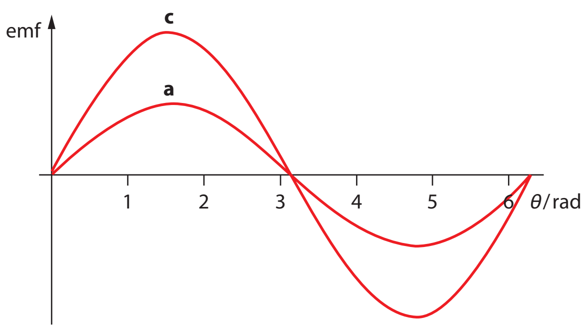
13 As the ring enters the region of magnetic field the flux will be increasing and so an emf will be induced in the ring. Since it is conducting a current will be established as well. The current must produce a field out of the page so as to oppose the increase in flux which means that the current is clockwise. On the lower part of the ring in the region of the field there will therefore be a magnetic force directed upward. Similarly there will be an upward force as the ring leaves the region of the magnetic field. In both cases the upward force delays the fsll of the ring so this ring land last.

11.2 Transmission of power

14 **a** The emf is the (negative) rate of change of the flux with time and so is a sine function with the same period as the graph for flux.

b This is a tricky question: we want the variation with angle and not time so the graph does not change.

c The maximum induced emf will now double since the rate of rotation doubles. But since we want the dependence on angle and not time the period of the graph will not change. This gives the graphs shown here.

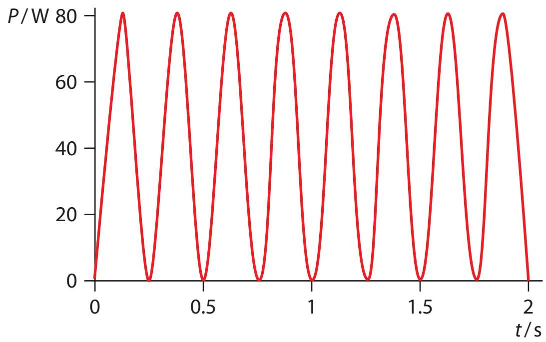


15 a The peak power is 20 W and so the average power is 10 W. Hence $\bar{P} = RI^2_{rms} \Rightarrow I_{rms} = \sqrt{\frac{10}{2.5}} = 2.0$ A.

b $R = \frac{V_{rms}}{I_{rms}} \Rightarrow V_{rms} = 2.0 \times 2.5 = 5.0$ V.

c The period is 1.0 s (there are two peaks within one period).

d At double the rotation speed the period will halve and the peak power will increase by a factor of 4 leading to the graph in the answers in the textbook. This is because at double the rotation speed the induced emf doubles and so the power (that depends on the square of the induced emf) increases by a factor of 4. This gives the graph shown here.



16 a From $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ we get $\frac{220}{V_S} = \frac{500}{200} \Rightarrow V_S = \frac{200 \times 220}{500} = 88$ V. The frequency is unchanged so it stays 50 Hz.

b The power in the primary is $P = V_P I_P = 220 \times 6.0 = 1320$W and so that in the secondary is $0.70 \times 1320 = 924$W.

Hence the current is $\frac{924}{88} = 10.5$ A.

17 a The current produced is $I = \frac{P}{V} = \frac{300 \quad 10^6}{80 \quad 10^3} = 3750$ A. The power lost in the cables is then

$5.0 \times 3750^2 = 7.0 \times 10^7$ W, a fraction $\frac{7.0 \quad 10^7}{300 \quad 10^6} = 0.23$ of the power produced.

b At 100 kV, the current is $I = \frac{P}{V} = \frac{300 \quad 10^6}{100 \quad 10^3} = 3000$ A and the power lost is $5.0 \times 3000^2 = 4.5 \times 10^7$ W

representing a smaller fraction $\frac{4.5 \quad 10^7}{300 \quad 10^6} = 0.15$ of the total power produced.

18 The r.m.s. voltage is given by $V_{rms} = \frac{\omega NBA}{\sqrt{2}}$ and $\omega = 2\pi f = 100\pi\,\text{s}^{-1}$.

Hence $B = \frac{V_{rms}\sqrt{2}}{\omega NA} = \frac{220\sqrt{2}}{100\pi \times 300 \times 0.20^2} = 0.0825$ T.

19 There is error in the question! The vertical axis is flux axis not power. The flux is given by

$\Phi = 10\sin(2\pi \frac{t}{0.9 \times 10^{-3}})$ and so the induced emf is $\varepsilon = \frac{10 \times 2\pi}{0.9 \times 10^{-3}} \cos(2\pi \frac{t}{0.9 \times 10^{-3}})$. The peak value is

$\varepsilon = \frac{10 \times 2\pi}{0.9 \times 10^{-3}} = 69813$ V and so the rms value is $\varepsilon_{rms} = \frac{69813}{\sqrt{2}} = 4.9 \times 10^4$ V.

20 a The current produced is $I = \frac{P}{V} = \frac{150 \quad 10^3}{1.0 \quad 10^3} = 150$ A. The power lost in the cables is then

$2.0 \quad 150^2 = 4.5 \quad 10^4$ W, a fraction $\frac{4.5 \quad 10^4}{150 \quad 10^3} = 0.30$ of the power produced.

b At 5000 V, the current is $I = \frac{P}{V} = \frac{150 \quad 10^3}{5.0 \quad 10^3} = 30$ A and the power lost is $2.0 \quad 30^2 = 1.8 \quad 10^3$ W representing a smaller fraction $\frac{1.8 \quad 10^3}{150 \quad 10^3} = 0.012$ of the total power produced.

21 The maximum power is $\frac{140^2}{24} = 816.7$ W. The average power is $\frac{816.7}{2} = 408 \approx 410$ W.

11.3 Capacitance

22 $C = \varepsilon_0 \frac{A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0}$. Thus $A = \frac{1.0 \times 1 \times 10^{-2}}{8.85 \times 10^{-12}} = 1.1 \times 10^9\,\text{m}^2 = 1.1 \times 10^3\,\text{km}^2$. This is a huge area and shows that a 1 F capacitor is an enormous value for a capacitor.

23 The capacitance is $C = \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \times \frac{0.25}{8.0 \times 10^{-3}} = 2.8 \times 10^{-10}$ F. The charge is then $Q = CV = 2.8 \times 10^{-10} \times 24 = 6.6 \times 10^{-9}$ C.

24 The charge on the fully charged capacitor is $Q = CV = 12 \times 10^{-6} \times 220 = 2.64 \times 10^{-3}$ C. The average current is then $\bar{I} = \frac{Q}{t} = \frac{2.64 \times 10^{-3}}{15 \times 10^{-3}} = 0.18$ A.

25 **a** $Q = CV = 20 \times 10^{-3} \times 9.0 = 180$ mC

b $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-3} \times 9.0^2 = 810$ mJ

c $P = \frac{E}{t} = \frac{810 \times 10^{-3}}{50 \times \times 10^{-3}} = 16.2 \approx 16$ W

26 **a** $C_T = C_1 + C_2 = V = 120 + 240 = 360\,\mu\text{C}$

b The charges are: $Q_1 = C_1 V = 120 \times 10^{-6} \times 6.0 = 720\,\mu\text{C}$ and $Q_2 = C_2 V = 240 \times 10^{-6} \times 6.0 = 1440\,\mu\text{C}$.

c $E_1 = \frac{1}{2}C_1 V^2 = \frac{1}{2} \times 120 \times 10^{-6} \times 6.0^2 = 2160\,\mu\text{J} \approx 2.2 \times 10^{-3}$ J

and $E_2 = \frac{1}{2}C_2 V^2 = \frac{1}{2} \times 240 \times 10^{-6} \times 6.0^2 = 4320\,\mu\text{J} \approx 4.3 \times 10^{-3}$ J.

27 **a** $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = \frac{120 \times 240}{360} = 80\,\mu\text{F}$

b The charge on each capacitor is the same and equals $Q = C_T V = 80 \times 10^{-6} \times 6.0 = 480\,\mu\text{C}$.

c $E_1 = \frac{Q^2}{2C_1} = \frac{(480 \times 10^{-6})^2}{2 \times 120 \times 10^{-6}} = 960\,\mu\text{J}$ and $E_1 = \frac{Q^2}{2C_2} = \frac{(480 \times 10^{-6})^2}{2 \times 240 \times 10^{-6}} = 480\,\mu\text{J}$.

28 The capacitor has a charge of $Q = CV = 25 \times 10^{-12} \times 24 = 600$ pC. After connecting the charged capacitor to the uncharged charge will move form to the other until the voltage across each is the same. The total charge on both capacitors will be 600 pC by charge conservation.

a $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ and so $\frac{Q_1}{25} = \frac{Q_2}{75} \Rightarrow Q_2 = 3Q_1$. $Q_1 + Q_2 = 600\,\text{pC} \Rightarrow Q_1 + 3Q_1 = 600$ pC. Hence $Q_1 = \frac{600}{4} = 150$ pC and so $Q_2 = 450$ pC.

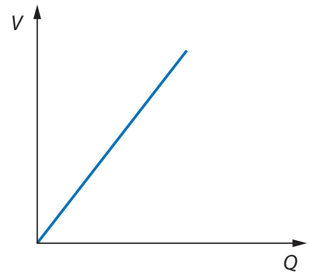
b Initially the energy stored was $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 25 \times 10^{-12} \times 24^2 = 7.2$ nJ. After the connection the total energy is $E_T = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{(150 \times 10^{-12})^2}{2 \times 25 \times 10^{-12}} + \frac{(450 \times 10^{-12})^2}{2 \times 75 \times 10^{-12}} = 1.8$ nJ. The difference is a "loss" of 5.4 nJ.

c The energy was dissipated as thermal energy in the connecting wires.

29 a $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 250 \times 10^{-3} \times 12^2 = 18\ \text{J}$

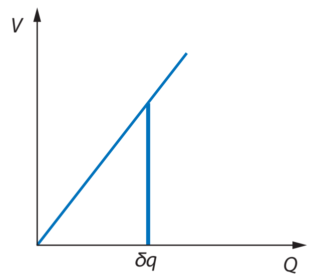
b The resistance of the lamp is found from: $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{12^2}{6.0} = 24\ \Omega$ and the time constant for the circuit is $\tau = RC = 6.0\,\text{s}$. This is, approximately, the time for which the current is appreciable enough to light the lamp.

30 a Since $V = \frac{Q}{C}$ the graph would be a straight line with positive slope through the origin:



To increase the charge on the capacitor by a small change of charge δq requires work $\delta q V$ to be done. This is represented by the area of the thin strip in the graph.

b



This work is stored as energy in the capacitor. Hence the total area is the total energy stored.

31 a and **c** The voltage across the capacitor is given by $V = V_0 e^{-\frac{t}{RC}} = 48e^{-\frac{0.20}{25\times10^{-5}\times15\times10^3}} = 28.159\ \text{V}$. The charge is then $Q = CV = 25.0\times10^{-6}\times28.159 = 7.0\times10^{-4}\,\text{C}$.

b The voltage across the resistor is $48 - 28.159 = 19.841\,\text{V}$ and so the current is $I = \frac{V}{R} = \frac{19.841}{15\times10^3} = 1.3\times10^{-3}\,\text{A}$.

32 a The "half-life" is 1.5 s and so from $\ln 2 = \frac{T_{1/2}}{\tau}$ we find $\tau = \frac{T_{1/2}}{\ln 2} = \frac{1.5}{\ln 2} = 2.16\ \text{s}$.

b From $\tau = RC$ we find $R = \frac{\tau}{C} = \frac{2.16}{50\times10^{-6}} = 43\ \text{k}\Omega$.

33 a The charge stored on the capacitor initially is $Q = CV = 250\times10^{-6}\times12 = 3.0\times10^{-3}\,\text{C}$. The charge t seconds later is $Q = Q_0 e^{-\frac{t}{RC}}$ and the voltage is $V = V_0 e^{-\frac{t}{RC}}$. Hence $\frac{Q}{V} = \frac{Q_0}{V_0} \Rightarrow Q = \frac{VQ_0}{V_0} = \frac{6.0\times3.0\times10^{-3}}{12} = 1.5\times10^{-3}\ \text{C}$.

b The current is $I = \frac{Q_0}{RC}e^{-\frac{t}{RC}}$ and again

$$\frac{I}{V} = \frac{Q_0}{RCV_0} \Rightarrow I = \frac{VQ_0}{RCV_0} = \frac{6.0\times3.0\times10^{-3}}{250\times10^{-6}\times75\times10^3\times12} = 8.0\times10^{-5}\times10^{-3}\,\text{A}.$$

34 a $I = \frac{Q_0}{RC}e^{-\frac{t}{RC}}$ where the initial charge is $Q = CV = 2.00\times10^{-6}\times9.0 = 1.80\times10^{-5}\,\text{C}$.

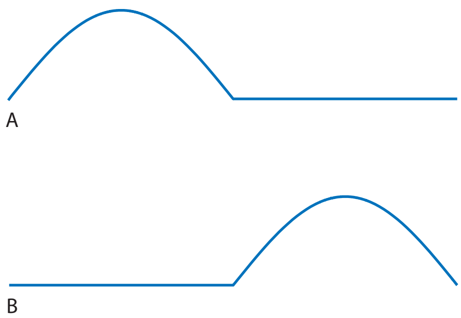
Then, $I = \frac{1.80\times10^{-5}}{5.00\times10^6\times2.00\times10^{-6}}e^{-\frac{1.00}{5.00\times10^6\times2.00\times10^{-6}}} = 1.63\times10^{-6}\ \text{A}.$

b We are asked to find the power in the resistor and so $P = RI^2 = 5.00 \times 10^6 \times (1.63 \times 10^{-6})^2 = 1.33 \times 10^{-5}$ W.

c This has to be the same as **b**.

35 Diode A is top left and diode B is top right!

a



b By using a higher value of the resistance or capacitance.

Answers to test yourself questions

Topic 12

12.1 The interaction of matter with radiation

1 **a** The emission of electrons from a metallic surface when light or other forms of electromagnetic radiation are incident on the surface.

b From the Einstein formula $E_{\max} = hf - \phi$. At the critical frequency, $E_{\max} = 0$ and so

$$hf_c - \phi = 0 \Rightarrow f_c = \frac{\phi}{h} = \frac{3.00 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 7.24 \times 10^{14}\,\text{HZ}.$$

2 **a** Evidence for photons comes from the photoelectric effect, Compton scattering and others.

b $\phi = hf_c$. Then $E_{\max} = hf - \phi = hf - hf_C = h(f - f_C) = 6.63 \times 10^{-34} \times (3.872.25) \times 10^{14} = 1.074 \times 10^{-19}$ J. The stopping voltage is $qV_s = E_{\max} \Rightarrow V_s = \frac{E_{\max}}{q} = \frac{1.0074 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.671V$.

3 **a** Light consists of photons. When light is incident on the metal an electron from within the metal may absorb one photon and so its energy will increase by an amount equal to the photon energy. If this energy is sufficient to overcome the potential well the electron finds itself in, the electron may be free itself from the metal.

b The number of electrons emitted per second is 10^{15} and so the charge that leaves the metal per second, i.e. the current, is $10^{15} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-4}$ A.

c From $E_{\max} = hf - \phi$ we get

$$\phi = hf - E_{\max} = \frac{hc}{\lambda} - E_{\max}$$
$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{5.4 \times 10^{-7}} - 2.1 \times 1.6 \times 10^{-19}$$
$$= 3.23 \times 10^{-19}\,\text{J}$$
$$= \frac{3.23 \times 10^{-20}}{1.6 \times 10^{-19}}$$
$$= 0.20\,\text{eV}$$

d The energy is independent of intensity and so we still have $E_{\max} = 2.1$ eV.

e The current will double since current is proportional to intensity.

4 **a**
- The electrons are emitted without delay. In the photon theory of light the energy is supplied to an electron by a single photon in an instantaneous interaction.
- There is a critical frequency below which no electrons are emitted. The energy of the photon depends on frequency so if the photon energy is less than the work function the electron cannot be emitted.
- The intensity of light has no effect on the energy of the emitted electrons. The intensity of light depends on the number of photons present and so this will affect the number of electrons emitted not their energy.

b **i** Stopping voltage is the voltage that makes the current in the photoelectric experiment zero.

ii $qV_S = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - qV_S$

$$\phi = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{2.08 \times 10^{-7}} - 1.6 \times 10^{-19} \times 1.40 = 7.32 \times 10^{-19}\,\text{J}$$

The longest wavelength corresponds to the smallest frequency i.e. the critical frequency:

$$\frac{hc}{\lambda_C} - \phi = 0 \Rightarrow \lambda_C = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\times3.0\times10^{8}}{7.32\times10^{-19}} = 2.72\times10^{-7}\,\text{m}.$$

5 a i As long as the wavelength stays the same, the energy of the emitted will stay the same.

ii Increasing the intensity of light increases the number of electrons emitted i.e. the photocurrent.

b We have that $qV_S = \frac{hc}{2.3\times10^{-7}} - \phi$ and $q(2V_S) = \frac{hc}{1.8\times10^{-7}} - \phi$.

Subtracting, $qV_S = \frac{hc}{1.8\times10^{-7}} - \frac{hc}{2.3\times10^{-7}} \Rightarrow V_S = 1.50\,\text{V}$. Hence,

$$\phi = \frac{hc}{2.3\times10^{-7}} - qV_S = 6.25\times10^{-19}\,\text{J} = \frac{6.25\times10^{-19}}{1.6\times10^{-19}} = 3.9\,\text{eV}.$$

6 a The work function is $\phi = 3.0\times1.6\times10^{-19} = 4.8\times10^{-19}$ J. The power incident on the given area is $P = 5.0\times10^{-4}\times1.0\times10^{-18} = 5.0\times10^{-22}$ W. To accumulate the energy equal to the work function we need a time of $5.0\times10^{-22}\times t = 4.8\times10^{-19}$ i.e. $t = \frac{4.8\times10^{-19}}{5.0\times10^{-22}} = 960\,\text{s}$ or 16 minutes.

b Since in photoelectric experiments there is no time delay in the emission of electrons, light cannot be treated as a wave as this calculation has.

c In the photon theory of light, the energy carried by a photon is given to the electron in one go, not gradually.

7 a i Extending the graph we find a horizontal intercept of about 5.0×10^{14} Hz.

ii The work function and the critical frequency are related through:

$hf_C - \phi = 0 \Rightarrow \phi = hf_C = 6.63\times10^{-34}\times5.0\times10^{14} = 3.3\times10^{-19}\,\text{J} = 2.1\,\text{eV}$.

b Reading off the graph we find about 2.0×10^{-19} J = 1.25 eV.

c It will be a line parallel to the original with a horizontal intercept at 6.0×10^{14} Hz.

8 The energies, in eV, of the hydrogen atom electron are found from $-\frac{13.6}{n^2}$ and so form the set {−13.6, −3.4, 1.51, 0.85, . . .}. The difference between excited levels and the ground state are {10.2, 12.1, 12.8, . . .}. Thus an electron with energy 11.5 eV can give 10.2 eV of its energy to a ground state electron that will make a transition to the level $n = 2$ and rebound with a kinetic energy 11.5 − 10.2 = 1.3 eV. Of course the electron may just lose no energy to the atom in which case it will have an elastic collision moving away with the same energy as the original, i.e. 11.5 eV.

9 a The intensity is $I = \frac{P}{A} = \frac{nhf}{A}$ where n is the number of photons incident per second. Then $I = \Phi hf$ where $\Phi = \frac{n}{A}$ is the number of photons per second per unit area.

b $I = \Phi hf = \frac{\Phi hc}{\lambda} = \frac{3.8\times10^{18}\times6.33\times10^{-34}\times3.0\times10^{8}}{5.0\times10^{-7}} = 1.5\ \text{Wm}^{-2}.$

c $\frac{\Phi hc}{\lambda} = \frac{\Phi' hc}{\lambda'} \Rightarrow \Phi' = \frac{\lambda'}{\lambda}\Phi = \frac{4.0\times10^{-7}}{5.0\times10^{-7}} = 3.0\times10^{18}\ \text{m}^{-2}s^{-1}.$

d Since the intensity is the same, the flux for the shorter wavelength is less and hence fewer electrons will be emitted since fewer photons are incident.

e This assumes that the fraction of photons that eject electrons is the same for both wavelengths. This fraction is called the quantum efficiency, which in general does depend on wavelength in a complex way.

10 a The existence of absorption and emission spectra – the wavelengths of the emitted or absorbed photons are specific to specific elements and correspond to specific energies. This can be understood if we accept that the energy in atoms has specific values so that differences in levels also have specific values.

b We list the energy differences between the ground sate and the excited states: $\Delta E_{12} = 10.2$ eV, $\Delta E_{13} = 12.1$ eV, $\Delta E_{14} = 12.8$ eV, $\Delta E_{15} = 13.1$ eV, $\Delta E_{16} = 13.2$ eV, $\Delta E_{17} = 13.3$ eV. Hence: **i** not enough energy for an excitation, **ii** the electron can reach $n = 4$ and **iii** the electron can reach $n = 6$.

11 a This is the energy that must be supplied to an atom so that an electron can be ejected from the atom.

b The energy in the $n = 3$ level is $E_3 = -\dfrac{13.6}{3^2} = -1.51$ eV and this is the ionization energy for this level.

12 a The smallest wavelength corresponds to the largest energy difference and this theoretically is 13.6 eV. Hence

$$E = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3.0\times10^{8}}{13.6\times1.6\times10^{-19}} = 9.1\times10^{-8}\ \text{m}.$$

b The kinetic energy of the electron must be at least 13.6 eV, i.e.

$$\frac{1}{2}mv^2 = E_K \Rightarrow v = \sqrt{\frac{2\times13.6\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 2.2\times10^{6}\ \text{m s}^{-1}.$$

13 a $\lambda = \dfrac{h}{p} = \dfrac{6.63\times10^{-34}}{0.250\times10} = 2.7\times10^{-34}\,\text{m}$

b No because to show diffraction effects the brick would have to go through openings of size similar to the wavelength and this is not possible.

14 a The Davisson-Germer experiment – see text.

b $p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}}{680\times10^{-9}} = 9.75\times10^{-28}$ N s. Hence $v = \dfrac{p}{m} = \dfrac{9.75\times10^{-28}}{9.1\times10^{-31}} = 1.1\times10^{3}\,\text{ms}^{-1}$.

15 a The work done in accelerating the electron will go into kinetic energy and so $E_K = qV$. Then $\dfrac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV}$. Then $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$.

b $$\frac{\lambda_p}{\lambda_\alpha} = \frac{\dfrac{h}{\sqrt{2m_p q_p V}}}{\dfrac{h}{\sqrt{2m_\alpha q_\alpha V}}} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} \approx \sqrt{4\times2} = \sqrt{8}$$

c From $E_K = \dfrac{p^2}{2m}$ we find $p = \sqrt{2mE_K} = \sqrt{2\times9.1\times10^{31}\times520\times1.6\times10^{-19}} = 1.23\times10^{-23}$ N s. Hence

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{1.23\times10^{-23}} = 5.4\times10^{-11}\,\text{m}.$$

16 a From $E_K = \dfrac{p^2}{2m}$ we find $p = \sqrt{2mE_K} = \sqrt{2\times1.67\times10^{-27}\times200\times10^{6}\times1.6\times10^{-19}} = 3.27\times10^{-19}$ N s. Hence

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{1.23\times10^{-23}} = 5.4\times10^{-11}\,\text{m}.$$

b The total energy of the electron in the state $n = 2$ is $E = \dfrac{-13.6}{2^2} = -5.44\times10^{-19}$ J. The kinetic energy of the electron is the negative of the total energy and so $E_K = +5.44\times10^{-19}$ J. Since $E_K = \dfrac{p^2}{2m}$ we find $p = \sqrt{2mE_K} = \sqrt{2\times9.1\times10^{-31}\times5.44\times10^{-19}} = 9.95\times10^{-25}$ N s. Hence $\lambda = \dfrac{h}{p} = \dfrac{6.63\times10^{-34}}{9.95\times10^{-25}} = 6.66\times10^{-10} \approx 6.7\times10^{-10}$ m. An alternative way is to use $mvr = \dfrac{nh}{2\pi} \Rightarrow 2\pi r = \dfrac{nh}{p} = n\lambda$ and so $2\pi r = 2\lambda \Rightarrow \lambda = \pi r$. The $n = 2$ state has $r = 4\times0.5\times10^{-10}$ m and so $\lambda = 6.3\times10^{-10}$ m. (The difference with the previous answer is a question of significant figures.)

17 We may take the uncertainty in the electron's position to be $\Delta x \approx 1\times10^{-10}$ m, the "size" of the atom.

Then $\Delta p \geq \frac{h}{4\pi\Delta x} = \frac{6.63\times10^{-34}}{4\pi\times1\times10^{-10}} = 5.27\times10^{-25}$ N s. The corresponding kinetic energy is then of order

$E_K = \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} = \frac{(5.27\times10^{-25})^2}{2\times9.1\times10^{-31}} = 1.53\times10^{-19}\text{ J} = \frac{1.53\times10^{-19}}{1.6\times10^{-19}} = 0.96 \approx 1\text{ eV}$ which is the correct order of magnitude.

18 **a** There is a wave associated with every moving particle, of wavelength equal to Planck's constant divided by the momentum of the particle.

b The kinetic energy of the electron will be $E_K = qV$ and so $\frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV} = 1.21\times10^{-24}$ N s. Then $\lambda = \frac{6.63\times10^{-34}}{1.21\times10^{-24}} = 5.5\times10^{-10}$ m.

c Precise knowledge of the wavelength implies precise knowledge of the momentum. By the uncertainty principle the uncertainty in position must be large.

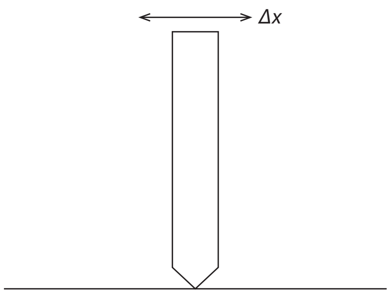
19 **a** As the opening decreases there will be more and more diffraction and so the beam will not be thin – it will spread.

b To reduce diffraction the wavelength must be as small as possible (and smaller than *d*). This requires fast electrons.

20 The de Broglie wavelength of the tennis ball is $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{6} \approx 1\times10^{-34}$ m. The tennis ball wave will diffract through the opening. The angle at which the first diffraction minimum occurs is of order

$\theta_D \approx \frac{\lambda}{b} = \frac{1\times10^{-34}}{1} = 1\times10^{-34}$ rad. The angle is insignificantly small. The tennis ball will move on a straight line without any deviation.

21 There will always be an uncertainty Δx in the position of the top of the pencil and so there will be a corresponding uncertainty in momentum. Hence the top of the pencil will have to move and hence the pencil will fall.



22 **a** The top graph allows precise determination of the wavelength and hence the momentum. The uncertainty in momentum will then be small.

b The bottom diagram shows that the probability of finding the particle is large within a small area of space.

23 **a** The wavelength will be given by $\lambda = \frac{2L}{n}$ and for the fundamental (i.e. the first harmonic), $\lambda = 2L = 2\times10^{-15}$ m.

b $\Delta p \geq \frac{h}{4\pi\Delta x} = \frac{6.63\times10^{-34}}{4\pi\times1\times10^{-15}} = 5.27\times10^{-20}$ N s

$E_K = \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} = \frac{(5.27\times10^{-20})^2}{2\times9.1\times10^{-31}} = 1.53\times10^{-9}\text{ J} = \frac{1.53\times10^{-9}}{1.6\times10^{-19}} \approx 10^{10}\text{ eV} = 10^4\text{ MeV}.$

c This is far larger than the binding energy of a nucleus and so the electron would rip the nucleus apart. The electron cannot be confined within a nucleus.

12.2 Nuclear physics

24 By conservation of energy

$$\frac{1}{2}mv^2 = \frac{k(2e)(79e)}{d} \Rightarrow v = \sqrt{\frac{2k(2e)(79e)}{md}}$$

$$v = \sqrt{\frac{2\times 9\times 10^9(2\times 1.6\times 10^{-19})(79\times 1.6\times 10^{-19})}{6.4\times 10^{-27}\times 8.5\times 10^{-15}}}$$

$$v = 3.7\times 10^7 \text{ m s}^{-1}$$

25 The idea is that since the nucleus is very massive it will not recoil. Then at the point of closest separation the kinetic energy will be a minimum and will increase as the separation increases. The potential energy is given by $E_P = \frac{kZe^2}{r}$ and so will be a maximum at the point of closet separation and will tend to zero as the separation increases. These observations give the graphs in the answers in the textbook.

26 **a** As the energy increases the alpha particle can approach closer and closer to the nucleus. Eventually it will be within the range of the strong nuclear force and some alphas will be absorbed by the nucleus and will not scatter.

b Since the nuclear charge of aluminum is smaller than that of gold the alphas will get closer to aluminum and so will experience the nuclear force first. Hence deviations will first be seen for aluminum.

27 The radius of a nucleus of mass number A is $R = 1.2\times A^{1/3}\times 10^{-15}$ m and its mass is $M = Am_n$ (here m_n is the mass of a nucleon). The density is therefore $\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{A\times m_n}{\frac{4}{3}\pi(1.2\times A^{1/3}\times 10^{-15})^3} = \frac{m_n}{\frac{4}{3}\pi(1.2\times 10^{-15})^3}$ and so is independent of A. An estimate of this density is $\rho = \frac{1.67\times 10^{-27}}{\frac{4}{3}\pi(1.2\times 10^{-15})^3} \approx 10^{17}\text{ kg m}^{-3}$.

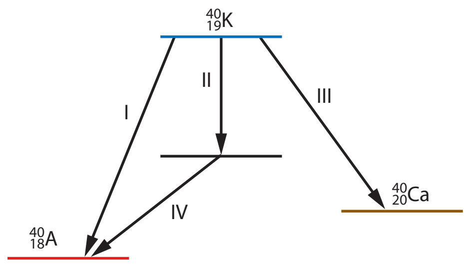
28 **a** The main evidence is the discrete energies of alpha particles and gamma partivcles in alpha and gamma decay.

b ${}^{226}_{88}\text{Ra} \rightarrow {}^{226}_{88}\text{Ra} + {}^{0}_{0}\gamma$

c $hf = \frac{hc}{\lambda} = \Delta E \Rightarrow \lambda = \frac{hc}{\Delta E}$. Hence $\lambda = \frac{6.63\times 10^{-34}\times 3.0\times 10^8}{0.0678\times 10^6\times 1.6\times 10^{-19}} = 1.83\times 10^{-11}$ m.

29 Plutonium $\left({}^{242}_{94}\text{Pu}\right)$ decays into uranium $\left({}^{238}_{92}\text{U}\right)$ by alpha decay. The energy of the alpha particles takes four distinct values, 4.90 MeV, 4.86 MeV, 4.76 MeV and 4.60 MeV. In all cases a gamma ray photon is also emitted except when the alpha energy is 4.90 MeV. Use this information to suggest a possible nuclear energy level diagram for uranium.

30



The four indicated transitions are:

I: ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + e^+ + \nu$ (beta plus decay)

II: ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{19}\text{K} + \gamma$ (gamma decay)

III: ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + e^+ + \nu$ (beta plus decay)

IV: ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{20}\text{Ca} + e^- + \nu$ (beta minus decay)

31 **a** We know that $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{3.00} = 0.231\text{ s}^{-1}$.

b We start with $\frac{1}{100}$ 6.02 10^{23} = 6.02 10^{21} nuclei and so:

i $N = 6.02 \times 10^{21} \times e^{-0.231\times 1} = 4.78 \times 10^{21}$;
ii $N = 6.02 \times 10^{21} \times e^{-0.231\times 2} = 3.79 \times 10^{21}$;
iii $N = 6.02 \times 10^{21} \times e^{-0.231\times 3} = 3.01 \times 10^{21}$.

32 **a** The probability of decay within a half-life is always $\frac{1}{2}$.

b The probability that the nucleus will not decay after the passage of three half – lives is $\underbrace{\frac{1}{2}}_{\text{no decay}} \quad \underbrace{\frac{1}{2}}_{\text{no decay}} \quad \underbrace{\frac{1}{2}}_{\text{no decay}} = \frac{1}{8}$.

Hence the probability that the nucleus will decay some time within three half – lives is $1 - \frac{1}{8} = \frac{7}{8} = 0.875$.

c The probability of decay in any one half – life interval is 0.5.
More mathematically, we want to find $P(D|N)$ where we use the notation of conditional probability and the events D and N stand for D = decay in the next half-life and N = no decay in the first 4 half lives. Then $P(D|N) = \frac{P(D\cap N)}{P(N)}$. Now, $P(N) = \frac{1}{2^4} = \frac{1}{16}$ and $P(D\cap N) = \frac{1}{32}$. Hence $P(D|N) = \frac{1}{2}$.

33 The half-life is so long so that what we are really asked to find is the initial activity of 1.0 g of pure radium. We have that $A = \lambda N_0 e^{-\lambda t}$ so that the initial activity is λN_0. A mass of 1.0 g of radium corresponds to $\frac{1.0}{226.025} = 0.0044243$ moles and hence $N_0 = 0.0044243 \quad 6.02 \quad 10^{23} = 2.6634 \quad 10^{21}$ nuclei. Since $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1600\times 365\times 24\times 60\times 60} = 1.3737\times 10^{-11}\ \text{s}^{-1}$ we find an activity of $1.3737\times 10^{-11} \times 2.6634\times 10^{21} = 3.66\times 10^{10}$ Bq.

34 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12} = 0.0578\ \text{d}^{-1}$ and so $A = \lambda N_0 e^{-\lambda t} = 3.5 \times e^{-0.0578\times 20} = 1.1$ MBq.

35 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{6\times 24\times 60\times 60} = 1.34\times 10^{-6}\ \text{s}^{-1}$. From $A = \lambda N_0 e^{-\lambda t}$ we find

$0.50\times 10^{6} = 1.34\times 10^{-6} \times N_0 e^{-1.34\times 10^{-6}\times 24\times 60\times 60} \Rightarrow N_0 = 4.2\times 10^{11}$.

36 After time t the number of uranium atoms remaining in the rocks is $N = N_0 e^{-\lambda t}$ and so the number that decayed (and hence eventually became lead) is $N - N_0 = N_0(1 - e^{-\lambda t})$. Hence we have that $\frac{N_0(1-e^{-\lambda t})}{N_0 e^{-\lambda t}} = 0.80$.
This means that $1 - e^{-\lambda t} = 0.80 e^{-\lambda t} \Rightarrow 1 = 1.80 e^{-\lambda t} \Rightarrow e^{\lambda t} = 1.80$. Hence $\lambda t = \ln(1.80) = 0.5878$. Since $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.5\times 10^{9}} = 1.54\times 10^{-10}\ \text{yr}^{-1}$ we find $t = \frac{0.5878}{1.54\times 10^{-10}} = 3.8\times 10^{9}$ yr.

37 The method of question **36** may be used but here, clearly, a ratio of 1:7 corresponds to three half – lives and so the age is about $t = 3 \quad 1.37 \quad 10^{9} = 4.1 \quad 10^{9}$ yr.

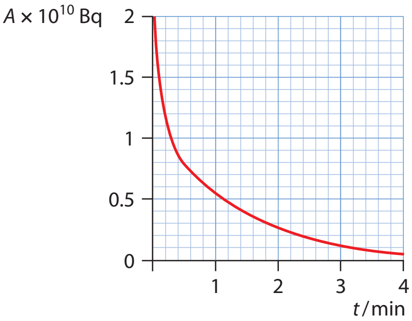
38 The activity is given by $A = \lambda N = \lambda N_0 e^{-\lambda t}$ where $\lambda = \frac{\ln 2}{T_{1/2}}$ is the decay constant.

a $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A}}{\lambda_B N_{0B}} = \frac{3}{4} \times 1 = \frac{3}{4} = 0.75$

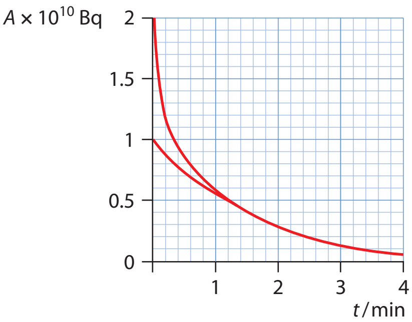
b $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A} e^{-\lambda_A \times 4}}{\lambda_B N_{0B} e^{-\lambda_B \times 4}} = \frac{3}{4} \times \frac{e^{-\frac{\ln 2}{4}\times 4}}{e^{-\frac{\ln 2}{3}\times 4}} = 0.95$

c $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A} e^{-\lambda_A \times 12}}{\lambda_B N_{0B} e^{-\lambda_B \times 12}} = \frac{3}{4} \times \frac{e^{-\frac{\ln 2}{4}\times 12}}{e^{-\frac{\ln 2}{3}\times 12}} = 1.5$

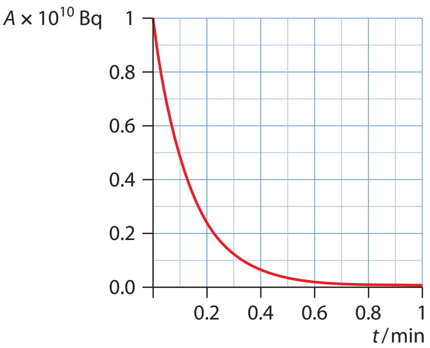
39 This is a very difficult question and many different possibilities must be considered. Essentially we must be able to determine form a graph of activity versus time the initial activities of the two isotopes and their respective half – lives. One possibility is represented by the following graph in which a short (S) and a long (L) half – life isotopes are present. The shape of the curve is not a pure exponential.



We see that after about 1 minute we have a smooth exponential curve which implies that one of the isotopes has essentially decayed away, leaving behind just one isotope. This is justified by estimating a half-life for times greater than 1 minute. We get consistently a half-life of 1 minute for the long half – life isotope. Extending smoothly the exponential curve backwards, we intercept the vertical axis at about 1 10^{10} Bq.



Thus the activity of isotope L is given by $A_L = 10^{10} \quad 0.5^{t/1}$. This means that the initial activity of the other isotope is also 1 10^{10} Bq. Subtracting from the data points of the given graph the activity of this isotope we get the following graph.



This represents the decay of just isotope S. From this graph we find a half-life of about 0.1 minute.
Obviously, this analysis gets more complicated when the half- lives are not so different or when the initial activities are very different.

40 **a** If the mass (in grams) is m and the molar mass is ∝, the number of moles of the radioactive isotope is $\frac{m}{\propto}$. The initial number of nuclei is then $N_0 = \frac{m}{\propto} \quad N_A$ since one mole contains Avogadro's number of molecules.

b The activity is $A = \lambda N = \lambda N_0 e^{-\lambda t} = \lambda \frac{m}{\mu} N_A e^{-\lambda t}$ and the initial activity is thus $A_0 = \lambda \frac{m}{\mu} N_A x$. Measuring the initial activity then allows determination of the decay constant and hence the half-life from $\lambda = \frac{\ln 2}{T_{1/2}}$.

Additional Topic 1 questions

Test yourself

1.1 Measurement in physics

1 What is the radius of the Earth (6380 km) expressed in units of the Planck length?

2 Assuming the entire universe to be made up of hydrogen gas, how many molecules of hydrogen are there?

3 How many apples do you need to make up the mass of an average elephant?

4 Give an order-of-magnitude estimate for the time taken by light to travel across the diameter of the Milky Way galaxy.

5 Give an order-of-magnitude estimate for the gravitational force of attraction between two people 1 m apart.

6 Calculate the acceleration of a block of mass 2.42 kg that is acted upon by a force of 15 N. ($F = ma$)

1.2 Uncertainties and errors

7 The mass of a rectangular block is measured to be 2.2 kg with an uncertainty of 0.2 kg. The sides are measured as 60 ± 3 mm, 50 ± 1 mm and 40 ± 2 mm. Find the density of the cube in kilograms per cubic metre, giving the uncertainty in the result.

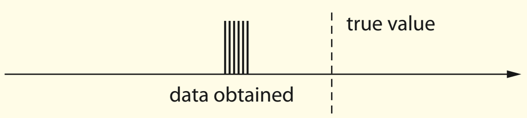
8 The radius r of a sphere is measured to be $22.7\text{ cm} \pm 0.2\text{ cm}$. Find the uncertainty in:

a the surface area of the sphere

b the volume of the sphere.

9 The length of a pendulum is measured with a percentage uncertainty of 5% and the period with a percentage uncertainty of 6%. Find the percentage uncertainty in the measured value of the acceleration due to gravity.

10 A student measured a given quantity many times and got the results shown in the diagram. The true value of the quantity is indicated by the dotted line.



a Discuss whether she should continue accumulating more data in the hope of getting a result that agrees with the true value.

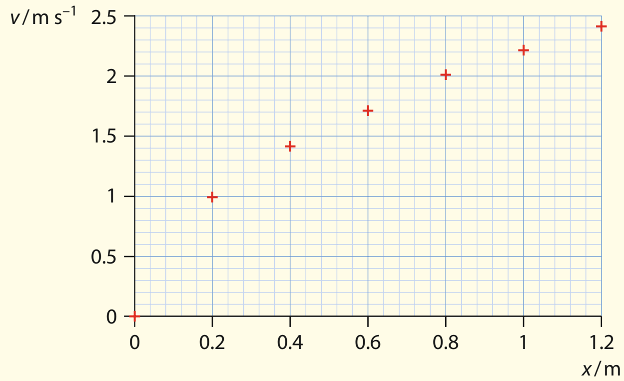
b Suggest whether the source of error is systematic or random.

11 In yet another experiment, the following data was collected for current and voltage: $(V, I) = \{(0.1, 29), (0.2, 46), (0.3, 62), (0.4, 80)\}$, with uncertainty of ± 4 mA in the current.

a Plot the current versus the voltage and draw the best-fit line. Suggest whether the current is proportional to the voltage.

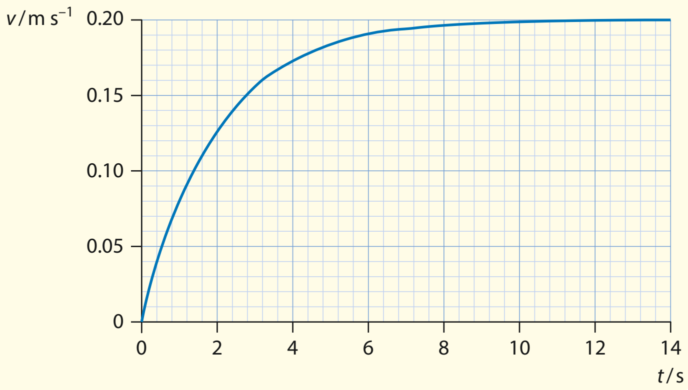
b The experimenter is convinced that the straight line fitting the data should go through the origin. What can allow for this?

12 The velocity of an object after a distance x is given by $v^2 = 2ax$, where a is the constant acceleration. The graph shows the results of an experiment in which velocity and distance travelled were measured. Copy the graph and draw a smooth curve through the points. Estimate the acceleration and the velocity of the object after a distance of 2.0 m.



13 A sphere and a cube have the same surface area. Which shape has the larger volume?

14 The graph shows how the velocity of a steel ball depends on time as it falls through a viscous medium. Find the equation that gives the velocity as a function of time.



15 The table shows the data collected in an experiment.

$x/\pm 0.1$	y
1.0	2.0 ± 0.1
2.0	11.3 ± 0.8
3.0	31 ± 3
4.0	64 ± 6
5.0	112 ± 10
6.0	176 ± 20

a Plot y against x and draw the best-fit line.

b Assuming the suspected relationship between the variables is $y = cx^{2.5}$, plot the data in order to get a straight line and then find the value of the constant c.

1.3 Vectors and scalars

16 A person walks 5.0 km due east, then 3.0 km due north and finally stops after walking an additional 2.0 km due north east. How far and in what direction relative to her starting point is she?

17 Vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ have components ($A_x = 3.00$, $A_y = 4.00$) and ($B_x = -1.00$, $B_y = 5.00$). Find the magnitude and direction of the vector $\boldsymbol{C}$ such that $\boldsymbol{A} - \boldsymbol{B} + \boldsymbol{C} = 0$.

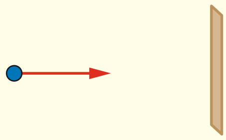
18 Points P and Q have coordinates $P = (x_1, y_1)$, $Q = (x_2, y_2)$.

a Find the components of the vector from P to Q.

b What are the components of the vector from Q to P?

c What is the magnitude of the vector from the origin to P?

19 A molecule with a velocity of $352\,\text{m}\,\text{s}^{-1}$ collides with a wall as shown in the diagram, and bounces back with the same speed.



a What is the change in the molecule's velocity?

b What is the change in the speed?

Additional Topic 1 answers

Topic 1 Measurements and uncertainties

1.1 Measurement in physics

Many of the calculations in the problems of this section have been performed without a calculator and are estimates. Your answers may differ.

1 6.4×10^{41}

2 3×10^{79}

3 16 000 (assuming a 4000 kg elephant)

4 100 000 years

5 2×10^{-7} N

6 $6.2 \times 10^{9}\,\text{m s}^{-2}$

1.2 Uncertainties and errors

7 $(1.8 \pm 0.4) \times 10^{4}\,\text{kg m}^{-3}$

8 **a** $(6.5 \pm 0.1) \times 10^{3}\,\text{cm}^{2}$

b $(4.9 \pm 0.1) \times 10^{4}\,\text{cm}^{3}$

9 17%

10 **a** no

b systematic

11 The line of best fit intersects at 12 mA. The extreme line within the error bars intersects at 6 mA. So no line can be made to go through the origin for this data. A systematic error of about 1 mA is required.

12 $2.4\,\text{m s}^{-2}$; $3.1\,\text{m s}^{-1}$

13 sphere

14 $v = 0.2(1 - e^{-0.5t})$

15 b $c = 2$

1.3 Vectors and scalars

16 7.79 km at 34.5°

17 $\boldsymbol{C} = (-4.00, 1.00)$

18 **a** $(x_2 - x_1, y_2 - y_1)$

b $(x_1 - x_2, y_1 - y_2)$

c $\sqrt{x_1^2 + y_1^2}$

19 **a** $704\,\text{m s}^{-1}$ in magnitude

b zero

Additional Topic 2 questions

Test yourself

2.1 Motion

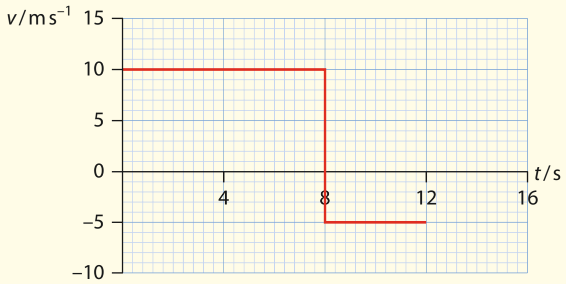
Uniform motion

1 A person walks a distance of 3.0 km due south and then a distance of 2.0 km due east. If the walk lasts for 3.0 h, find:

a the average speed for the motion

b the average velocity.

2 An object moving in a straight line according to the velocity–time graph shown below has an initial position of 8.00 m.

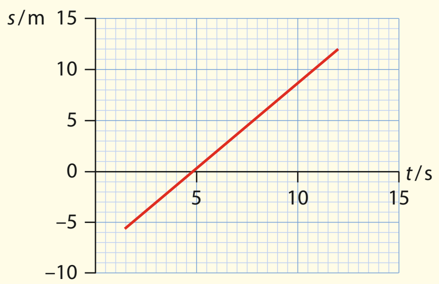


a Find the position after 8.00 s.

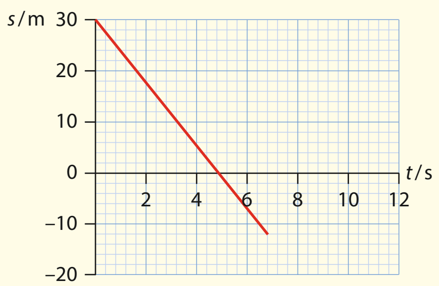
b Find the position after 12.00 s.

c Calculate the average speed and average velocity for this motion.

3 Find the velocity of the two objects whose displacement–time graphs are shown below.



a



b

Accelerated motion

4 The acceleration of a car is assumed constant at 1.5 m s^{-2}. Determine how long it will take the car to accelerate from 5.0 m s^{-1} to 11 m s^{-1}.

5 A body has an initial velocity of 3.0 m s^{-1} and after travelling 24 m the velocity becomes 13 m s^{-1}. Determine how long this took.

6 A ball is thrown upwards with a speed of 24 m s^{-1}.

a Find the time when the velocity of the ball is 12 m s^{-1}.

b Find the time when the velocity of the ball is −12 m s^{-1}.

c Calculate the position of the ball at the times found in **a** and **b**.

d Determine the velocity of the ball 1.5 s after launch.

e Predict the maximum height reached by the ball.

(Take the acceleration of free fall to be 9.8 m s^{-2}.)

7 A stone is thrown vertically upwards with an initial speed of 10.0 m s^{-1} from a cliff that is 50.0 m high.

a Find the time when it reaches the bottom of the cliff.

b Find the speed just before hitting the ground.

c Determine the total distance travelled by the stone.

(Take the acceleration of free fall to be 9.81 m s^{-2}.)

8 A rock is thrown vertically down from the roof of a 25.0 m high building with a speed of 5.0 m s^{-1}.

a Find the time when the rock hits the ground.

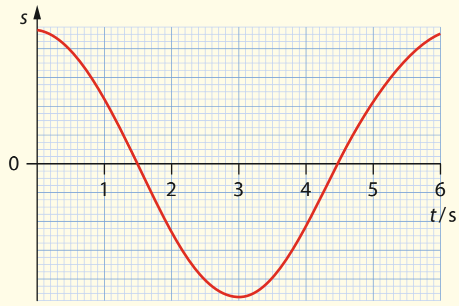
b Find the speed with which it hits the ground.

(Take the acceleration of free fall to be 9.81 m s^{-2}.)

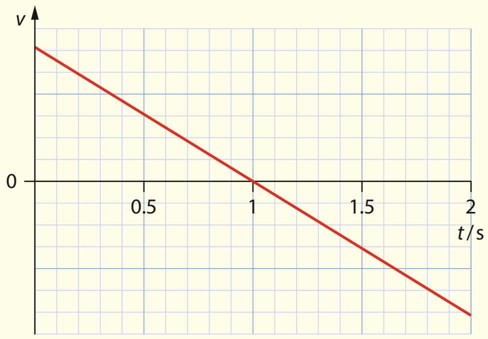
9 A window is 1.50 m high. A stone falling from above passes the top of the window with a speed of 3.00 m s^{-1}. Find the time when it will pass the bottom of the window. (Take the acceleration of free fall to be 9.81 m s^{-2}.)

10 A ball is dropped from rest from a height of 20.0 m. One second later a second ball is thrown vertically downwards. The two balls arrive on the ground at the same time. Determine the initial velocity of the second ball.

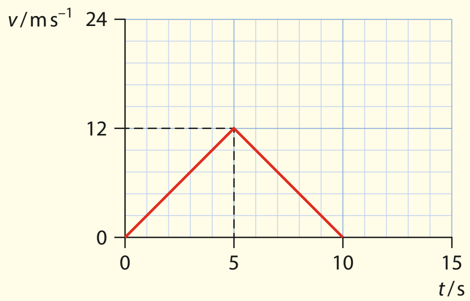
11 The graph shows the variation of the position of a particle with time. Draw the graph showing the variation of the velocity of the object with time.



12 The graph shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the position of the object with time.



13 The graph shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the position of the object with time (assuming a zero initial displacement).



14 A hiker starts climbing a mountain at 08:00 in the morning and reaches the top at 12:00 (noon). He spends the night on the mountain and the next day at 08:00 starts on the way down following exactly the same path. He reaches the bottom of the mountain at 12:00 (noon). Show that there must be a time between 08:00 and 12:00 when the hiker was at the same spot along the route on the way up and on the way down.

15 A stone is thrown vertically up from the edge of a cliff 35.0 m from the ground. The initial velocity of the stone is $8.00\,\mathrm{m\,s^{-1}}$.

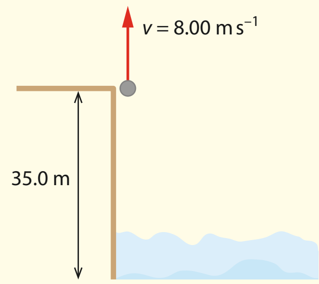
Sketch:

f a graph to show the variation of velocity with time

g a graph to show the variation of position with time.

(Take the acceleration of free fall to be $10.0\,\mathrm{m\,s^{-2}}$.)

16 A rocket accelerates vertically upwards from rest with a constant acceleration of $4.00\,\mathrm{m\,s^{-2}}$. The fuel lasts for 5.00 s.



a Find the maximum height achieved by this rocket.

b Determine the time when the rocket reaches the ground again.

c Sketch a graph to show the variation of the velocity of the rocket with time from the time of launch to the time it falls to the ground.

(Take the acceleration of free fall to be $10.0\,\mathrm{m\,s^{-2}}$.)

Projectile motion

17 A ball is kicked horizontally with a speed of $5.0\,\mathrm{m\,s^{-1}}$ from the roof of a house 3.0 m high.

a Calculate the time when the ball hits the ground.

b Determine the speed of the ball just before hitting the ground.

18 A particle is launched horizontally with a speed of $8.0\,\mathrm{m\,s^{-1}}$ from a point 20 m above the ground.

a Calculate the time when the particle lands on the ground.

b Determine the speed of the particle 1.0 s after launch.

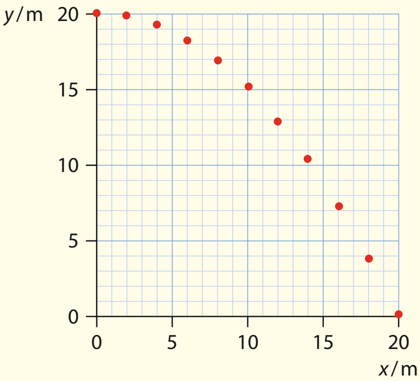
c Find the angle between the velocity and the horizontal 1.0 s after launch.

d Determine the velocity with which the particle hits the ground.

19 A plane flying at a constant speed of $50.0\,\text{m s}^{-1}$ and a constant height of 200 m drops a package of emergency supplies to a group of hikers. The package is released just as the plane flies over a fir tree. Find at what distance from the tree the package will land.

20 A stone is thrown with initial speed $6.0\,\text{m s}^{-1}$ at 35° to the horizontal. Find the direction of the velocity vector 1.0 s later.

21 A ball is launched horizontally from a height of 20 m above ground on Earth and follows the path shown in the diagram. Air resistance and other frictional forces are neglected. The position of the ball is shown every 0.20 s.



a Determine the horizontal component of velocity of the ball.

b The ball is now launched under identical conditions on the surface of a planet where the acceleration of free fall is $20\,\text{m s}^{-2}$. Draw the position of the ball on the diagram at time intervals of 0.20 s.

22 A soccer ball is kicked so that it has a range of 30 m and reaches a maximum height of 12 m. Determine the initial velocity (magnitude and direction) of the ball.

23 A projectile is launched horizontally. The force of air resistance is proportional to speed. Explain why the projectile's velocity is more vertical than it would have been in the absence of air resistance.

2.2 Forces

Equilibrium

24 A block rests on an inclined plane. Draw the forces on the block.

25 Sketch a diagram showing a mass hanging at the end of a vertical spring that is attached to the ceiling. On the diagram, draw the forces on:

a the hanging mass

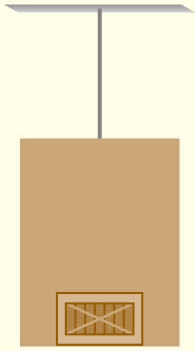
b the ceiling.

26 A force of 125 N is required to extend a spring by 2.8 cm. Estimate the force required to stretch the same spring by 3.2 cm.

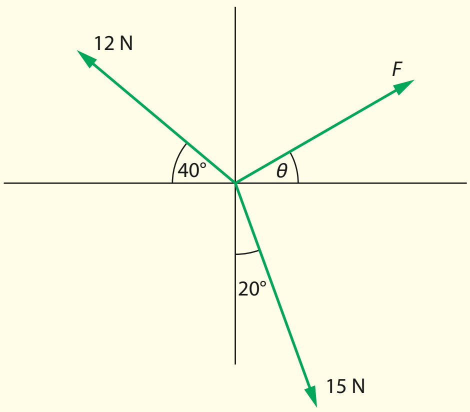
27 A block rests on an elevator floor, as shown in the diagram. The elevator is held in place by a cable attached to the ceiling. On a copy of the diagram, draw the forces on:

a the block

b the elevator.



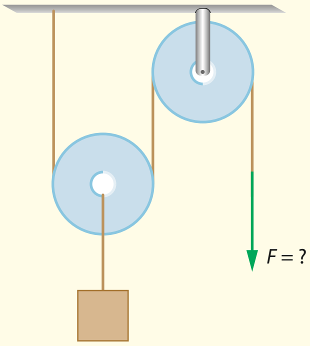
28 Determine F and θ in the diagram such that the net force is zero.



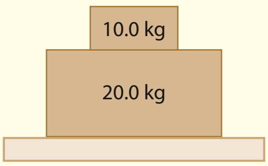
29 A force of 10.0 N is acting along the negative x-axis and a force of 5.00 N at an angle of 20° with the positive x-axis. Find the net force.

30 A force has components 2.45 N and 4.23 N along two perpendicular axes. Determine the magnitude of the force.

31 A block of mass 12.5 kg hangs from very light, smooth pulleys as shown in the diagram. Determine the magnitude of the force that must be applied to the rope so the system is in equilibrium.

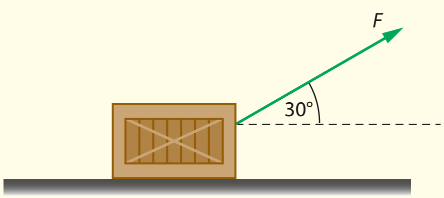


32 A block of mass 10.0 kg rests on top of a bigger block of mass 20.0 kg, which in turn rests on a horizontal table (see the diagram).

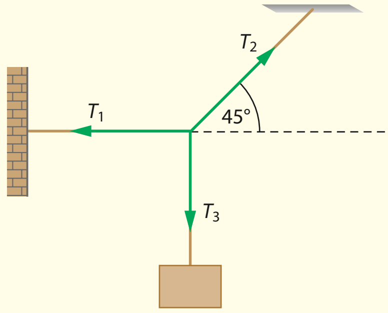


- **a** Find the individual forces acting on each block.
- **b** Identify force pairs according to Newton's third law.
- **c** A vertical downward force of 50.0 N acts on the top block. Calculate the forces on each block now.

33 A 460 kg crate is being pulled at constant velocity by a force directed at 30° to the horizontal as shown in the diagram. The coefficient of dynamic friction between the crate and the floor is 0.24. Calculate **a** the magnitude of the pulling force and **b** the reaction force from the floor on the crate.



34 A block of mass 5.00 kg hangs attached to three strings as shown in the diagram. Find the tension in each string. (Hint: Consider the equilibrium of the point where the strings join.)



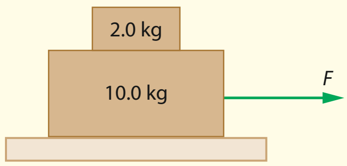
Accelerated motion

35 A mass of 2.00 kg is acted upon by two forces of 4.00 N and 10.0 N. What is the smallest and largest acceleration these two forces can produce on the mass?

36 A bird is in a glass cage that hangs from a spring scale. Compare the readings of the scale in the following cases.

- **a** The bird is sitting in the cage.
- **b** The bird is hovering in the cage.
- **c** The bird is moving upward with acceleration.
- **d** The bird is accelerating downward.
- **e** The bird is moving upward with constant velocity.

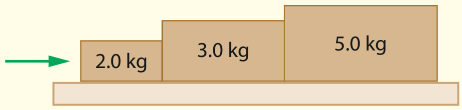
37 A block of mass 2.0 kg rests on top of another block of mass 10.0 kg that itself rests on a frictionless table (see diagram). The coefficient of static friction between the two blocks is 0.80. Calculate the largest force with which the bottom block can be pulled so that both blocks move together without sliding on each other.



38 A small passenger car and a fully loaded truck collide head-on.

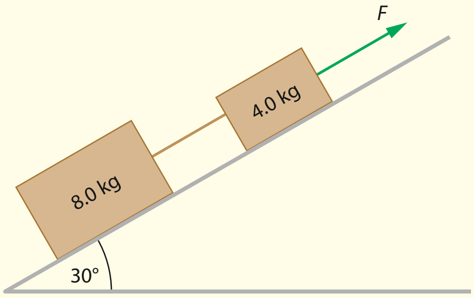
- **a** State and explain which vehicle experiences the greater force.
- **b** If you had to be in one of the vehicles, which one would you rather be in? Explain your answer.

39 Three blocks rest on a horizontal frictionless surface, as shown in the diagram. A force of 20.0 N is applied horizontally to the right on the block of mass 2.0 kg.



a Find the individual forces acting on each mass.
b Identify force pairs according to Newton's third law.

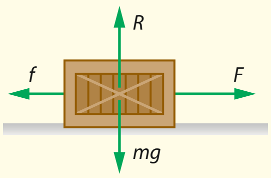
40 Two bodies are joined by a string and are pulled up an inclined plane that makes an angle of 30° to the horizontal, as shown in the diagram.



a Calculate the tension in the string when:
i the bodies move with constant speed
ii the bodies move up the plane with an acceleration of $2.0\,\mathrm{m\,s^{-2}}$.
b What is the value of F in each case?

2.3 Work, energy and power

41 A block of mass 4.0 kg is pushed to the right by a force $F = 20.0$ N. A frictional force of 14.0 N is acting on the block while it is moved a distance of 12.0 m along a horizontal floor. The forces acting on the mass are shown in the diagram.

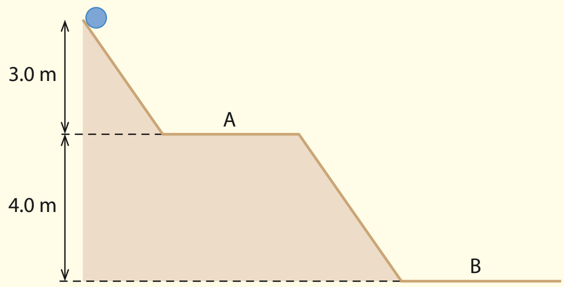


a Calculate the work done by each of the **four** forces acting on the mass.
b Hence find the net work done.
c State by how much the kinetic energy of the mass changes.

42 A weightlifter slowly lifts a 100 kg mass from the floor up a vertical distance of 1.90 m and then slowly lets it down to the floor again.
a Find the work done by the weight of the mass on the way up.
b Find the work done by the force exerted by the weightlifter when lifting the weight up.
c Find the total work done by the weight on the way up and the way down.

43 A spring of spring constant $k = 150\,\mathrm{N\,m^{-1}}$ is compressed by 4.0 cm. The spring is horizontal and a block of mass of 1.0 kg is held against the right end of the spring. The mass is released. Calculate the speed with which the block moves away.

44 A ball is released from rest from the position shown in the diagram. What will its speed be as it goes past positions A and B?



45 A 25.0 kg block is very slowly raised up a vertical distance of 10.0 m by a rope attached to an electric motor in a time of 8.2 s. Calculate the power developed in the motor.

46 For cars having the same shape but different size engines the maximum power developed by the car's engine is proportional to the third power of the car's maximum speed. Predict the dependence on speed of the wind resistance force.

47 Describe the energy transformations taking place when a body of mass 5.0 kg:
a falls from a height of 50 m without air resistance
b falls from a height of 50 m with constant speed
c is being pushed up an incline of 30° to the horizontal with constant speed.

48 A car of mass 1200 kg starts from rest, accelerates uniformly to a speed of 4.0 m s^{-1} in 2.0 s and continues moving at this constant speed in a horizontal straight line for an additional 10 s. The brakes are then applied and the car is brought to rest in 4.0 s. A constant resistance force of 500 N is acting on the car during its entire motion.

a Calculate the force accelerating the car in the first 2.0 s of the motion.

b Calculate the average power developed by the engine in the first 2.0 s of the motion.

c Calculate the force pushing the car forward in the next 10 s.

d Calculate the power developed by the engine in those 10 s.

e Calculate the braking force in the last 4.0 s of the motion.

f Describe the energy transformations that have taken place in the 16 s of the motion of this car.

49 A bungee jumper of mass 60 kg jumps from a bridge 24 m above the surface of the water. The rope is 12 m long and is assumed to obey Hooke's law.

a Estimate the spring constant of the rope so that the jumper just reaches the water surface.

b The same rope is used by a man whose mass is more than 60 kg. Explain why the man will not stop before reaching the water. (Treat the jumper as a point and ignore any resistance to motion.)

50 For the bungee jumper of mass 60 kg in question **49**, calculate:

a the speed of the jumper after falling 12 m

b the maximum speed attained by the jumper during their fall.

c Explain why the maximum speed is reached after falling more than a distance of 12 m (the unstretched length of the rope).

d Sketch a graph to show the variation of the speed of the jumper with distance fallen.

2.4 Momentum and impulse

51 Two bodies of mass 2.00 kg and 4.00 kg are kept on a frictionless horizontal table with a compressed spring between them. The masses are released. The heavier body moves away with velocity 3.50 m s^{-1}. Find the velocity of the other body.

52 A body of mass 0.500 kg moving at 6.00 m s^{-1} strikes a wall normally and bounces back with a speed of 4.00 m s^{-1}. The mass was in contact with the wall for 0.200 s. Find:

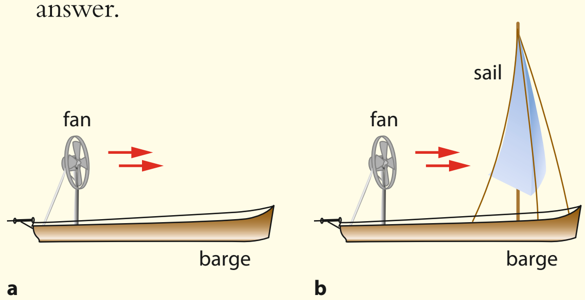
a the change of momentum of the mass

b the average force the wall exerted on the mass.

53 A person holds a book stationary in his hand and then releases it. As the book falls, state and explain whether the momentum of the book is conserved.

54 **a** A fan on a floating barge blows air at high speed toward the right, as shown in the diagram. State and explain whether the barge will move.

b A sail is now put up on the barge so that the fan blows air toward the sail, as shown in the diagram. Will the barge move? Explain your answer.



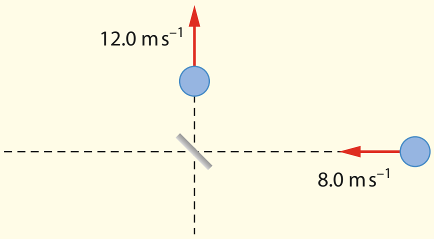
55 You jump from a height of 1.0 m from the surface of the Earth. The Earth will actually move up a bit as you fall.

a Explain why.

b Estimate the distance the Earth moves, listing any assumptions you make.

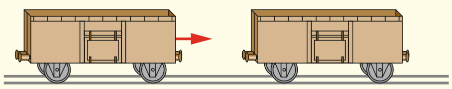
c State and explain whether the Earth would move more, less or the same if a heavier person jumped.

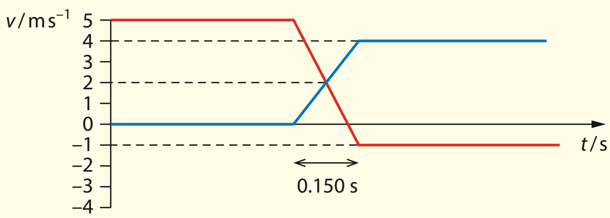
56 A 0.350 kg mass is approaching a moving plate with speed 8.00 m s^{-1}. The ball leaves the plate at right angles with a speed of 12.0 m s^{-1} as shown in the diagram. What impulse has been imparted to the ball?



57 A body of mass M, initially at rest, explodes and splits into two pieces of mass $\frac{M}{3}$ and $\frac{2M}{3}$, respectively. Find the ratio of the kinetic energies of the two pieces.

58 A wagon of mass 800 kg moving at 5.0 m s^{-1} collides with another wagon of mass 1200 kg that is initially at rest. Both wagons are equipped with buffers. The graph shows the velocities of the two wagons before, during and after the collision.





Use the graph to:

a show that the collision has been elastic

b calculate the average force on each wagon during the collision

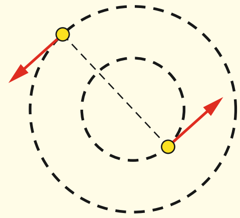
c calculate the impulse given to the heavy wagon.

d If the buffers on the two wagons had been stiffer, the time of contact would have been less but the final velocities would be unchanged. Predict how your answers to **b** and **c** would change (if at all).

e Calculate the kinetic energy of the two wagons at the time during the collision when both have the same velocity and compare your answer with the final kinetic energy of the wagons. How do you account for the difference?

59 A mass of 6.0 kg moving at 4.0 m s^{-1} collides with a mass of 8.0 kg at rest on a frictionless surface and sticks to it. How much kinetic energy was lost in the collision?

60 A binary star system consists of two stars that are orbiting a common centre, as shown in the diagram. The only force acting on the stars is the gravitational force of attraction in a direction along the line joining the stars.



a Explain carefully why the total momentum of the binary star is constant.

b Explain why the two stars are always in diametrically opposite positions.

c Hence explain why the two stars have a common period of rotation and why the inner star is the more massive of the two.

61 You have a mass of 60 kg and are floating weightless in space. You are carrying 100 coins each of mass 0.10 kg.

a If you throw all the coins at once with a speed of 5.0 m s^{-1} (relative to you) in the same direction, calculate the velocity with which you will recoil.

b If instead you throw the coins one at a time with a speed of 5.0 m s^{-1} relative to you, discuss whether your final speed will be different from before. (Use your graphics display calculator to calculate the speed in this case.)

Additional Topic 2 answers

Topic 2 Mechanics

2.1 Motion

1 **a** $1.67\,\text{km h}^{-1}$

b $1.2\,\text{km h}^{-1}$ at 34° east of south

2 **a** 88 m

b 68 m

c $8.33\,\text{m s}^{-1}$; $5.0\,\text{m s}^{-1}$

3 **a** $1.7\,\text{m s}^{-1}$

b $-6.0\,\text{m s}^{-1}$

4 4.0 s

5 3.0 s

6 **a** 1.2 s

b 3.7 s

c 22 m

d $9.3\,\text{m s}^{-1}$

e 29 m

7 **a** 4.37 s

b $32.9\,\text{m s}^{-1}$

c 60.2 m

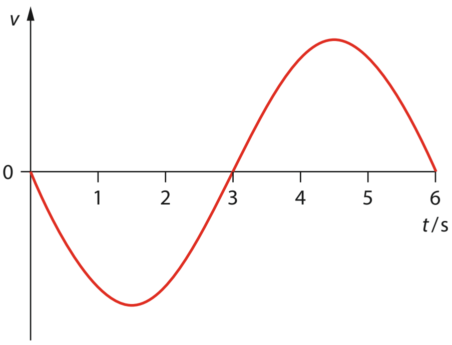
8 **a** 1.8 s

b $23\,\text{m s}^{-1}$

9 0.326 s

10 $-14.6\,\text{m s}^{-1}$

11



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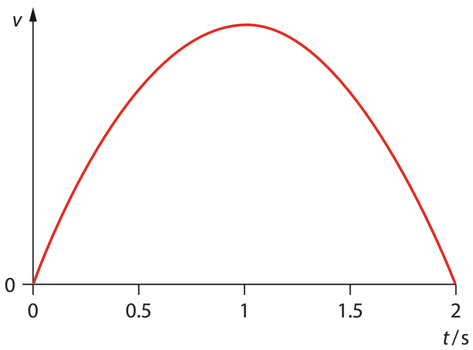

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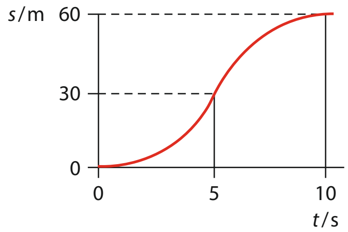
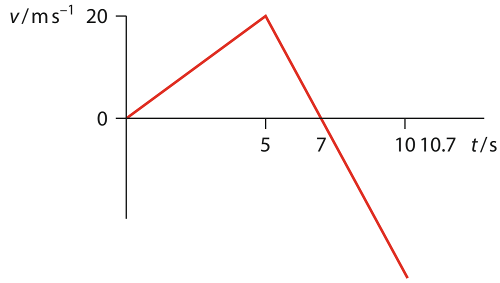


14 Make graphs of position against time; the graphs must cross.

16 **a** 70 m

b 10.7 s from the start

c



17 **a** 0.78 s

b $9.2\,\text{m s}^{-1}$

18 **a** 2.0 s

b $13\,\text{m s}^{-1}$

c −51°

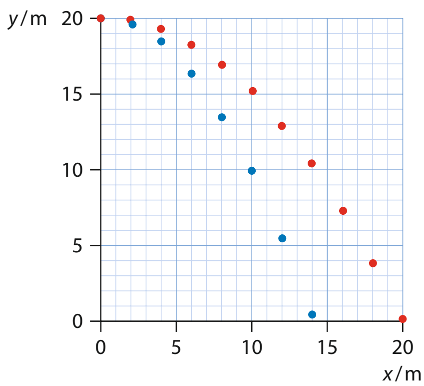
d $21\,\text{m s}^{-1}$ at −68°

19 320 m

20 52° below the horizontal

21 **a** $10\,\text{m s}^{-1}$

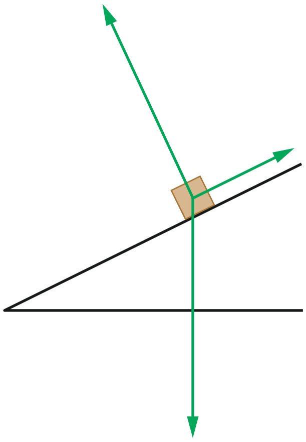
b



22 $18\,\text{m s}^{-1}$ at 58°

2.2 Forces

24



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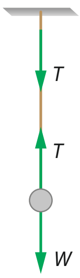
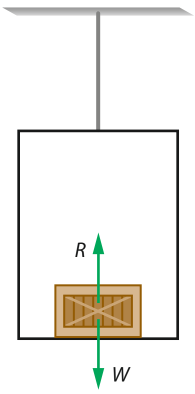


26 143 N

27



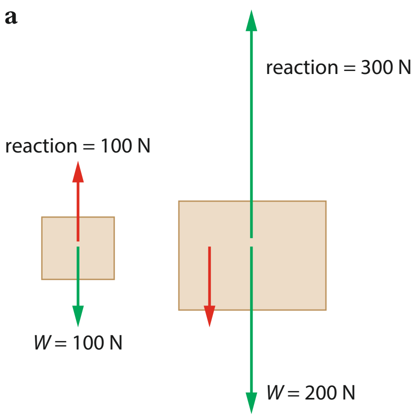
28 7.6 N at 58°

29 5.57 N at 162°

30 4.89 N

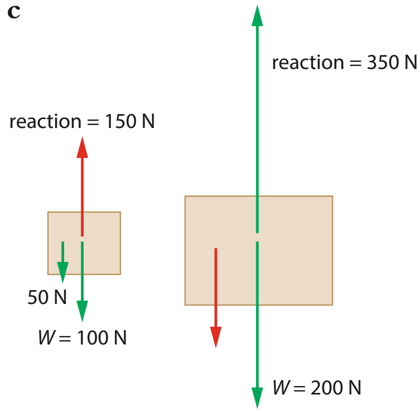
31 62.5 N

32 **a**



b Forces in red are an action–reaction pair.

c



33 **a** 1220 N

b 4040 N

34 < AQ:This is not the right answer: should be T_1, T_2 and T_3>

35 7.00 m s^{-2} and 3.00 m s^{-2}

36 **a** *mg*

b *mg*

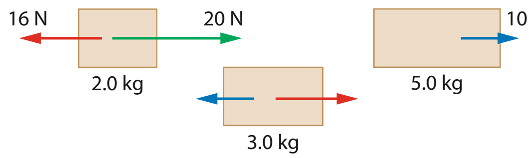
c greater than *mg*

d less than *mg*

e *mg*

37 94 N

39 **a** see free-body diagrams (vertical forces are excluded)



b forces in the same colour are action–reaction pairs

40 **a** **i** 39 N **ii** 55 N

b 59 N in **i** and 83 N in **ii**

2.3 Work, energy and power

41 **a** Work done by weight and reaction force is zero. Work done by F is 240 J and by friction is −168 J.

b 72 J

c The kinetic energy increases by 72.0 J.

42 **a** −1900 J

b +1900 J

c z ero

43 0.49 m s^{-1}

44 7.7 m s^{-1}; 12 m s^{-1}

45 3.0×10^2 W

46 $F \propto v^2$

47 a The potential energy the mass has at the top is converted into kinetic energy. As the mass lands, all its potential energy has been converted to kinetic energy.

b Some of the initial potential energy has been converted to kinetic energy. The kinetic energy remains constant during the fall. The remaining potential energy decreases as the mass falls and gets converted into thermal energy. As the mass lands, all the initial potential energy gets converted into thermal energy (and perhaps a bit of sound energy and deformation energy during impact with the ground).

c The kinetic energy remains constant. The potential energy is increasing at a constant rate equal to the rate at which the pulling force does work.

48 a 2900 N

b 5.8 kW

c 500 N

d 2.0 kW

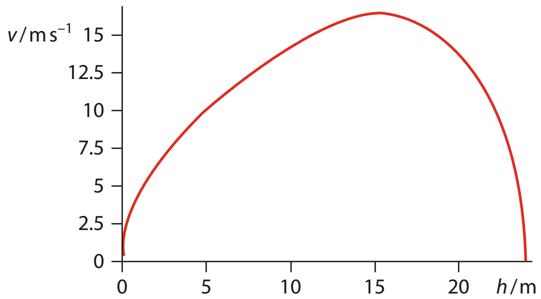
e 700 N

49 a $200\,\mathrm{N\,m^{-1}}$

50 a $15\,\mathrm{m\,s^{-1}}$

b $16\,\mathrm{m\,s^{-1}}$

d



2.4 Momentum and impulse

51 $7.00\,\mathrm{m\,s^{-1}}$

52 a −5.00 N s

b 25.0 N

54 a yes

b no

55 b The order of magnitude is about 10^{-23} m.

56 5.05 N s at 56.3°

57 ratio of light to heavy = 2

58 b 38 400 N on both

c 4800 N s

d The force would be larger but the impulse would be the same.

e 4000 J; the final kinetic energy is 10 000 J

59 27 J

60 a There are no external forces on the binary star system.

b The momentum of the system is not just constant but also zero; otherwise it would change direction as the stars moved. Hence the stars must have opposite momenta, i.e. they have to be diametrically opposite each other.

c Since they are opposite each other at all times, they complete orbits in the same time.

61 a $0.71\,\mathrm{m\,s^{-1}}$

b $0.77\,\mathrm{m\,s^{-1}}$

Additional Topic 3 questions

Test yourself

3.2 Modelling a gas

1 The density of copper is $8.96\,\text{g}\,\text{cm}^{-3}$ and its molar mass is $64\,\text{g}\,\text{mol}^{-1}$.

a Calculate the mass of an atom of copper.

b Determine the number of copper atoms per cubic metre.

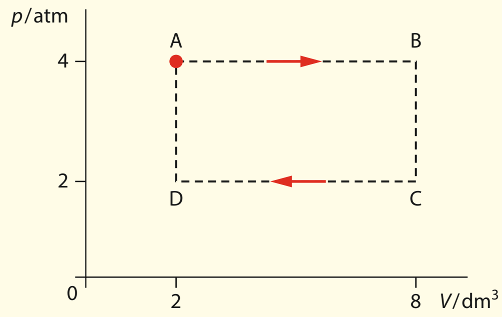
2 A volume of $2.00 \times 10^{-4}\,\text{m}^3$ of a gas is heated from 20.0 °C to 80.0 °C at constant pressure. Calculate the new volume.

3 Determine the number of moles in a gas kept at a temperature of 350 K, volume $0.20\,\text{m}^3$ and pressure $4.8 \times 10^5\,\text{Pa}$.

4 A gas is kept at a pressure of $4.00 \times 10^5\,\text{Pa}$ and a temperature of 30.0 °C. When the pressure is reduced to $3.00 \times 10^5\,\text{Pa}$ and the temperature raised to 40.0 °C, the volume is measured to be $0.45 \times 10^{-4}\,\text{m}^3$. Estimate the original volume of the gas.

5 An air bubble exhaled by a diver doubles in radius by the time it gets to the surface of the water. Assuming that the air in the bubble stays constant in temperature, predict by what factor the pressure of the bubble is reduced.

6 The point labelled A in the diagram shows the state of a fixed quantity of ideal gas kept at a temperature of 300 K. The state of the gas changes and is represented by the arrowed route in the pressure–volume diagram. The gas is eventually returned to its original state.



a Calculate the temperature of the gas at the corners of the rectangle on the pressure–volume diagram.

b Predict at what point on the dotted path the internal energy of the gas is greatest.

Additional Topic 3 answers

Topic 3 Thermal physics

3.2 Modelling a gas

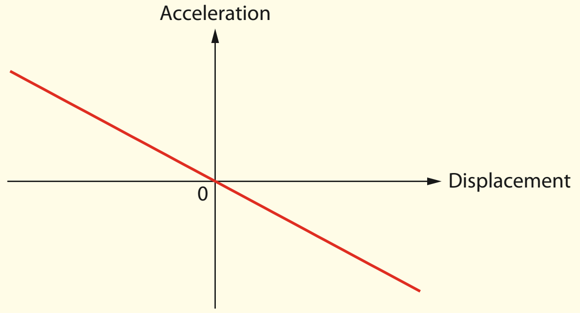
1 **a** 1.0×10^{-25} kg
b 8.4×10^{28} m^{-3}
2 2.41×10^{-4} m^3
3 33 mol
4 1.1×10^{-5} m^3
5 by a factor of 8
6 **a** at B, 1200 K; at C, 600 K; at D, 150 K
b at B

Additional Topic 4 questions

Test yourself

4.1 Oscillations

1 The graph shows the variation with displacement of the acceleration of a particle that is performing oscillations.

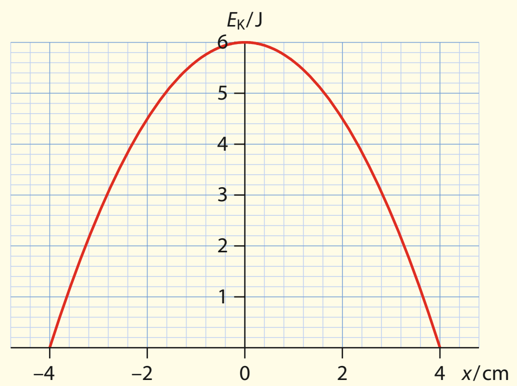


a Explain why the oscillations are simple harmonic.

b Make a copy of the graph and mark with the letter V a point on the graph where the speed is a maximum.

c The amplitude of oscillations is reduced from 2.0 cm to 1.0 cm. On your graph draw the variation with displacement of the acceleration of the particle.

2 The graph shows the variation with displacement of kinetic energy of a particle of mass 0.25 kg that is undergoing simple harmonic oscillations.



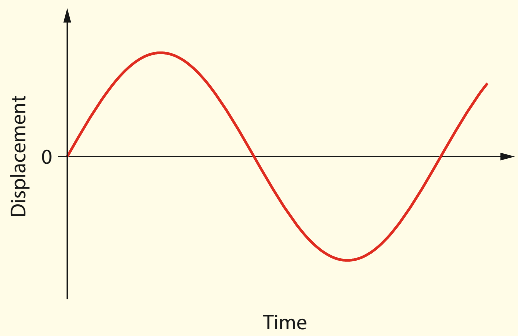
a Use the graph to calculate:

i the maximum speed of the particle

ii the potential energy when the displacement is 2.0 cm.

b On a copy of the axes draw a graph to show the variation of the potential energy with displacement.

3 The graph shows the variation with time of the displacement of a particle undergoing simple harmonic oscillations.



a On a copy of the graph mark a point where:

i the velocity is zero (mark this with the letter V)

ii the acceleration has maximum magnitude (mark this with the letter A)

iii the kinetic energy is maximum (mark this with the letter K)

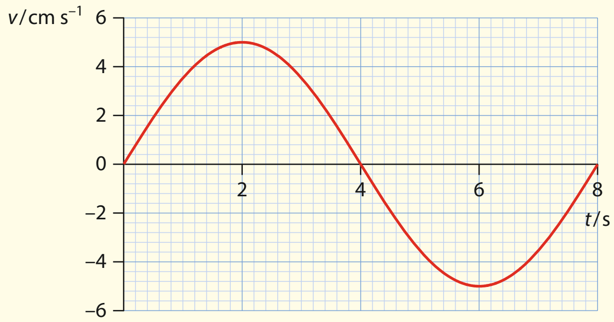
iv the potential energy is maximum (mark this with the letter P).

b For the motion shown, sketch a graph of:

i velocity versus time (no numbers on the axes are necessary)

ii acceleration versus time (no numbers on the axes are necessary).

4 The graph shows how the velocity of a particle undergoing simple harmonic oscillations varies with time.

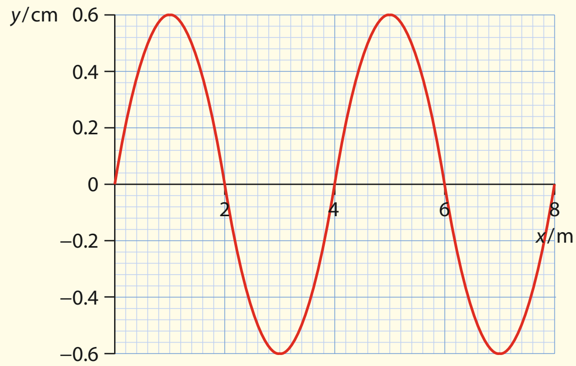


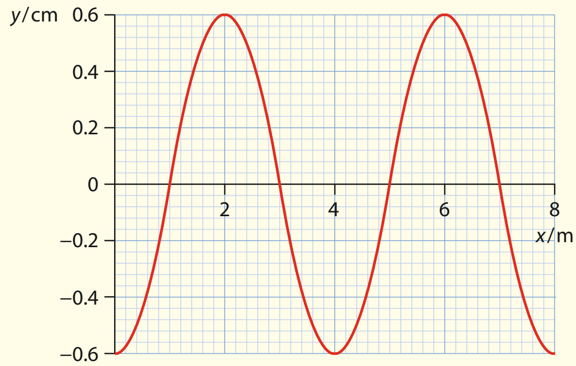
a On a copy of the graph mark a point where:
  i the displacement is zero (mark this with the letter Z)
  ii the displacement has maximum magnitude (mark this with the letter M).
b Sketch an acceleration versus time graph for this motion (no numbers on the axes are necessary).
c The mass of the particle is 0.20 kg. Draw a graph to show the variation with time of the kinetic energy of the particle.

4.2 Travelling waves

5 The speed of ocean waves approaching the shore is given by the formula $v = \sqrt{gh}$, where h is the depth of the water. It is assumed here that the wavelength of the waves is much larger than the depth (otherwise a different expression gives the wave speed).
a Calculate the speed of water waves near the shore where the depth is 1.0 m.
b Assuming that the depth of the water decreases uniformly, draw a graph of the water wave speed as a function of depth from a depth of 1.0 m to a depth of 0.30 m.

6 These displacement–position graphs show the same wave at two different times. The wave travels to the right and the bottom graph represents the wave 0.20 s after the time illustrated in the top graph.

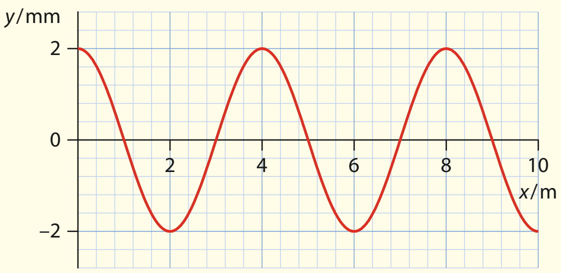




a For this wave determine:
  i the amplitude
  ii the wavelength
  iii the speed
  iv the frequency.
b Suggest whether the graphs may be used to determine if the wave is transverse or longitudinal.

7 An earthquake creates waves that travel in the Earth's crust; these can be detected by seismic stations.
a Explain why three seismic stations must be used to determine the position of the earthquake.
b Describe **two** differences in the signals recorded by three seismic stations, assuming they are at different distances from the centre of the earthquake.

8 The graph shows the variation with distance *x*, and of the displacement, *y*, of a sound wave travelling towards the right along a metal rod. This is the displacement at $t = 0$. The frequency of the wave is 1250 Hz.



a Calculate the speed of the wave.

b Determine the displacement of a point on the rod:

i at $x = 129$ m and $t = 0$

ii at $x = 212$ m and $t = 10$ ms.

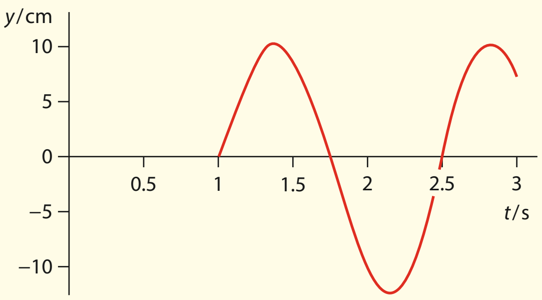
9 A stone is dropped on a still pond at $t = 0$. The wave reaches a leaf floating on the pond a distance of 3.00 m away. The leaf then begins to oscillate. The graph shows how its displacement *y* varies with time *t*.

a Calculate the speed of the water waves.

b Determine the period and frequency of the wave.

c Calculate the wavelength of the wave.

d State the initial amplitude of the wave.



10 A sound wave of frequency 500 Hz travels from air into water. The speed of sound in air is $330\,\text{m s}^{-1}$ and in water $1490\,\text{m s}^{-1}$. Calculate the wavelength of the wave in:

a air

b water.

11 A ship sends a sonar pulse of frequency 30 kHz and duration 1.0 ms towards a submarine and receives a reflection of the pulse 3.2 s later. The speed of sound in water is $1500\,\text{m s}^{-1}$. Calculate:

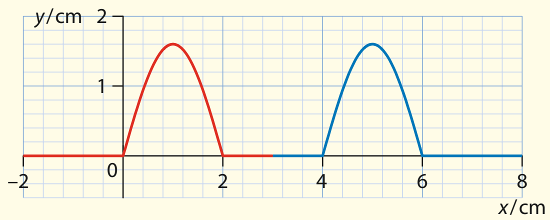
a the distance of the submarine from the ship

b the wavelength of the pulse

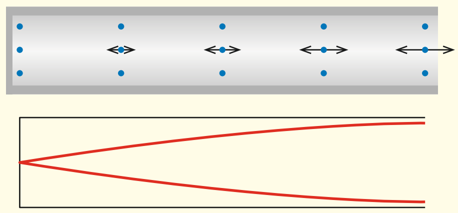
c the number of full waves emitted in the pulse.

4.3 Wave characteristics

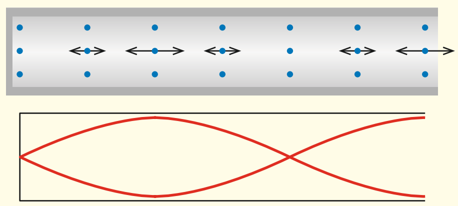
12 Two pulses are moving towards each other. The graph shows the variation of displacement *y* with distance *x* at $t = 0$ s. Both pulses have a speed of $1\,\text{cm s}^{-1}$. Draw the shape of the string at $t = 2$ s.



13 Two pulses are moving towards each other. The diagram shows the variation of displacement *y* with distance *x* at $t = 0$ s. Both pulses have a speed of $1\,\text{cm s}^{-1}$. Draw the shape of the string at $t = 2$ s.

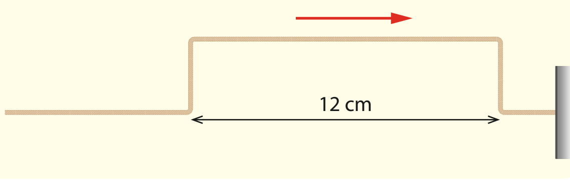


a



b

14 A pulse with the shape shown in the diagram travels on a string at $40\,\text{m s}^{-1}$ towards a fixed end. Taking $t = 0$ ms to be when the front of the pulse first arrives at the fixed end, draw the shape of the string at: $t = 1.0$ ms; $t = 1.5$ ms; $t = 2.0$ ms; $t = 2.5$ ms; $t = 3.0$ ms; $t = 4.0$ ms.



15 Polarised light is incident on a polariser whose transmission axis makes an angle θ with the direction of the electric field of the incident light. Sketch a graph to show the variation with angle θ of the transmitted intensity of light.

16 Unpolarised light of intensity I_0 is incident on a polariser. The transmitted light is incident on a second polariser whose transmission axis is at 60° to that of the first. Calculate, in terms of I_0, the intensity of light transmitted through the second polariser.

17 Unpolarised light of intensity I_0 is incident on a polariser. A number of other polarisers will be placed in line with the first so that the final transmitted intensity is $\frac{I_0}{100}$. Each polariser has its transmission axis rotated by 10° with respect to the previous one. Determine how many additional polarisers are required.

18 Light is incident on two analysers whose transmission axes are at right angles to each other. No light gets transmitted. Discuss whether it can be deduced whether the incident light is polarised or not.

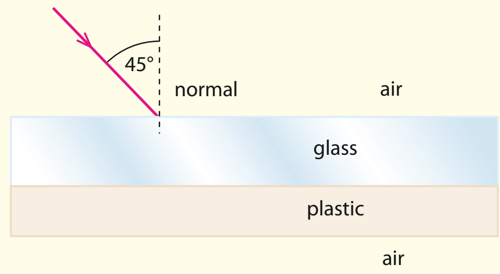
19 Unpolarised light is incident on two polarisers whose transmission axes are parallel to each other. Calculate the angle by which one of them must be rotated so that the transmitted intensity is half of the intensity incident on the second polariser.

20 A fisherman is fishing in a lake. Explain why it would be easier for him to see fish in the lake if he was wearing Polaroid sunglasses.

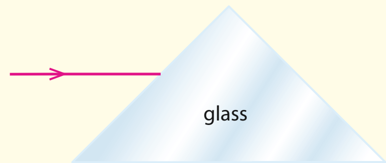
21 You stand next to a lake on a bright morning with one sheet of Polaroid. You don't know the orientation of its transmission axis. Suggest how you can determine it. (You may not use other Polaroid sheets with known transmission axes.)

4.4 Wave behaviour

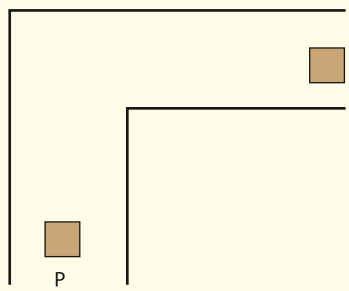
22 A ray of light enters glass from air at an angle of incidence equal to 45°, as shown in the diagram. Draw the path of this ray assuming that the glass has a refractive index of 1.420 and the plastic has a refractive index of 1.350.



23 A ray of light moving in air parallel to the base of a glass prism of angles 45°, 45° and 90° enters the prism, as shown in the diagram. Investigate the path of the ray as it enters the glass. The refractive index of glass is 1.50.



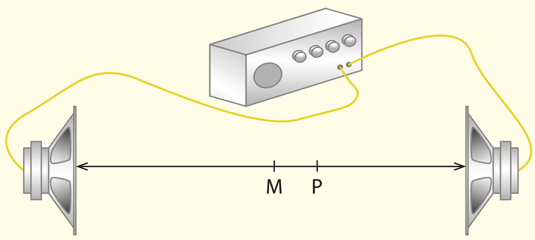
24 In the corridor shown in the diagram an observer at point P can hear someone at point Q but cannot see them. State the name(s) of the physical phenomena that may account for this. How could someone at P see Q?



25 Two loudspeakers are connected to the same audio oscillator. An observer walks along the straight line joining the speakers (see diagram). At a point M halfway between the speakers he hears a loud sound. By the time he gets to point P, a distance of 2.00 m from M, he hears no sound at all.

a Explain how this is possible.

b Determine the largest possible wavelength of sound emitted by the loudspeakers.



26 A car moves along a road that joins the twin antennas of a radio station that is broadcasting at a frequency of 90.0 MHz (see diagram). When in position A, the reception is good but it drops to almost zero at position B. Determine the minimum distance AB.

27 Two sources emit identical sound waves with a frequency of 850 Hz.

a An observer is 8.2 m from the first source and 9.0 m from the second. Describe and explain what this observer hears.

b A second observer is 8.1 m from the first source and 8.7 m from the second. Describe and explain what this observer hears. (Take the speed of sound to be 340 $\mathrm{m\,s^{-1}}$.)

28 In the context of wave motion, state what you understand by the term **superposition**. Illustrate constructive and destructive interference by suitable diagrams.

Additional Topic 4 answers

Topic 4 Waves

4.1 Oscillations

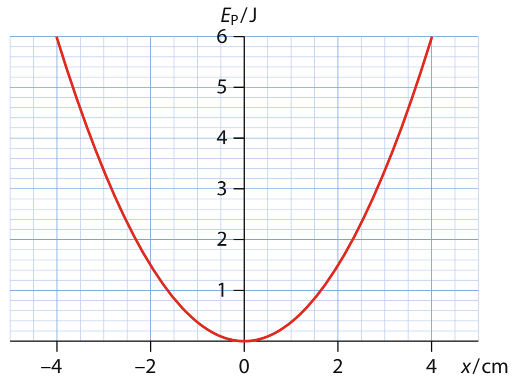
1 **b** V at the origin

c same graph but from $x = -1$ cm to $xv = 1$ cm

2 **a** **i** $6.9\,\mathrm{m\,s^{-1}}$

ii 1.5 J

b



3 **a** **i** V at any maximum or minimum of the graph

ii A at any maximum or minimum of the graph

iii K at any point where $x = 0$

iv P at any maximum or minimum of the graph

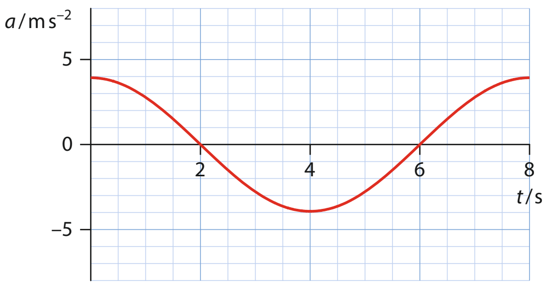
b **i** a negative cosine curve

ii a negative sine curve

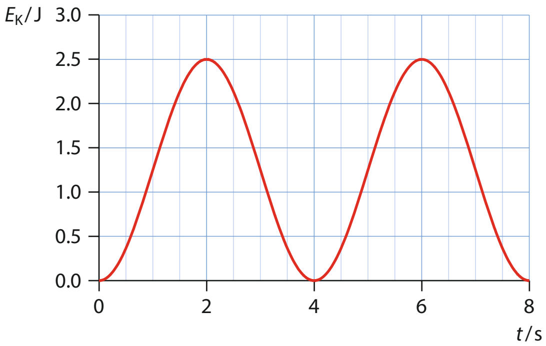
4 **a** **i** Z at any maximum or minimum of the graph

ii M at any point where $v = 0$

b



c



4.2 Travelling waves

5 **a** $3.16\,\mathrm{m\,s^{-1}}$

b K at any point where $x = 0$

6 **a** **i** 0.6 cm

ii 4.0 m

iii $5.0\,\mathrm{m\,s^{-1}}$

iv 1.25 Hz

b no

8 **a** $5.0 \times 10^3\,\mathrm{m\,s^{-1}}$

b **i** 0 mm

ii –2.00 mm

9 **a** 3.0 m s−1

b T = 1.5 s; f = 0.667 Hz

c 4.5 m

d 12 cm

10 **a** 0.66 m

b 2.98 m

11 **a** 2400 m

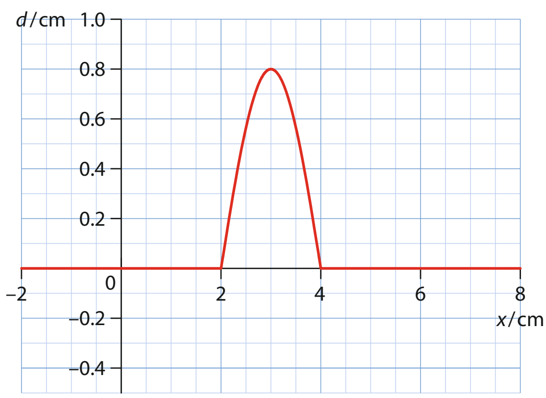
b 0.050 m

c 30

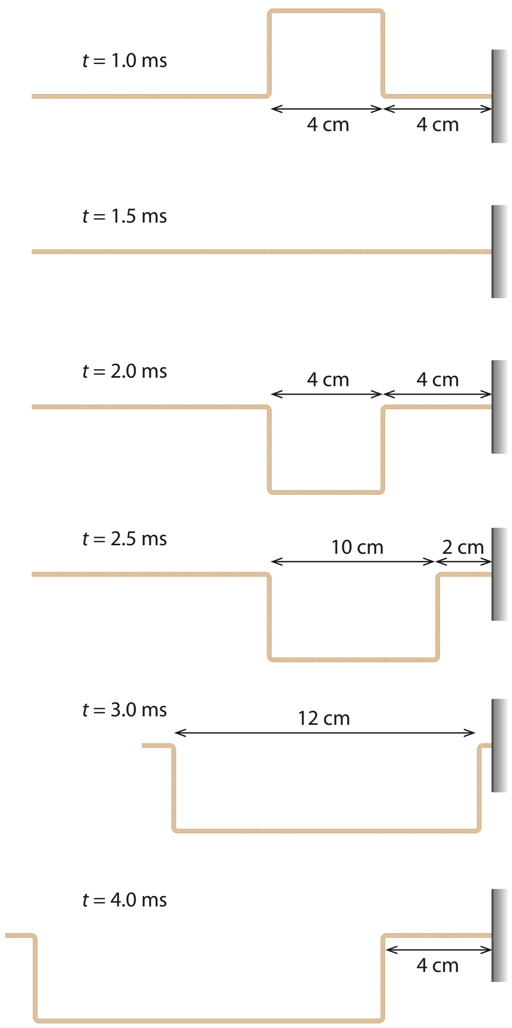
4.3 Wave characteristics

12

13



14



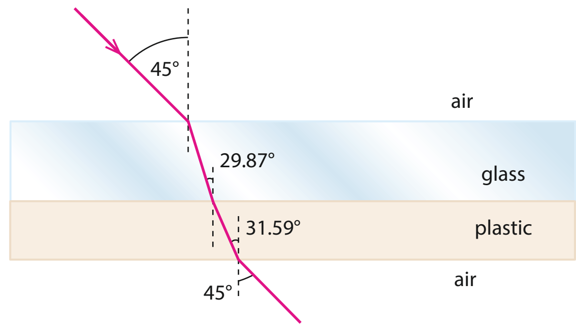
16 $\frac{I_0}{8}$

17 128 additional polarisers

19 45°

4.4 Wave behaviour

22



24 Reflection and diffraction of sound; absence of these for light. P could see Q by using a mirror at the corner.

25 **b** 8.0 m

26 0.83 m

27 **a** The path difference is two wavelengths, so the observer hears a loud sound because of constructive interference.

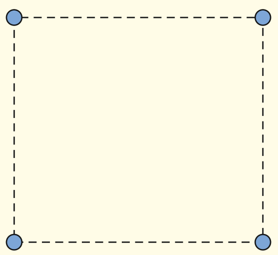
b The path difference is one and a half wavelengths, so the observer hears no sound because of destructive interference.

Additional Topic 5 questions

Test yourself

5.1 Electric fields

1 Four equal charges $q = -5.0\,\mu C$ are placed at the vertices of a square of side 12 cm, as in the diagram. Determine the force on the charge at the top right vertex.



2 A small plastic sphere is suspended from a fine insulating thread near, but not touching, a large sphere that is being charged. As the charge on the big sphere increases it is observed that **i** the plastic sphere is slowly attracted toward the large sphere, **ii** eventually touching it, **iii** at which point it is violently repelled. Carefully explain these observations.

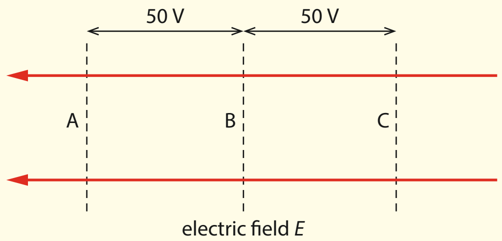
3 The electric field at a point in space has magnitude $100\,N\,C^{-1}$ and is directed to the right. An electron is placed at that point. For this electron, calculate **a** the force and **b** the acceleration.

4 The number of electrons per second moving through the cross-sectional area of a copper wire is 4.0×10^{19}.

- **a** Determine the current in the wire.
- **b** The diameter of the wire is 1.5 mm and the number of free electrons per unit volume for copper is $8.5 \times 10^{28}\,m^{-3}$. Estimate the drift speed for the electrons.

5 Give an estimate for the number of free electrons per unit volume for gold (density $19\,390\,kg\,m^{-3}$; molar mass $197\,g\,mol^{-1}$). Assume that each atom contributes just one electron to the set of free electrons.

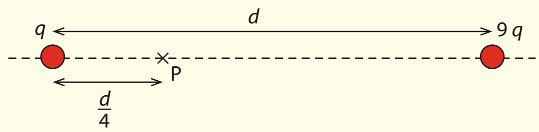
6 The potential difference between consecutive dotted lines in the diagram is 50 V. The red arrows indicate the electric field.



- **a** Calculate the work that must be done by an external agent in moving a charge of $+5.0\,\mu C$ from A to B.
- **b** Repeat the calculation in **a** when the same charge is moved from A to C.
- **c** The $+5.0\,\mu C$ charge is moved from A to C and then from C to B. Calculate how much work would be required then. Compare your answer to that in part **a** and comment.
- **d** An electron is released from rest from a point on line B. State whether the electron will reach line A or line C and calculate its speed there.

7
- **a** An electron is accelerated by a potential difference of 100.0 V. Determine the speed of the electron after acceleration.
- **b** Determine the speed a proton would attain if accelerated by the same potential difference as the electron.

8 Two positive point charges of magnitude q and $9q$ are a distance d apart, as shown in the diagram.



- **a** Calculate the electric field strength at point P, a distance $\frac{d}{4}$ from q.
- **b** Sketch a graph of the electric field as a function of the distance x from the charge q. (Take the field to be positive if it is directed to the right.)
- **c** How do the answers to **a** and **b** change if the charges are both negative?

5.2 Heating effect of electric currents

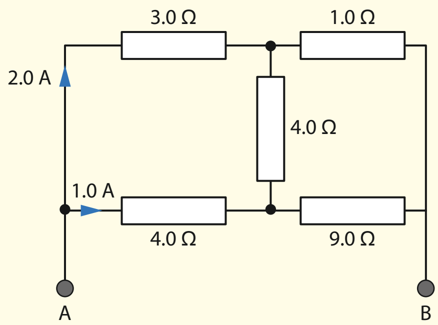
9 Explain why a light bulb is most likely to burn out when it is first turned on rather than later.

10 State the factors that affect the resistance of a metal wire.

11 Determine the factor by which the resistance of a wire changes when its radius is doubled.

12 The resistance of a fixed length of wire of circular cross-section is 10.0 Ω. Predict the resistance of a wire of the same length made of the same material but with only half the radius.

13 Look at the arrangement of resistors shown in the diagram.



a Find the current in, and potential difference across, each resistor. The potential at A is 12 V.

b What is the potential difference between A and B?

14 A light bulb is rated as 60 W at 220 V.

a Calculate the current flows in the light bulb when it is connected in series to a 220 V source of voltage.

b The lamp is connected in series to a 110 V source of voltage. Calculate the current flows in the lamp. (Assume the resistance stays the same.)

c Determine the power output of the light bulb when it is connected to the 110 V source.

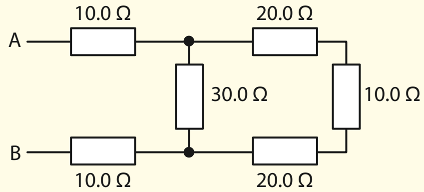
15 Determine the energy used when a 1500 W kettle is used for four minutes:

a in kW h

b in joules.

16 In country X the voltage supplied by the electricity companies is 110 V and in country Y it is 220 V. Consider a light bulb rated as 60 W at 110 V in X and a light bulb rated as 60 W at 220 V in Y. Take the cost of electricity per kW h to be the same. Suggest where it costs more to operate a light bulb for one hour.

17 Determine the resistance between A and B in the diagram.



18 Six light bulbs, each of constant resistance 3.0 Ω, are connected in parallel to a battery of emf = 9.0 V and negligible internal resistance. The brightness of a light bulb is proportional to the power dissipated in it. Compare the brightness of one light bulb when all six are on, to that when only five are on, the sixth having burnt out.

19 One light bulb is rated as 60 W at 220 V and another as 75 W at 220 V.

a Both of these are connected in parallel to a 110 V source. Determine the current in each light bulb. (Assume that the resistances of the light bulbs are constant.)

b Would it cost more or less (and by how much) to run these two light bulbs connected in parallel to a 110 V or a 220 V source?

20 Three appliances are connected (in parallel) to the same outlet, which provides a voltage of 220 V. A fuse connected to the outlet will blow if the current drawn from the outlet exceeds 10 A. The three appliances are rated as 60 W, 500 W and 1200 W at 220 V. Suggest whether the fuse blows.

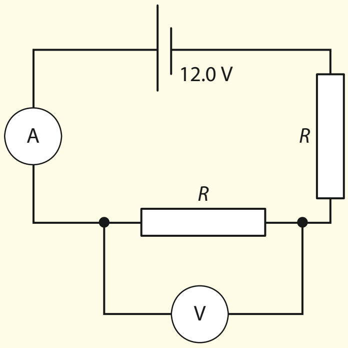
21 An electric kettle rated as 1200 W at 220 V and a toaster rated at 1000 W at 220 V are both connected in parallel to a source of 220 V. The fuse connected to the source blows when the current exceeds 9.0 A. Determine whether both appliances can be used at the same time.

22 At a given time a home is supplied with 100.0 A at 220 V. How many 75 W (rated at 220 V) light bulbs could be on in the house at that time, assuming they are all connected in parallel?

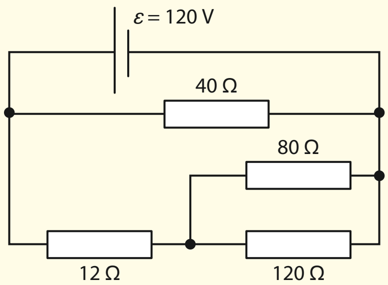
23 **a** Determine the reading of the voltmeter in the circuit shown in the diagram if both resistances are 200 Ω and the voltmeter also has a resistance of 200 Ω.

b Determine the reading of the ammeter.

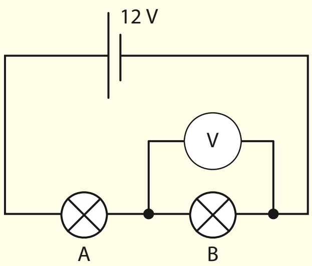
c The voltmeter is replaced by an ideal voltmeter. Determine the readings of the voltmeter and ammeter.



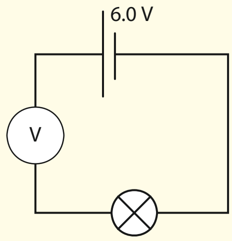
24 For the circuit shown in the diagram, calculate the current taken from the supply.



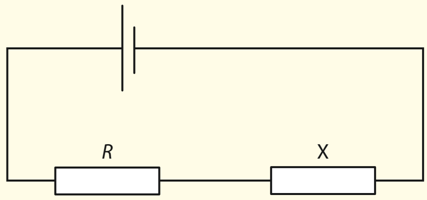
25 Two identical lamps are connected to a cell of emf 12 V and negligible internal resistance, as shown in the diagram. Calculate the reading of the (ideal) voltmeter when lamp B burns out.



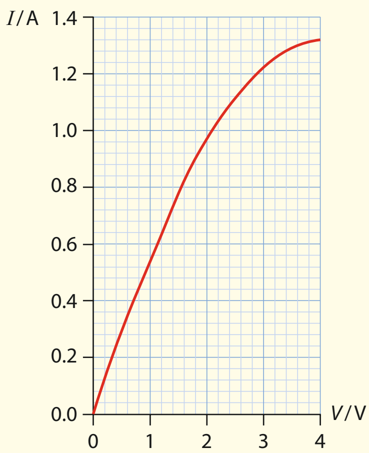
26 State the reading of the ideal voltmeter in the circuit in the diagram.



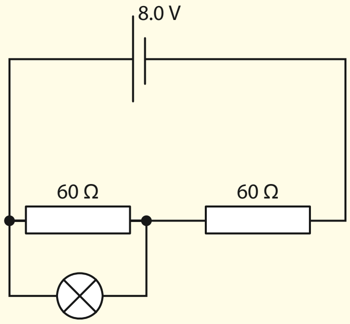
27 Two resistors are connected in series as shown in the diagram. The cell has negligible internal resistance. Resistor R has a constant resistance of 1.5 Ω.



The current–voltage (I–V) characteristic of resistance X is shown in the diagram.
The potential difference across resistor R is 1.2 V.
Calculate the emf of the cell.



28 A lamp of constant resistance operates at normal brightness when the potential difference across it is 4.0 V and the current through it is 0.20 A. To light up the lamp, a student uses the circuit shown in the diagram.



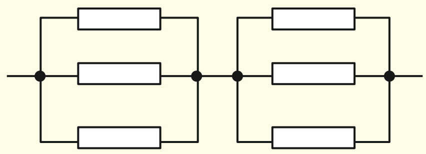
a Calculate the resistance of the light bulb at normal brightness.

b Calculate the potential difference across the light bulb in the circuit in the diagram.

c Calculate the current through the light bulb.

d Hence explain why the light bulb will not light.

24 Each resistor in the diagram has a value of 6.0 Ω. Calculate the resistance of the combination.



25 You are given one hundred 1.0 Ω resistors. Determine the smallest and largest resistance you can make in a circuit using these resistors.

26 A wire that has resistance R is cut into two equal pieces. The two parts are joined in parallel. What is the resistance of the combination?

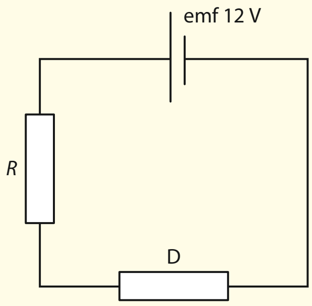
27 A toaster is rated as 1200 W and a mixer as 500 W, both at 220 V.

a Both appliances are connected (in parallel) to a 220 V source. Determine the current in each appliance.

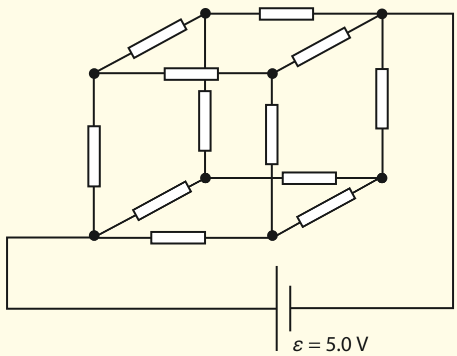
b How much energy do these appliances use if both work for 1 hour?

28 Two light bulbs are rated as 60 W and 75 W at 220 V. If these are connected in series to a source of 220 V, what will the power in each be? Assume a constant resistance for the light bulbs.

29 A device D, of constant resistance, operates properly when the potential difference across it is 8.0 V and the current through it is 2.0 A. The device is connected in the circuit shown, in series with an unknown resistance R. Calculate the value of the resistance R. (The cell has negligible internal resistance.)



35 Twelve 1.0 Ω resistors are placed on the edges of a cube and connected to a 5.0 V battery, as shown in the diagram. Determine the current leaving the battery.



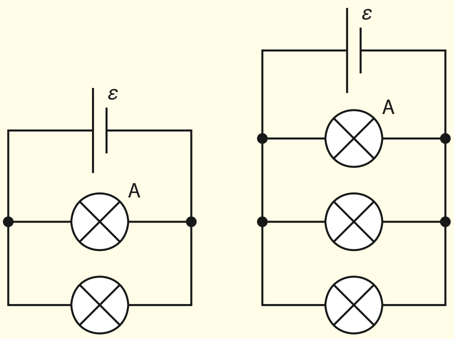
5.3 Electric cells

36 A direct current supply of constant emf 12.0 V and internal resistance 0.50 Ω is connected to a load of constant resistance 8.0 Ω. Find:

a the power dissipated in the load resistance

b the energy lost in the internal resistance in 10 min.

37 Two identical lamps, each of constant resistance *R*, are connected as shown in the circuit on the left. A third identical lamp is connected in parallel to the other two.

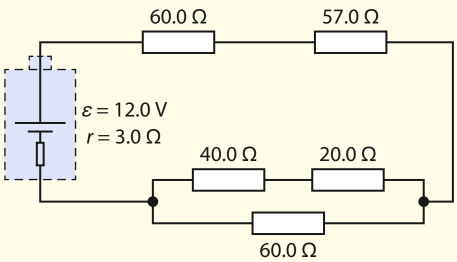


Compare the brightness of lamp A in the original circuit (left) with its brightness in the circuit with three lamps (right), when:

a the battery has no internal resistance

b the battery has an internal resistance equal to *R*.

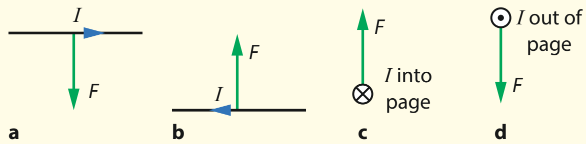
38 Find the current in each of the resistors in the circuit in the diagram. What is the total power dissipated in the circuit?



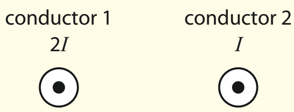
5.4 Magnetic fields

39 An electron is shot along the axis of a solenoid that carries current. Suggest whether it will experience a magnetic force.

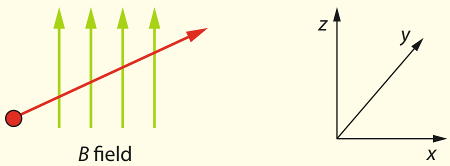
40 The diagram shows four different wires carrying current and the magnetic force on each. Determine the direction of the magnetic field in each case.



41 The diagram shows two parallel conductors carrying current out of the page. Conductor 1 carries double the current of conductor 2. On a copy of the diagram, draw to scale the magnetic fields created by each conductor at the position of the other and the forces on each conductor.



42 A proton of velocity $1.5 \times 10^6\,\mathrm{m\,s^{-1}}$ enters a region of uniform magnetic field $B = 0.50\,\mathrm{T}$. The magnetic field is directed vertically up (along the positive z-direction) and the proton's velocity is initially on the z–x plane, making an angle of 30° with the positive x-axis.



a Show that the proton will follow a helical path around the magnetic field lines.

b Calculate the radius of the helix.

c Determine the number of revolutions per second the proton makes.

d Determine the velocity of the proton along the field lines.

e Calculate the vertical separation of the coils of the helix.

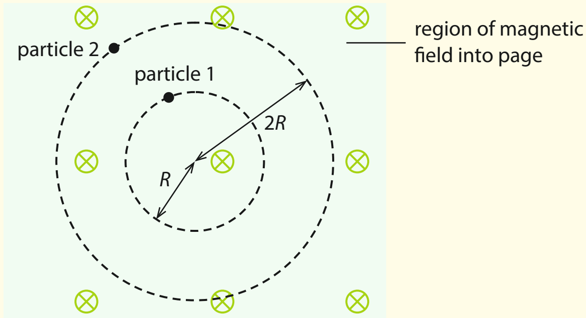
43 An electron enters a region of uniform magnetic field $B = 0.50$T. Its velocity is normal to the magnetic field direction. The electron is deflected into a circular path and leaves the region of magnetic field after being deflected by an angle of 30° with respect to its original direction. Determine the time for which the electron was in the region of magnetic field.

44 Two identical charged particles move in circular paths at right angles to a uniform magnetic field as shown in the diagram. The radius of particle 2 is twice that of particle 1.

Determine the following ratios:

a $\dfrac{\text{period of particle 2}}{\text{period of particle 1}}$

b $\dfrac{E_K \text{ of particle 2}}{E_K \text{ of particle 1}}$



Additional Topic 5 answers

Topic 5 Electricity and magnetism

5.1 Electric fields

1 30 N north east

3 **a** 1.60×10^{-17} N left

b 1.76×10^{13} m s^{-2}

4 **a** 6.4 A

b 2.7×10^{-4} m s^{-2}

5 6×10^{28} electrons per cubic metre

6 **a** 2.5×10^{-4} J

b 5.0×10^{-4} J

c 2.5×10^{-4} J

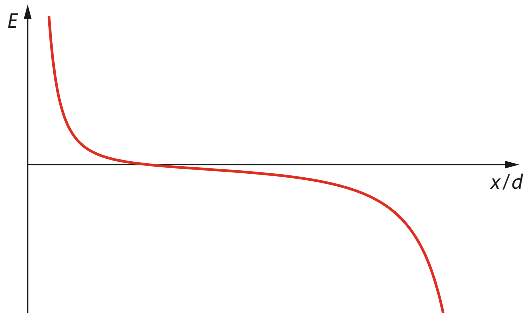
d C; 4.2×10^{6} m s^{-1}

7 **a** 5.9×10^{6} m s^{-1}

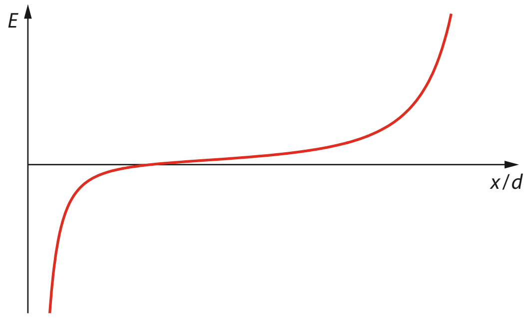
b 1.4×10^{5} m s^{-1}

8 **a** 0

b



c Field is still 0 at $\frac{d}{4}$.

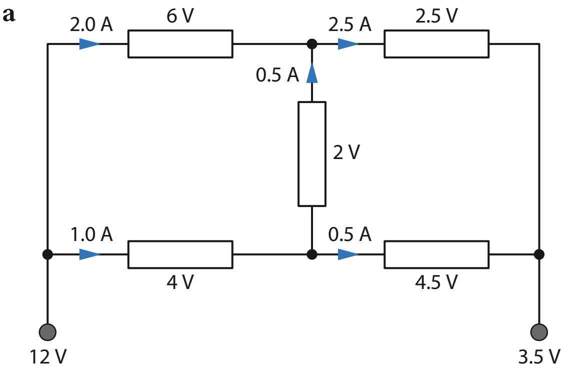


5.2 Heating effect of electric currents

11 decreases by a factor of 4

12 40 Ω

13 **a**



b 8.5 V

14 **a** 0.27 A

b 0.136 A

c 15 W

15 **a** 0.1 kW h

b 3.6×10^{5} J

16 cost is the same

17 38.75 Ω

18 same

19 **a** 0.14 A and 0.17 A

b Costs more at 220 V by a factor of 4

20 does not blow

21 cannot be used at the same time

22 293

23 **a** 4.0 V

b 40 mA

c 6.0 V, 30 mA

24 5.0 A

25 12 V

26 6.0 V

27 2.8 V

28 **a** 20 Ω

b 1.6 V

c 0.080 A

29 4.0 Ω

30 0.01 Ω, 100 Ω

31 $\frac{R}{4}$

32 **a** toaster, 5.45 A; mixer, 2.27 A

b 6.1 MJ

33 19 W; 15 W

34 2.0 Ω

35 6.0 A

5.3 Electric cells

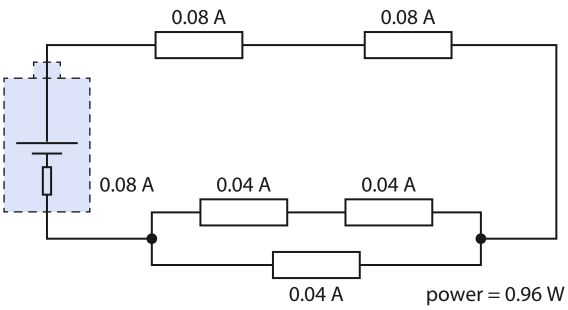
36 **a** 16 W

b 600 J

37 **a** same

b $\frac{16}{9}$ times as bright

38



5.4 Magnetic fields

39 no

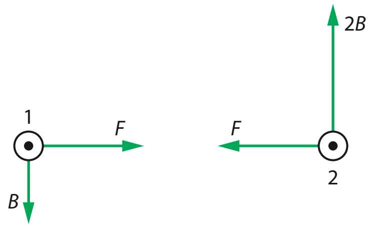
40 **a** out of page

b out of page

c left

d left

41



42 **b** 2.7 cm

c 7.6×10^{6} per second

d $7.5 \times 10^{5}\,\mathrm{m\,s^{-1}}$

e 0.098 m

43 6.0×10^{-12} s

44 **a** 1

b 4

Additional Topic 7 questions

Test yourself

7.1 Discrete energy and radioactivity

1 Calculate the number of neutrons in these nuclei: ${}^{3}_{1}H$, ${}^{23}_{11}Na$, ${}^{48}_{22}Ti$, ${}^{179}_{72}Hf$.

2 Tritium (${}^{3}_{1}H$) is a radioactive isotope of hydrogen and decays by beta minus decay. State the equation for the reaction and the names of the products of the decay.

3 Nitrogen (${}^{14}_{7}N$) is produced in the beta minus decay of a radioactive isotope. State the equation for this reaction and the names of the particles in the reaction.

4 A nucleus (${}^{A}_{Z}X$) decays by emitting two positrons and one alpha particle. State the atomic and mass numbers of the resulting nucleus.

5 Name the two missing particles in the reaction ${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + ? + ?$

6 The initial activity of a radioactive sample is 120 Bq. After 24 hours the activity is measured to be 15 Bq. Determine the half-life of the sample.

7 Discuss how you could confirm that a particular element emits:

a positively charged particles

b negatively charged particles

c electrically neutral particles.

8 The track of an alpha particle is measured to be 30 mm. The energy required to produce an ion is about 32 eV, on average. Assuming that alpha particles create 6000 ions per mm along their path, estimate the energy of the alpha particle.

9 Discuss what is meant by the statement that the strong nuclear force has a short range.

10 Compare the gravitational force between two electrons a distance of 10^{-10} m apart with the electrical force between them at the same separation.

11 An unstable nucleus has too many neutrons. Suggest the likely way in which it will decay.

7.2 Nuclear reactions

12 Calculate the energy released in the beta minus decay of a neutron.

13 Calculate the energy released in the alpha decay:

$${}^{234}_{90}Th \rightarrow {}^{230}_{88}Ra + {}^{4}_{2}He$$

(The atomic mass of thorium is 234.043596 u; that of radium is 230.03708 u.)

14 One possible outcome in the fission of a uranium nucleus is the reaction:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{95}_{42}Mo + {}^{139}_{57}La + 2{}^{1}_{0}n + ?$$

a Write down what is missing in this reaction.

b Calculate the energy is released. (Atomic masses: U = 235.043922 u; Mo = 94.905841 u; La = 138.906349 u.)

15 The reaction by which hydrogen in stars is converted into helium is:

$$4{}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2{}^{0}_{1}e + 2\nu_e + {}^{0}_{0}\gamma$$

The reaction releases about 26.7 MeV of energy. The Sun radiates energy at the rate of 3.9×10^{26} W and has a mass of about 1.99×10^{30} kg, of which 75% is hydrogen. Calculate how long it will take the Sun to convert 12% of its hydrogen into helium.

16 Outline the role in nuclear fusion reactions of:

a temperature

b pressure.

7.3 The structure of matter

17 Describe the Rutherford–Geiger–Marsden experiment and explain how its results led to the Rutherford model of the atom.

18 Explain, in terms of quarks, what is meant by the terms **a** hadron, **b** meson and **c** baryon.

19 Discuss whether it is correct that all electrically neutral particles are their own anti-particles. Give examples to support your answer.

20 In the reaction $p + p \rightarrow p + p + X$, determine whether X can be a baryon.

21 Write down the charge and strangeness of the baryon $\lambda = (uds)$.

22 Identify the reactions that conserve lepton number.

a $p^+ \rightarrow e^+ + p^0$

b $p^0 \rightarrow e^+ + \mu^-$

c $\tau^+ \rightarrow \pi^+ + \bar{\nu}_\tau$

d $\pi^- \rightarrow e^- + \bar{\nu}_e$

23 Explain whether the weak force acts:

a on mesons.

b on baryons.

24 Neutrinos are electrically neutral. Are neutrinos identical to anti-neutrinos?

25 a The positive pion π^+ has the quark content $(d\bar{u})$ and rest mass $140\,\text{MeV}\,c^{-2}$. Explain why there exists a different meson (the ρ^+ of rest mass $770\,\text{MeV}\,c^{-2}$) with the same quark content as the π^+.

b The negative pion π^- has quark content $(d\bar{u})$. Explain how it may be deduced that there exists a meson with the same quark content as the π^- and rest mass $770\,\text{MeV}\,c^{-2}$.

26 Outline how the exchange of gluons by quarks results in the strong nuclear force between nucleons.

27 Using the weak interaction vertices, draw a Feynman diagram for the reaction $\mu^+ + e^- \rightarrow \bar{\nu}_\mu + \nu_e$.

28 a Describe what is meant by a Feynman diagram.

b Draw Feynman diagrams to represent the electromagnetic processes:

i $e^- + e^+ \rightarrow e^- + e^+$

ii $e^- + e^+ \rightarrow \gamma + \gamma$.

29 A meson has quark content $u\bar{u}$.

a State the electric charge of the meson. The meson is at rest and decays into photons.

b Explain why the meson cannot decay into just one photon. The meson in fact decays into two photons.

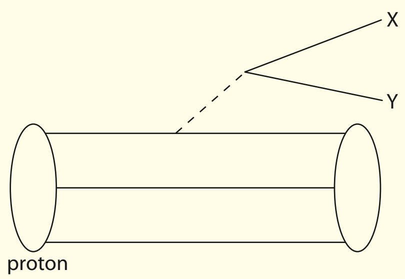
c Draw the Feynman diagram for this decay.

30 The diagram represents the beta plus (e^+) decay of a proton.

a Identify the quarks making up the neutron.

b State the name of the particle represented by the wavy line.

c Identify the particles denoted by X and Y in the diagram.



Additional Topic 7 answers

Topic 7 Atomic, nuclear and particle physics

7.1 Discrete energy and radioactivity

1 2, 12, 26, 107

2 ${}^{3}_{1}\text{H} \rightarrow {}^{0}_{-1}\text{e} + \overline{\nu}_{e} + {}^{3}_{2}\text{He}$

3 ${}^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + \overline{\nu}_{e} + {}^{14}_{7}\text{N}$

4 ${}^{A}_{Z}\text{X} \rightarrow 2\,{}^{0}_{-1}\text{e} + {}^{4}_{2}\alpha + {}^{A-4}_{Z}\text{X}$

5 ${}^{22}_{11}\text{Na} \rightarrow {}^{0}_{-1}\text{e} + \overline{\nu}_{e} + {}^{22}_{10}\text{Ne}$

6 8.0 h

8 5.8 MeV

10 $\frac{F_e}{F_g} = 4 \times 10^{42}$

7.2 Nuclear reactions

12 0.783 MeV

13 3.65 MeV

14 **a** seven electrons

b 208 MeV

15 8.9×10^{9} yr

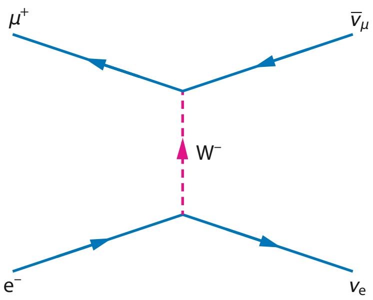
7.3 The structure of matter

20 it cannot; baryon number would not be conserved

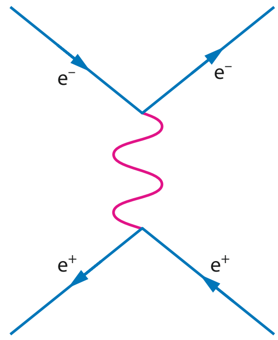
21 $Q = 0$, $S = -1$

22 **c** and **d**

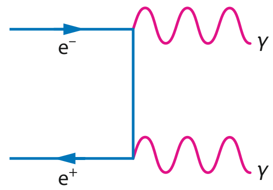
27



28 **b** **i**



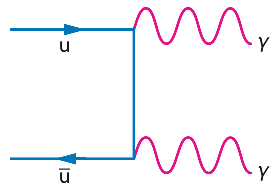
ii



29 **a** 0

b violates momentum conservation

c



30 **a** $u \rightarrow d + e^{+} + \nu_{e}$

b W^{+}

c positron and electron neutrino

Additional Topic 9 questions

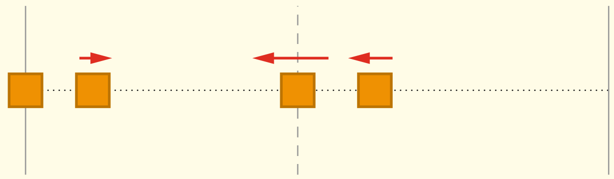
Test yourself

9.1 Simple harmonic motion

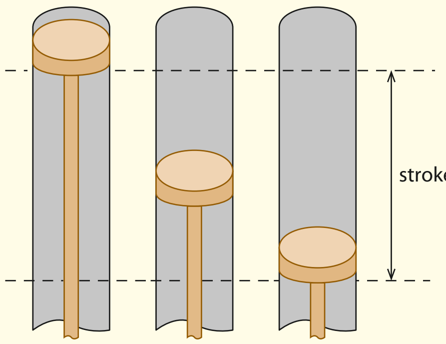
1 A body performs SHM along a horizontal straight line between the extremes shown by the solid grey lines in the diagram.
The arrows represent the direction of motion of the body. The body is shown in four positions: A, B, C and D. Copy the diagram and, in each position, draw arrows to represent the direction and relative magnitude of:

a the acceleration of the body

b the net force on the body.



2 The piston (of mass 0.25 kg) of a car engine has a **stroke** (i.e. distance between extreme positions) of 9.0 cm and operates at 4500 rev min^{-1}, as shown in the diagram.

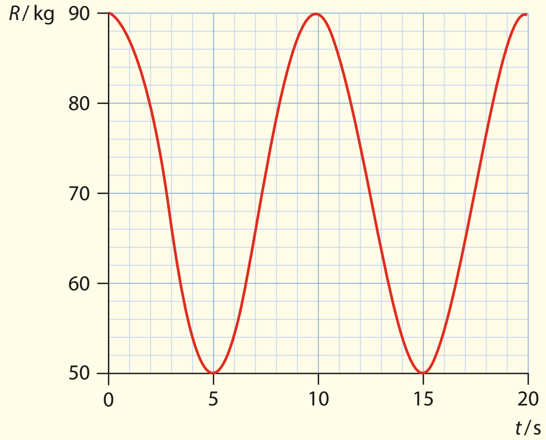


a Calculate the acceleration of the piston at maximum displacement.

b Calculate the velocity as the piston moves past its equilibrium point.

c What is the net force exerted on the piston at maximum displacement?

3 A passenger on a cruise ship in rough seas stands on a set of 'weighing scales'. The reading R of the scales (in kilograms) as a function of time is shown in the diagram.



Use the graph to determine:

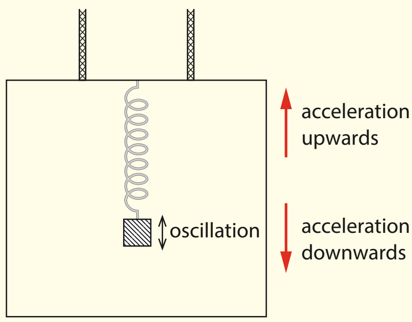
a the mass of the passenger

b the amplitude of the waves in the sea.

4 A body is suspended vertically at the end of a spring that is attached to the ceiling of an elevator, as shown in the diagram. The elevator moves with constant acceleration. Discuss **qualitatively** the effect, if any, of the acceleration on the period of oscillations of the mass when the acceleration is:

a upwards

b downwards.



5 A body of mass 1.80 kg executes SHM such that its displacement from equilibrium is given by $x = 0.360 \cos(6.80t)$, where x is in metres and t is in seconds. Determine:

a the amplitude, frequency and period of the oscillations

b the total energy of the body

c the kinetic energy and the elastic potential energy of the body when the displacement is 0.125 m.

9.2 Single-slit diffraction

6 In a single-slit diffraction experiment the slit width is 0.025 mm and the wavelength of light is 625 nm.

a Calculate the angle of the first diffraction minimum.

b State an approximate value for the ratio of the intensity of the central maximum to the intensity of the first maximum to the side.

7 White light illuminates a slit. The first diffraction minimum for a wavelength of 625 nm is observed at 14°.

a Calculate the slit width.

b Determine the wavelength of light for which the first secondary maximum occurs at an angle of 14°. (The first secondary maximum appears at an angle (in radians) of approximately $\frac{3\lambda}{2b}$.)

c Explain why the central maximum will be white but the spot at 14° will be coloured.

9.3 Interference

8 Discuss the effect on the bright spots in a Young's two-slit experiment of:

a decreasing the separation of the slits

b increasing the wavelength of light

c increasing the distance to the screen

d increasing the distance of the source from the slits

e using white light as the source.

9 A diffraction grating with 350 lines per mm produces first-order maxima at angles 8.34° and 8.56° for two separate wavelengths of light.

a Determine these wavelengths.

b Calculate the angle that separates the second-order maxima of these wavelengths.

10 In a two-slit interference experiment with slits of negligible width, five maxima are observed on each side of the central maximum. When the slits are replaced by two slits of finite width separated by the same distance as before, the third maximum on either side of the central maximum is missing (i.e. the intensity of light there is zero). Calculate the width of the slits in terms of their separation, *d*.

11 When a thin soap film of uniform thickness is illuminated with white light, it appears purple in colour. Explain this observation carefully.

12 A car moves along a road that is parallel to the twin antennas of a radio station broadcasting at a frequency of 95.0 MHz (see diagram). The antennas are 30.0 m apart and the distance of A from the mid-point of the antennas is 2.0 km. When in position A, the reception is good, but it drops to almost zero at position B. Calculate the distance AB.

13 Two radio transmitters are 80.0 m apart on a north–south line. They emit coherently at a wavelength of 1.50 m. A satellite in a north–south orbit travelling at 7.50 km s^{-1} receives a signal that alternates in intensity with a frequency of 0.560 Hz. Assuming that the signal received by the satellite is the superposition of the waves from the individual transmitters, find:

a the distance between two consecutive points where the satellite receives a strong signal

b the height of the satellite from the Earth's surface.

14 A soap film will appear dark if it is very thin and will reflect all colours when thick. Carefully justify these statements using interference from thin films.

Additional Topic 9 answers

Topic 9 Wave phenomena (HL)

9.1 Simple harmonic motion

1 a A, right and long; B, right and shorter; C, zero; D, left and shortest

b same as **a**

2 a $1.0 \times 10^4\,\mathrm{m\,s^{-2}}$

b $21\,\mathrm{m\,s^{-1}}$

c $2.5 \times 10^3\,\mathrm{N}$

3 a 70 kg

b 7.1 m

5 a $A = 0.360\,\mathrm{m}$, $f = 1.08\,\mathrm{Hz}$, $T = 0.924\,\mathrm{s}$

b 5.39 J

c 4.74 J, 0.650 J

9.3 Interference

9 a $4.14 \times 10^{-7}\,\mathrm{m}$; $4.25 \times 10^{-7}\,\mathrm{m}$

b 0.462°

10 $3d$

12 105 m

13 a 13.4 km

b 714 km